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MISSILE AERODYNAMIC PARAMETER ESTIMATION

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ABSTRACT

Aerodynamic characteristics of missiles depend strongly on wind angles, that is, angle of attack and sideslip angle. However it is impractical to measure these angles during missile testing. Therefore, without direct information of the wind angles, it becomes a difficult problem to be able to accurately estimate the missile aerodynamic parameters from flight tests. This paper addresses this problem and suggests an alternative approach to estimate missile aerodynamic parameters successfully without wind angles measurements. Instead of reconstructing wind angles with post-process calculations prior to estimation, reconstruction process is handled within the estimation. Suggested approach is also tested with real flight data of a surface to surface missile for successful evaluation of aerodynamic parameters without wind angles measurements.

INTRODUCTION

Flight vehicle systems are designed with initial predictions based on similar systems. Throughout the design stage characteristics of the system are also needed to be represented with higher fidelities as the design evolves. One of the most difficult parts of the modeling involves postulating an accurate aerodynamic model for successful evaluation of system behavior.

Aerodynamic modeling starts with analytical calculations and continues with wind tunnel tests for fine tuning of aerodynamic parameters. In the end, postulated model is verified with flight tests. The easiest and most straight forward way of aerodynamic model validation is comparing the simulation results with real flight tests carried out for performance demonstration. However those tests are usually held with the autopilot in the (closed) loop. Match between simulation and flight test results does not necessarily mean that postulated model is accurate enough. This brings the necessity of separate flight tests specifically designed for aerodynamic model validation. For this reason estimating aerodynamic parameters from flight tests has always been a major interest for flight vehicles.

Parameter estimation methods have been extensively applied to flight tests for decades [1]. However most of the studies appeared in literature involve aircraft systems. The advantage of studying such systems is having reliable sensors in addition to Inertial Measurement Unit (IMU) such as airflow angle vane, integrating gyro and dynamic pressure sensor [3]. This is not the case for missile applications. For practical reasons, most of the time missiles have only IMU which measures linear accelerations and angular rates only. The required states are obtained by integrating IMU measurements during flight. Bias and scale errors in IMU measurements however, cause the integrated data to drift. Launch angles may also have uncertainty or IMU may not be able to detect attitude and velocity changes with enough accuracy during launch. These errors can be either neglected when their affect is minimal or handled

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with post process data reconstruction techniques [4]. Nevertheless, they can cause a poor representation of the true states.

In short instrumentation of missiles during flight test lacks high amount of information when compared with aircraft. Usually, angle of attack and sideslip angle are the most important data among the missing information for aerodynamic parameter estimation studies for missiles. Response of the system is characterized with wind angles and, therefore, parameter estimation procedure must be primarily focused on matching the predicted response and real response using these angles.

Since real wind angles are not available in most cases for missile applications, a workaround solution must be held. Morelli has recently suggested a way of doing this in frequency domain [5]. It is demonstrated that high frequency content of both reconstructed wind angles and real measurements are almost same. Making use of this information, wind angles are calculated with integrating IMU measurements, passed through a high pass filter and then used in frequency domain estimation.

This paper focuses on an alternative solution in time domain and proposes an approach with output error method to estimate aerodynamic parameters of a missile from control surface deflections and IMU measurements only. Instead of reconstructing wind angles with post-process calculations prior to estimation, reconstruction process is handled within the estimation. Output error method [2] is utilized for this purpose. Efficiency of the algorithm developed is demonstrated with real flight test data.

METHOD

The equations of motion of a missile after burn-out are given as follows:

$$\dot{p} = \bar{q} S_{ref} l_{ref} C_l / J_x$$

$$\dot{q} = (1 - J_x / J_y) pr + \bar{q} S_{ref} l_{ref} C_m / J_y$$

$$\dot{r} = (J_x / J_y - 1) pq + \bar{q} S_{ref} l_{ref} C_n / J_y$$

$$\dot{u} = -qw + rv - g \sin\theta + \bar{q} S_{ref} C_X / m$$

$$\dot{v} = -ru + pw + g \cos\theta \sin\phi + \bar{q} S_{ref} C_Y / m$$

$$\dot{w} = -pv + qu + g \cos\theta \cos\phi + \bar{q} S_{ref} C_z / m$$

$$\dot{\phi} = p + \tan\theta (q \sin\phi + r \cos\phi)$$

$$\dot{\theta} = q \cos\phi - r \sin\phi$$

where dynamic pressure, $\bar{q}$ is given as:

$$\bar{q} = \rho (u^2 + v^2 + w^2) / 2$$

The very first approach taking place in most of the aerodynamic parameter estimation studies is using the aerodynamic parameters to be estimated as variables in state equations given above while trying to match the model outputs with measurements iteratively with an optimization technique [2][3]. Since IMU is the only unit for measurements, model outputs should be selected as linear accelerations and angular rates. By restricting the estimation study in pitch and yaw planes, *p* and a_x measurements are used as inputs. The output equations in this case are given by:

$$a_y = \bar{q} S_{ref} C_Y / m$$

$$a_z = \bar{q} S_{ref} C_Z / m$$

$$\dot{q} = (1 - J_x / J_y) pr + \bar{q} S_{ref} l_{ref} C_m / J_y$$

$$\dot{r} = (J_x / J_y - 1) pq + \bar{q} S_{ref} l_{ref} C_n / J_y$$

Angular accelerations given by output equations are either integrated to match with gyro measurements or used as they are and matched with derivatives of the measurements. Both approaches come with different disadvantages. Integrating the outputs might end up with incorrect results due to the accumulated minor modeling errors and disturbances occurred during the flight. This might risk the convergence of estimated aerodynamic parameters. Taking derivative of measurements on the other hand is another problem which must be carefully handled and requires data with high signal to noise ratio. Experiences from missile flight tests showed that second approach, taking derivatives of gyro measurements is a guaranteed way of handling moment derivatives.

Non-dimensional aerodynamic coefficients used in equations can be modeled using linear expansions in wind angles, control surface deflections and angular rates:

$$C_Y = C_{Y_\beta} \beta + C_{Y_{\delta_r}} \delta_r + C_{Y_r} r$$

$$C_Z = C_{Z_\alpha} \tan^{-1} \alpha + C_{Z_{\delta_e}} \delta_e + C_{Z_q} q$$

$$C_m = C_{m_\alpha} \tan^{-1} \alpha + C_{m_{\delta_e}} \delta_e + C_{m_q} q$$

$$C_n = C_{n_\beta} \tan^{-1} \beta + C_{n_{\delta_r}} \delta_r + C_{n_r} r$$

Note that in order the linear assumption to be true, system must be held in linear side in terms of aerodynamic coefficients during the maneuvers. Therefore control surface deflections as inputs must provide enough perturbed response with a good signal to noise ratio but also must not result the system to exceed the limits of linear regions. The judgment over the input design is usually made by using the aerodynamic database before the flight test.

Assuming low-level wind conditions, angle of attack can be represented as:

$$\alpha = \tan^{-1}(w / u)$$

At low angles of attack, sideslip angle and flank angle are approximately the same. Sideslip angle then reduces to:

$$\beta = \tan^{-1}(v / u)$$

Missile studied in this paper is symmetric in pitch and yaw, therefore same aerodynamic derivatives are used for both planes. Putting all together, output equations appear as:

$$a_y = \rho(u^2+v^2+w^2)S_{ref}(C_{Z_\alpha}\tan^{-1}(v/u)+C_{Z_{\delta_e}}\delta_r - C_{Z_q}r)/2/m$$

$$a_z = \rho(u^2+v^2+w^2)S_{ref}(C_{Z_\alpha}\tan^{-1}(w/u)+C_{Z_{\delta_e}}\delta_e + C_{Z_q}q)/2/m$$

$$\dot{q} = (1-J_x/J_y)pr + \rho(u^2+v^2+w^2)S_{ref}l_{ref}(C_{m_\alpha}\tan^{-1}(w/u)+C_{m_{\delta_e}}\delta_e + C_{m_q}q)/2/J_y$$

$$\dot{r} = (J_x/J_y-1)pq + \rho(u^2+v^2+w^2)S_{ref}l_{ref}(-C_{m_\alpha}\tan^{-1}(v/u)-C_{m_{\delta_e}}\delta_r + C_{m_q}r)/2/J_y$$

Practical difficulty of using states in above output equations is that integrating the response of the model to be estimated may still give inaccurate results due to minor modeling errors that might possibly exist in the postulated model. Integrating any error creates eventually inaccurate results. This can be avoided by forcing the integrated states to match with measurements by using a state filter (filter-error method) or alternatively by using the states of the system in output equations directly from measurements (equation-error method) [2]. Both approaches might be still unpractical for missiles due to the lack of wind angle measurements. Therefore angular rates and control surface deflections involved in output equations should be used from measurements as inputs of estimation whereas angle of attack and sideslip angle can be reconstructed from IMU measurements [4].

Rather than obtaining wind angles with post process calculations, reconstruction can be included in the estimation procedure. Using IMU measurements as inputs and starting from an initial state vector, below equations can be integrated with a numerical method to represent wind angles in output equations.

$$\dot{u} = -(q-b_q)w + (r-b_r)v - g\sin\theta + (a_x - b_{a_x})$$

$$\dot{v} = -(r-b_r)u + (p-b_p)w + g\cos\theta\sin\phi + (a_y - b_{a_y})$$

$$\dot{w} = -(p-b_p)v + (q-b_q)u + g\cos\theta\cos\phi + (a_z - b_{a_z})$$

$$\dot{\phi} = (p-b_p) + \tan\theta\left[(q-b_q)\sin\phi + (r-b_r)\cos\phi\right]$$

$$\dot{\theta} = (q-b_q)\cos\phi - (r-b_r)\sin\phi$$

Note that initial state vector and bias values used for correcting IMU errors are also included to unknowns in addition to aerodynamic parameters in the estimation algorithm. Unknown parameter vector now appears as follows:

$$\underline{\Theta} = [u_0 \quad v_0 \quad w_0 \quad \phi_0 \quad \theta_0 \quad C_{Z_\alpha} \quad C_{Z_{\delta_e}} \quad C_{Z_q} \quad C_{m_\alpha} \quad C_{m_{\delta_e}} \quad C_{m_q} \quad b_p \quad b_q \quad b_r \quad b_{a_x} \quad b_{a_y} \quad b_{a_z}]^T$$

Control surface deflections together with IMU measurements are used as inputs in both state and output equations. Inputs, states and outputs of the postulated model are defined as:

$$\underline{u} = [\delta_e \quad \delta_r \quad p \quad q \quad r \quad a_x \quad a_y \quad a_z]^T$$

$$\underline{x} = [u \quad v \quad w \quad \phi \quad \theta]^T$$

$$\underline{y} = [a_y \quad a_z \quad \dot{q} \quad \dot{r}]^T$$

Using Maximum Likelihood estimation technique [2], unknown parameters are estimated for *N* independent measurements as follows:

$$\hat{\underline{\Theta}} = \max_{\underline{\Theta}} \left[\prod_{i=1}^{N} p\left(\underline{z}_i \mid \underline{\Theta}\right) \right] = \min_{\underline{\Theta}} \left[\sum_{i=1}^{N} -\ln p\left(\underline{z}_i \mid \underline{\Theta}\right) \right]$$

Probability density function given above can be represented as probability of a Gaussian distributed random variable:

$$p\left(\underline{z}_i \mid \underline{\Theta}\right) = \frac{1}{\sqrt{\left(2\pi\right)^n \left|\underline{R}\right|}} \exp\left[-\frac{1}{2}\left(\underline{z}_i - \underline{y}_i\right)^T \underline{R}^{-1} \left(\underline{z}_i - \underline{y}_i\right) \right]$$

In that case, cost function to be minimized becomes:

$$J\left(\underline{\Theta}\right) = \frac{nN \ln\left(2\pi\right)}{2} + \frac{N \ln\left(\left|\underline{R}\right|\right)}{2} + \frac{1}{2} \sum_{i=1}^{N} \left(\underline{z}_i - \underline{y}_i\right)^T \underline{R}^{-1} \left(\underline{z}_i - \underline{y}_i\right)$$

Above optimization problem can be solved iteratively with modified Newton-Raphson algorithm using a relaxation technique [2]. In this technique, noise covariance matrix, *R* is assumed not to be effected from change in system parameters and is treated as constant. Parameter change [2] at each step is calculated as follows:

$$\Delta\underline{\Theta} = -\left[\sum_{i=1}^{N} \left\{ \left[\frac{\partial \underline{y}_i}{\partial \underline{\Theta}} \right]^T \underline{R}^{-1} \frac{\partial \underline{y}_i}{\partial \underline{\Theta}} \right\} \Bigg|_{\underline{\Theta}=\underline{\Theta}_0} \right]^{-1} \sum_{i=1}^{N} \left\{ \left[\frac{\partial \underline{y}_i}{\partial \underline{\Theta}} \right]^T \underline{R}^{-1} \left(\underline{z}_i - \underline{y}_i\right) \right\} \Bigg|_{\underline{\Theta}=\underline{\Theta}_0}$$

Covariance values of the estimated parameters [2] at each step are given by the diagonal elements of the parameter error covariance matrix, *P* :

$$P = \left[\sum_{i=1}^{N} \left\{ \left[\frac{\partial \underline{y}_i}{\partial \underline{\Theta}} \right]^T \underline{R}^{-1} \frac{\partial \underline{y}_i}{\partial \underline{\Theta}} \right\} \Bigg|_{\underline{\Theta}=\underline{\Theta}_0} \right]^{-1}$$

After each parameter update noise covariance matrix is evaluated [2] independently again for updated values of parameters:

$$\underline{R} = \frac{1}{N} \sum_{i=1}^{N} \left(\underline{z}_i - \underline{y}_i\right)\left(\underline{z}_i - \underline{y}_i\right)^T$$

Parameter update process is repeated until a convergence criterion is satisfied. Check of final convergence should be made for both relative changes in cost function and in parameters at the same time. The procedure can be summarized as follows:

1. Set initial values for unknown parameters.
2. Find model outputs and evaluate noise covariance matrix
3. Update the unknown parameter vector.
4. Iterate step 2 and 3 until convergence.

RESULTS AND CONCLUSION

Suggested method is tested with real flight test data of a surface to surface missile that is researched and developed in Roketsan Missiles Industries. Flight test was designed specifically for aerodynamic parameter estimation. Square wave inputs with modal frequency which was determined prior to test relying on the wind tunnel tests were applied to control surfaces to excite missile in pitch and yaw planes while the control system was in open loop. Inputs were chosen to provide enough signal-to-noise ratios and also to keep missile close to reference flight condition in order to ensure that system will stay on the linear side of the aerodynamic model.

The convergence plots of the estimation procedure are given in **Figure 1**. Plots are scaled independently according to the final estimated values so that final values of results appear as one and other values present the relative errors. Initial values which are indicated by red points in plots, are selected from wind tunnel database. Updates indicated by blue points are given with one sigma error bands.

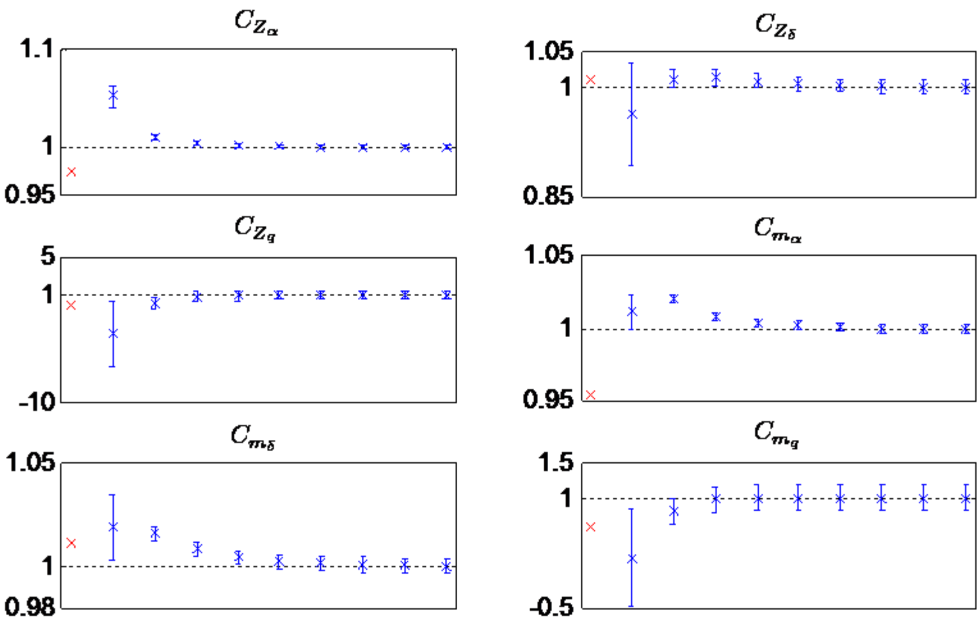

Figure 1 - Convergence of aerodynamic parameters

Match between model outputs and measurements can be seen from **Figure 2**. Note that angular accelerations given as measurements are locally smoothed derivatives [3] of IMU angular rate outputs. It is observed that navigation errors are successfully corrected during the estimation without any other information except IMU measurements.

As mentioned before same aerodynamic derivatives in pitch and yaw planes are used in estimation model. Even though the missile is symmetric in those planes, independent aerodynamic parameters may also be included in estimation model. Since there are slight offset errors due to the production in both control surfaces and mid-body wings, using different aerodynamic parameters in pitch and yaw planes might result better matches of model outputs with measurements. Similarly constant terms may also be included in the aerodynamic model to correct the bias errors between measurements and model outputs. This error can be clearly seen from yaw angular acceleration plot of estimated model in **Figure 2**. Postulated model doesn't involve any constant aerodynamic terms as it should be however it can be clearly seen that there is constant moment acting on the system in yaw plane due to the offset errors just mentioned.

Since the main focus of the estimation is verifying the aerodynamic model used in simulations, it is more appropriate to use the model as it is and searching for the best values of its parameters. Therefore minor modeling errors such as the ones mentioned above are commonly neglected in the estimation studies as long as they don't risk the convergence of unknown parameters. For that reason using kinematic equations in state equations instead of using original equations of motion given in the first place, provides more robust results even with a simple estimation algorithm studied in this paper. In fact it was observed that same estimation algorithm using the equations of motion of the missile as the system model fails to converge and doesn't give any results.

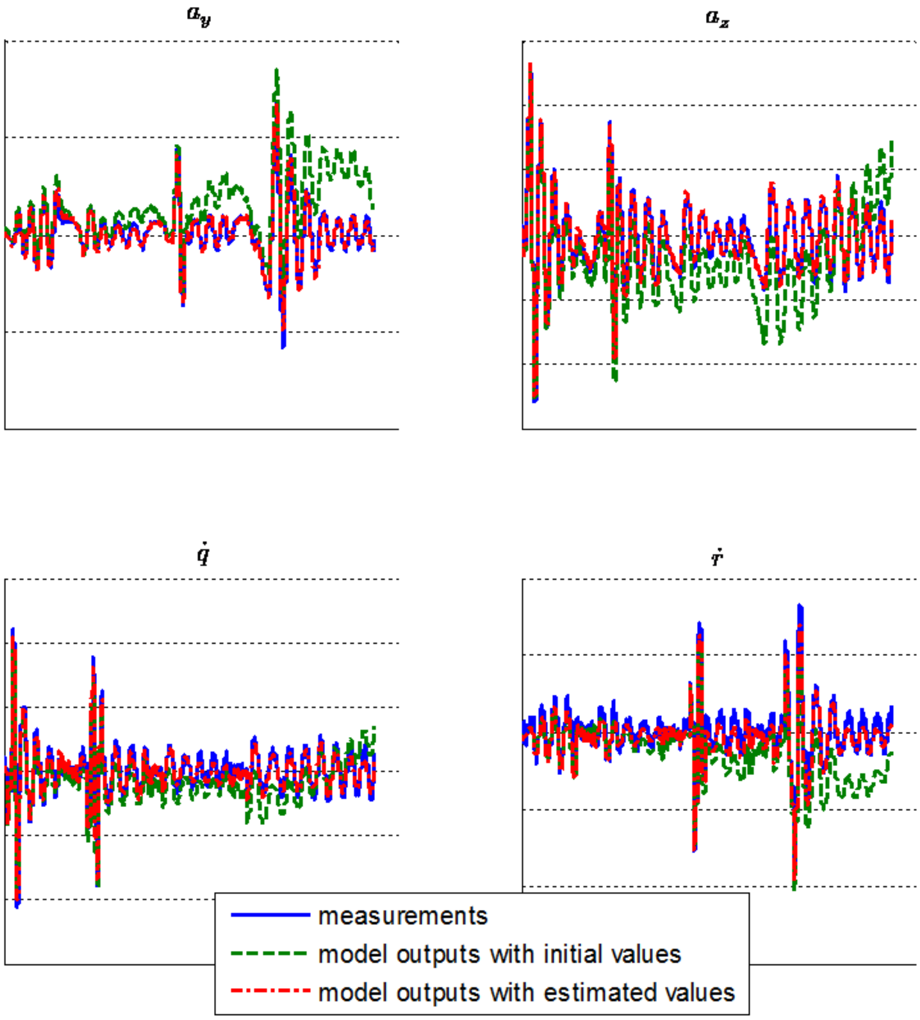

Figure 2 - Comparison of measurements and model outputs in pitch and yaw planes

References

[1] Hamel P. G., Jategaonkar R. V., "Evolution of Flight Vehicle System Identification", AIAA, Journal of Aircraft, Vol. 33, No. 1, 1996.

[2] Jategaonkar R. V., "Flight Vehicle System Identification: A Time Domain Methodology", AIAA, Progress in Astronautics and Aeronautics Volume 216, 2006.

[3] Klein V., Morelli E. A., "Aircraft System Identification Theory and Practice", AIAA Education Series, 2006.

[4] Kutluay Ü., "Aerodynamic Parameter Estimation Using Flight Test Data", Phd Thesis, METU, 2011.

[5] Morelli E. A., "Real-Time Aerodynamic Parameter Estimation Without Air Flow Angle Measurements", AIAA, Journal of Aircraft, Vol. 49, No. 4, 2012.