

## THE GROUND EFFECT ON THIN AIRFOLS AND WINGS

Ülgen Gülçat<sup>1</sup>  
Istanbul Technical University  
Istanbul, Turkey

### ABSTRACT

The effect of the ground on the aerodynamic performances of airfoils and wings are investigated. The lift, the propulsive force and the moment acting on those thin aerodynamic surfaces are predicted for steady and unsteady cases where arbitrary pitch and plunge motion are considered. The condition to ascend or descend of an airfoil is determined. For the purpose of achieving the largest range, the necessary maneuvering of a human powered flying wing near the ground is predicted. In all applications the flight path is solved to determine the new position, and hence the new aerodynamic forces and moments.

### INTRODUCTION

Air vehicles, micro or macro, one way or other, inevitably must perform near the ground which necessitates some analysis via mathematical modeling of the aerodynamics of the problem. In this respect, the mathematical modeling with vortices and their images enable us, in a convenient way, to impose the tangency boundary conditions at the ground. The lift coefficient under the effect of the ground for a thin airfoil is given by [Plotkin and Kennel, 1981] in a series form based on the Keldish-Lavrentiev expansion of the Kernel function as described in [Katz and Plotkin, 1991]. The detailed formulation of the problem and the application for the steady and the unsteady interaction of the airfoil with the ground are provided in [Gülçat, 2013].

For the wings, as is well known, the tip vortices and the spanwise variation of the bound and wake vortices create the lifting force according to the Prandtl's lifting line theory [Prandtl and Tietjens, 1934]. Under the presence of the ground, we have to consider the image vortex systems for the completion of the aerodynamic model which is numerically solved using the Glauert's Fourier series approach for the spanwise distribution of the circulation.

The additional lift created by the presence of the ground causes the airfoil or the wing to ascend which in turn changes the distance to the ground. The change in the distance, obviously, causes change in the lifting force. Therefore, it is necessary to solve the dynamic path equations of the airfoil or the wing step by step simultaneously with the aerodynamics involving the lift variation.

In this, study, first a thin airfoil of the ornithopter from [Mueller and Delaurier, 2003] interacting with the ground is studied by considering the lift change, either positive or negative, and flight path change simultaneously.

The second application concerns the performance of a Boeing 737-200 flying with a high velocity near to the ground where rapid ascending motion of the plane is observed.

Finally, near the ground flight of a lifting surface with a pilot is considered for annual flying competition involving the human powered air vehicles. Necessary maneuvering steps of the flying surface are determined in order to travel the largest range before landing to the sea surface.

---

<sup>1</sup> Emeritus Professor , Faculty of Aeronautics and Astronautics, gulcat@itu.edu.tr

In the following sections the formulation, applications and the conclusion for this study are going to be presented, respectively.

## FORMULATIONS

In this section necessary formulation for the ground effect on i) airfoils, ii) wings in steady motion, and wings in arbitrary unsteady motion will be provided.

### Airfoils

The simplest case, here, is the change in the lift of an airfoil under the presence of ground which is given by [Plotkin and Kennel, 1981], in terms of the series solution of Keldish and Lavrentiev, as follows:

$$\Delta C_l = 2\pi\alpha \left( \frac{1}{4h^2} - \frac{3}{32h^4} - \frac{1}{512h^6} \right) \quad (1)$$

Where,  $\alpha$  is the angle of attack, and  $h$  is the distance to the ground. The vertical position of the unrestrained airfoil then is given by the following ODE and the initial conditions

$$\frac{d^2 y}{ds^2} = \Delta C_l(s) \frac{b}{U^2} g, \quad y(0) = h_0, \quad \frac{dy}{ds} = v(0) = 0 \quad (2)$$

Here,  $b$  is the half chord,  $U$  is the free stream speed,  $g$  is the acceleration due to gravity and  $v$  is the vertical velocity.

If the ground effect is started with the presence of sharp corner the quasi-steady approach gives the lift change as follows:

$$\Delta C_{lc}(x) = \int_{-1}^x \frac{\gamma_2(x)}{h^2} dx = \frac{\alpha}{2h^2} \left[ \frac{1}{2}(x+1)\sqrt{1-x^2} + \sin^{-1}(x) + \pi/2 \right], \quad \gamma_2 = \frac{\alpha}{2} \sqrt{\frac{1-x}{1+x}} (x+3/2) \quad (3-a,b)$$

Where,  $x$  is the distance covered over the start of the sharp corner by the airfoil until  $x$  equals to the chord during the flight of the airfoil, and  $\gamma_2$  is the vortex sheet strength induced by image vortices.

The quasi-unsteady analysis, on the other hand, yields the following lift change using the Wagner function  $\varphi = \varphi(s=Ut/b)$  [Bisplinghoff, Ashly and Halfman, 1955] for the lift coefficient under arbitrary angle of attack change we have

$$\Delta C_l = 2\pi\alpha \left[ \frac{1}{8} + \frac{1}{4} \int_0^s 0.0455x0.165e^{-0.0455(s-\sigma)} + 0.3x0.335e^{-0.3(s-\sigma)} d\sigma \right] \quad (4)$$

With the downwash created by the ground effect which reads as

$$w_c = \frac{1}{2\pi} \int_{-1}^1 \frac{\gamma_2(\xi)}{x-\xi} d\xi = \frac{\alpha U}{4\pi} \int_{-1}^1 \sqrt{\frac{1-\xi}{1+\xi}} \frac{\xi+3/2}{x-\xi} d\xi = \frac{\alpha U}{4} (x+1/2)$$

The downwash at the quarter chord,  $x=1/2$ , becomes  $w_c = \alpha U/4$ .

### Wings

As the second study, the ground effect on a thin wing is studied using the Glauert's Fourier series solution for the Prandtl's lifting line theory. The spanwise circulation variation satisfies the following equation

$$U\alpha = \frac{1}{2\pi} \int_{-b}^b \frac{\gamma_a(\xi, y)}{x-\xi} d\xi - \frac{1}{4\pi} \int_{-l}^l \frac{d\Gamma}{d\eta} \frac{d\eta}{y-\eta} + \frac{1}{4\pi} \int_{-l}^l \Gamma \left\{ \frac{4h^2}{[(y-\eta)^2 + 4h^2]^2} - \frac{(y-\eta)^2}{[(y-\eta)^2 + 4h^2]^2} \right\} d\eta \quad (5)$$

The Glauert's Fourier series substitution:  $\Gamma(\theta) = U a_o b_o \sum_{n=1}^{\infty} A_n \sin n\theta$  with the following coordinate transformation  $y = l \cos \theta$  and  $\eta = l \cos \phi$  gives

$$\sum_{n=1}^{\infty} A_n \left\{ \sin n\theta \left( 1 + \frac{b\pi n}{2l \sin \theta} \right) - \frac{b}{2l} \int_0^{\pi} \sin n\phi \left[ \frac{4h^* b^2}{l^2 r^2} - \frac{(\cos \theta - \cos \phi)^2}{r^2} \right] \sin \phi d\phi \right\} = \frac{ab}{a_o b_o} \alpha \quad (6)$$

where, subscript o denotes the root values and  $r^2 = [(\cos \theta - \cos \phi)^2 + 4h^* b^2 / l^2]^2$ . Note that the integral in (4) represents the ground effect and needs to be evaluated numerically while obtaining its contribution to the lift. Shown in Fig.1 is the vortex systems and their images near the ground.

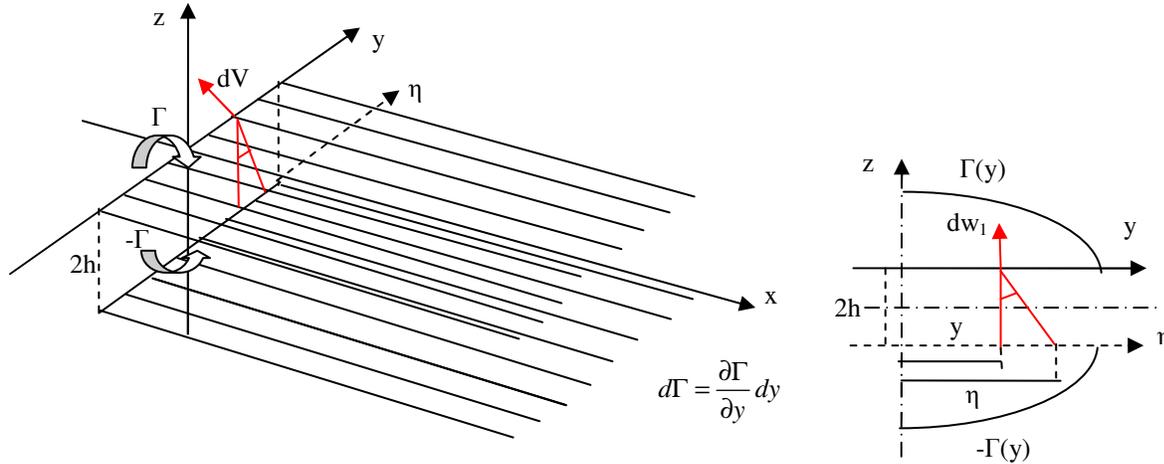


Figure 1: Vortex systems near the ground

**Wing over a corner**

As the third case, path of an elliptical wing under the ground effect which is allowed to be powered only by a human who flies with it is determined for its largest flight range. Shown in Figure 2 is the wing and the corner over which its flight takes place.

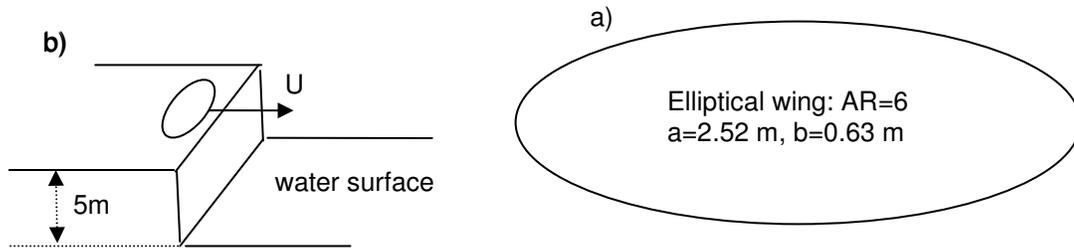


Figure 2. Elliptical wing (a), with AR=6, S=5m<sup>2</sup> over the corner (b).

Here, the elliptical wing is flown horizontally from left to right and descends to the water surface with a gliding motion. The path of the wing is determined by following equations with the prescribed initial conditions:

$$m \frac{d^2 x}{dt^2} = -D_x + P, \quad x(0) = 0, \quad \frac{dx}{dt} = u(0) = u_0$$

$$m \frac{d^2 y}{dt^2} = -mg - D_y + L(1 + 0.25 / h^2), \quad y(0) = h_0, \quad \frac{dy}{dt} = v(0) = 0 \quad (7-a,b)$$

Wherein,  $m$  is the total mass,  $L$  is the lifting force  $D$  and  $P$  are the drag and the propulsive forces respectively. The classical relation between the Lift and Drag for finite wings are given with

$$C_D = C_{D0} + aC_L^2 \quad (8)$$

For elliptical wings with  $AR$  being the aspect ratio we have  $a = 1/(\pi AR)$ , for which, however real effects will be considered here to make that value larger than the theoretical one.

Here, the elliptical wing, right after living the corner, is to perform an unsteady motion consist of plunging and pitching during its down stroke, and continue its flight with a constant angle of attack. The lift generated by the arbitrary unsteady motion is calculated with the aid of Wagner function. For the sake of simplicity, the plunge-pitch during down stroke is given by

$$\begin{aligned} h &= \bar{h} \cos(\omega t) \\ \alpha &= \alpha_0 - \alpha_0 \cos(\omega t + \varphi) \end{aligned} \quad (9-a,b)$$

The time wise variation of the equation for the airfoil reads as

$$z_a(x, t) = a b \alpha - h - \alpha x \quad (10)$$

The down wash then becomes

$$w_a(x, t) = \frac{\partial z_a}{\partial t} + U \frac{\partial z_a}{\partial x} \quad (11)$$

The lift and the moment coefficients in terms of the downwash at 3 quarter chord then read

$$C_l(s) = \frac{\pi b}{U^2} (\ddot{h} + U \dot{\alpha}) - \frac{2\pi}{U} \left[ w(b/2, s) / 2 + \int_0^s w(b/2, o) \varphi'(s-o) do \right] \quad (12)$$

$$C_m(s) = -\frac{\pi b}{2U^2} (b \dot{\alpha} (4 + U \ddot{\alpha})) - \frac{\pi}{U} \left[ w(b/2, s) / 2 + \int_0^s w(b/2, o) \varphi'(s-o) do \right] \quad (13)$$

## APPLICATIONS

The formulations given above are applied to the problems involving the ground effect on airfoils and wings as follows.

### Airfoils

The steady case is the simplest for a thin airfoil in ground effect. Here, the path of a thin airfoil which is  $h_0 = 0.63b$  away from the ground surface is solved with (1) and (2). The Runge-Kutta method is employed in (2) as the initial value problem with free stream speed  $U=15\text{m/sec}$  and half chord  $b=0.15\text{m}$ . The detailed path study is given in [Gülçat, 2013]. The effect of a sharp corner is also studied with the aid of (3-a) and (4) with quasi-steady and quasi unsteady approaches, respectively. As seen from Fig.3, quasi unsteady approach initially gives considerable elevation rise for the airfoil, however, the vertical speed gained, which is obtained by both approaches, merges with each other eventually. The viscous effects in Figure 3. is neglected. The inclusion of viscous effects indicates that first few seconds of the motion the effect of drag is insignificant. However, in later stages the elevation of the airfoil stabilizes as the vertical velocity goes to zero because of the viscous drag as seen in Figure 4.

### Wings

The application for the ground effect on a wing is chosen to simulate the performance of a Boeing 737-200 flying close the ground with  $h_0=2b_{av}=4.7\text{m}$   $U=134\text{m/sec}$ , and aspect ratio  $AR=6.5$ . Shown in Fig. 5-a,b are the vertical displacement with time and the path of the plane calculated, which nearly doubles its altitude in less than 4 seconds. Here, the path is obtained by solving (2) and (6) simultaneously. Since the solution for a few seconds is of interest, the viscous effects in such a short duration are negligible. If necessary, the drag polar for the plane can be utilized from [Fox and McDonald, 1992].

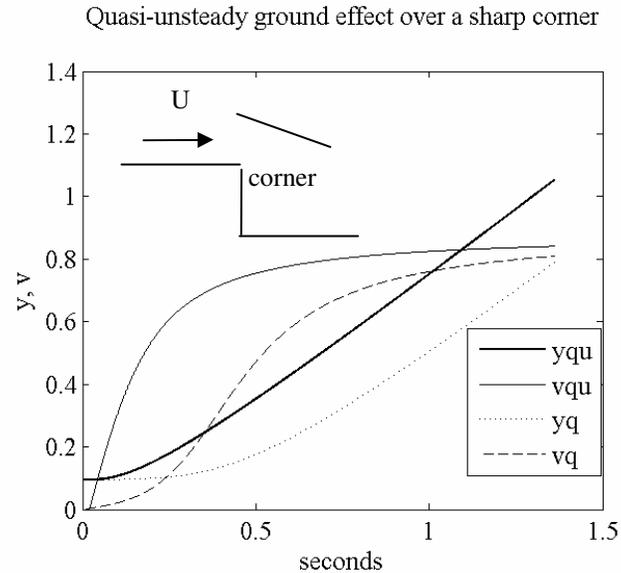


Figure 3. Quasi-steady and unsteady ascend of the airfoil flying with  $h=0.63b$  over a sharp corner.

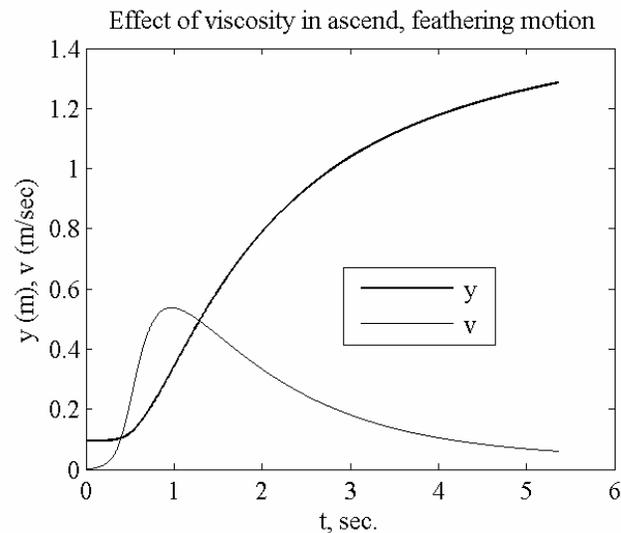


Figure 4. Vertical displacement  $y$ , and speed  $v$  under the viscous effect.

### Wing over a corner

As the last application, an elliptical wing shown in Fig 2., which is descending over a sharp corner of a fixed platform close to the water surface is studied. The aspect ratio of the elliptical wing is 6 and the area is  $5\text{m}^2$ . The wing is piloted by a person and right after it leaves the platform it is allowed to pitch about mid-chord and plunge during its down stroke. The wing starts its motion with a level flight, with  $U=6\text{m/sec}$ , 5m above the water surface. The lift generated with this level flight is not sufficient to overcome the weight of the wing and the pilot for which the total mass is 75kg. The down stroke of the wing with pitch-plunge is given by

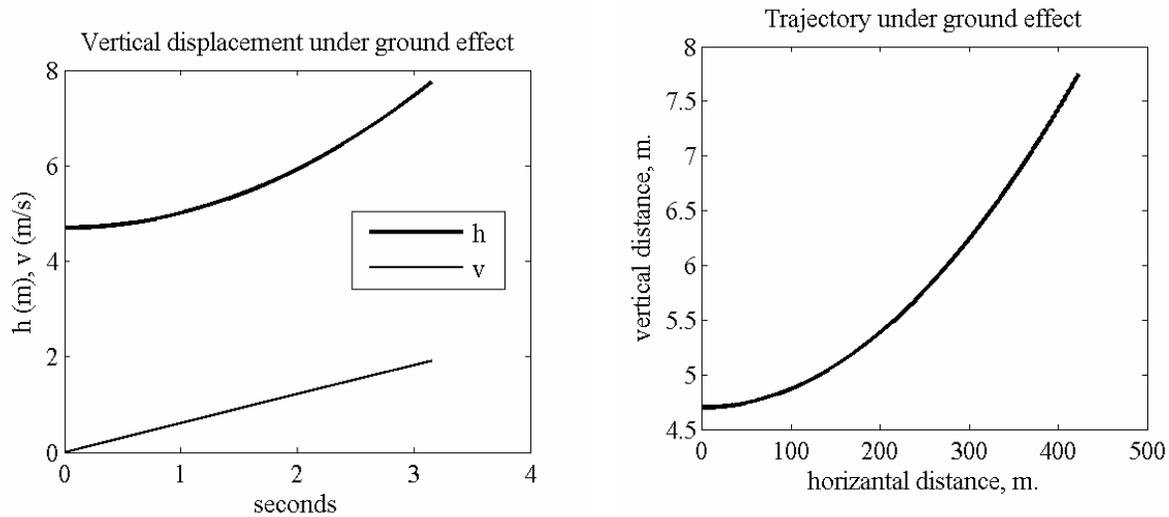


Figure 5: Vertical displacements of the plane, a) left: in time, b) right: in distance traveled

$$\begin{aligned} h &= 0.63 \cos(\omega t) \\ \alpha &= 10.4 - 10.4 \cos(1.7t + 79.9^\circ) \end{aligned} \quad (14-a,b)$$

The drag polar expression for the wing reads as [Kuethé and Chow, 1986]

$$C_D = 0.00742 + C_L^2 / 13 \quad (15)$$

For the steady portions  $C_L = 0.27 + 3.94\alpha$  is utilized, and the unsteady portion during down stroke Eqs.(12) and (13) is employed.

Shown in Fig.6 is the vertical position and the speed change, obtained by solving (7-a,b) and (12) simultaneously of the wing between the time it left the corner and the time just hitting the water surface.

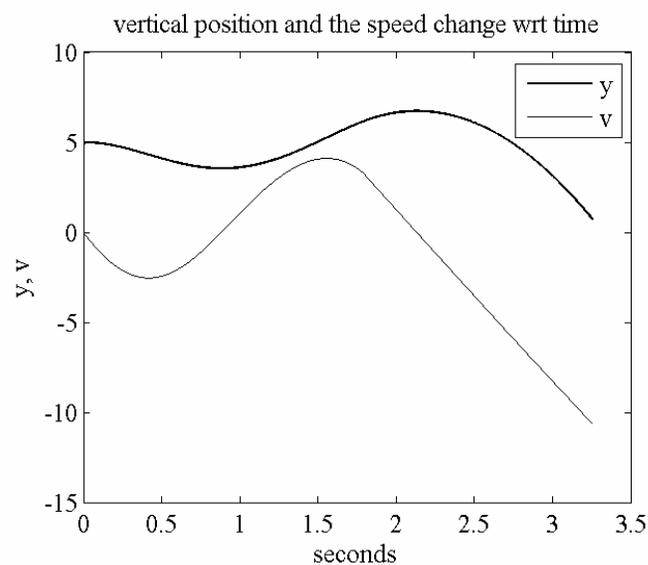


Figure 6. Time variation of vertical position  $y$ , and the speed  $v$  of the wing

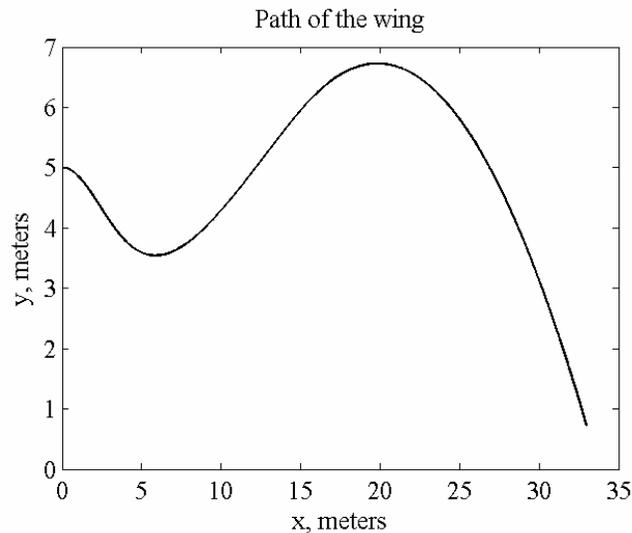


Figure 7. Path of the wing before hitting the water surface

If we also consider the drag due to the pilot who is attached to the wing, the range shown in Fig.7 is reduced about 5 meters. The drag coefficient for the pilot is calculated from [Schmitt, 1954] as  $C_{Dh}=0.1$  for the supine position.

### CONCLUSIONS

The ground effect on the aerodynamic performance of airfoils and wings under various steady and unsteady conditions are studied.

It has been observed that the extra lift over a sharp corner is felt much stronger with quasi unsteady approach as opposed to the quasi steady aerodynamics results.

The effect of viscosity is almost negligible within few seconds of the effect of the ground.

The pitch-plunge during the down stroke for the wing gives extra boost as well as lift.

### References

- Gülçat, Ü., *Flapping Thin Airfoil Interacting with the Ground*, SEECM III, 3<sup>rd</sup> South-East Conference on Computational Mechanics an ECCOMAS and IACM Special Interest Conference, M. Papadrakis, M. Kojic, I. Tuncer (eds.), Kos Island, Greece 12-14 June 2013
- Mueller, T.J. and DeLaurier, J.D., *Aerodynamics of Small Vehicles*, Ann. Rev. Fluid Mech., Vol. 35, p.89-111, 2003
- Fox, W.R. and McDonald, A.T. *Introduction to Fluid Mechanics*, Wiley, 1992
- Katz, J. and Plotkin, A, *Low-Speed Aerodynamics*, McGraw-Hill, 1991
- Kuethe, A.M. and Chow, C-Y., *Foundations of Aerodynamics*, John Wiley & Sons, 1986.
- Plotkin, A. and Kennel, C.G. Thickness-Induced Lift on a Thin Airfoil in Ground Effect, AIAA J., Vol. 19, p.1484-1486, Nov 1981.
- Bisplinghoff, R.L., Ashley, H. and Halfman, R.L., *Aeroelasticity*, Addison-Wesley, 1955
- Schmitt, T.J., *Wind-Tunnel Investigation of Air Loads on Human Beings*, Navy Department, R-892, 1954
- Prandtl, L. and Tietjens, O. G., *Applied Hydro-and Aeromechanics*, Dover Publications, 1934