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NONLINEAR PERFORMANCE EVALUATION OF ALTITUDE AUTOPILOT DESIGNED VIA ROOT LOCUS COMPENSATION

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ABSTRACT

The performance of an autopilot designed by linear methods such as root locus compensation has to be evaluated in a nonlinear simulation environment. The effect of omitted dynamics belonging to regulated flight variable's behavior may reveal itself through nonlinear simulation. In addition, the validity interval of the autopilot for the design value of the regulated variable can be obtained from nonlinear simulation trials. This study presents the evaluation of an altitude autopilot's acceptability to be employed to control the complete longitudinal flight dynamics governing the altitude of the aircraft.

INTRODUCTION

The altitude autopilot carries the aircraft to the reference altitude. In design, the transfer functions derived for short period linearized longitudinal flight dynamics are utilized. The sensitivity of the rate gyro in the inner loop, which senses the pitch rate, and the gain of the amplifier in the forward path of the outer loop are the control coefficients to be adjusted. The values of the mentioned coefficients, for which the output of the designed altitude hold system converges to the reference input with desirable transient and steady-state characteristics, are determined by applying root locus analysis and design for the inner and outer loops successively. A similar study is presented recently for displacement autopilot recently [Sofyalı and Caferov, 2012].

METHOD

The considered aircraft model belongs to the fighter F-94A and the steady straight level flight at the altitude of 15000 ft and with a velocity of 591 ft/s (403 mph) [Blakelock, 1991]. In short period approximation, the component of aircraft's velocity along the longitudinal body axis (U) is accepted to be constant and the deviations of pitch rate (q), pitch angle (θ), and angle of attack (α') from their steady state values as response to an elevator deflection (δ_e) are investigated. In root locus compensation, the following two short period transfer functions are used:

$$\frac{q(s)}{\delta_e(s)} = \frac{14.049s + 18.013}{s^2 + 1.326s + 6.6234} \tag{1}$$

$$\frac{h(s)}{q(s)} = \frac{-3.3931s^2 - 2.3643s + 757.747}{s^3 + 1.2822s^2} \tag{2}$$

In Figure 1, the block diagram of the altitude autopilot is presented [Blakelock, 1991]. In outer feedback path, both the altitude and its derivative are measured by a sensor with mathematical model of 10s+1.

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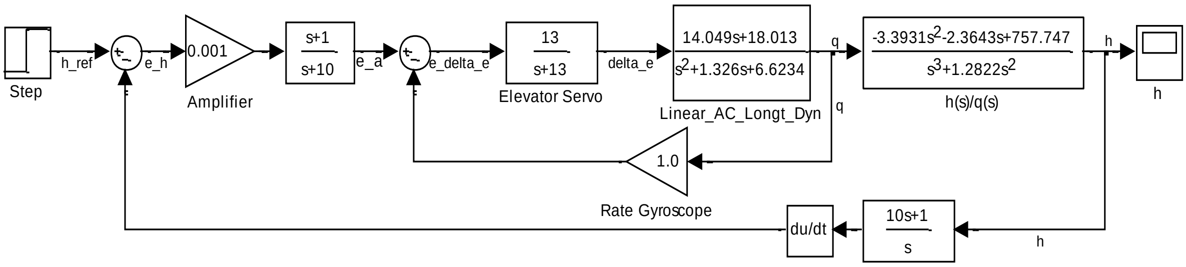

Figure 1: *Altitude hold system block diagram*

The nonlinear simulation environment is built by replacing the two transfer function blocks with a block including the following three differential equations of motion (see Appendix), which are nonlinear and represent the complete longitudinal motion of the aircraft:

$$\begin{aligned} \dot{U} &= -WQ - \left(W_g \sin\Theta + D\cos\alpha - L\sin\alpha - T\right)/m \\ \dot{Q} &= M/I_y \\ \dot{W} &= UQ + \left(W_g \cos\Theta - D\sin\alpha - L\cos\alpha\right)/m \end{aligned} \tag{3}$$

Here, W is the component of aircraft's velocity along the vertical body axis, Q is the pitch rate, W_g is the aircraft's weight, Θ is the pitch angle, D is the drag force, α is the angle of attack, L is the lift force, T is the thrust force, m is the aircraft's mass, M is the pitching moment, and I_y is the pitching moment of inertia. For the sake of the comparison, the second control input for the longitudinal motion, the throttle input δ_T, has to be taken as zero. The block diagram of the altitude hold system controlling the nonlinear longitudinal dynamics is given in Figure 2.

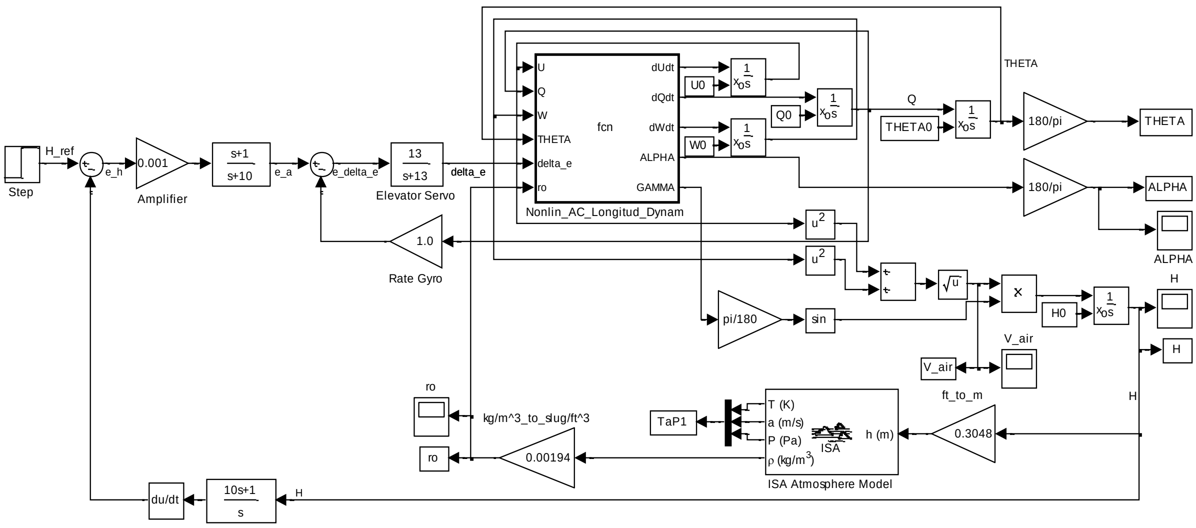

Figure 2: *Block diagram of altitude hold system controlling nonlinear flight dynamics*

Five initial conditions have to be input to the nonlinear simulation environment:

$$U_0 = 591\,\text{ft/s},\ Q_0 = 0\ \text{deg/s},\ W_0 = 0\ \text{ft/s},\ \Theta_0 = 0\ \text{deg},\ \text{and}\ H_0 = 15000\ \text{ft} \tag{4}$$

The values are taken as equal to the steady state values of the considered flight regime to make the comparison possible.

Firstly, a positive step reference with magnitude of 150 ft is input to both systems at t=20 s. The outputs of the linear and nonlinear simulations are depicted in Figure 3 and 4.

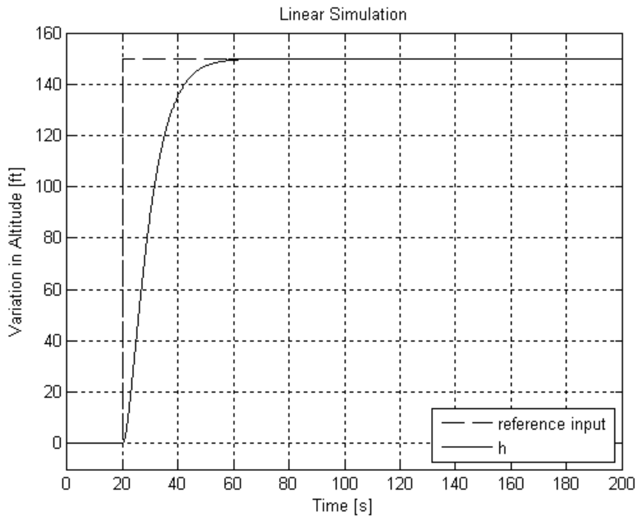

Figure 3: *The variation in altitude as output of the linear simulation environment in Figure 1*

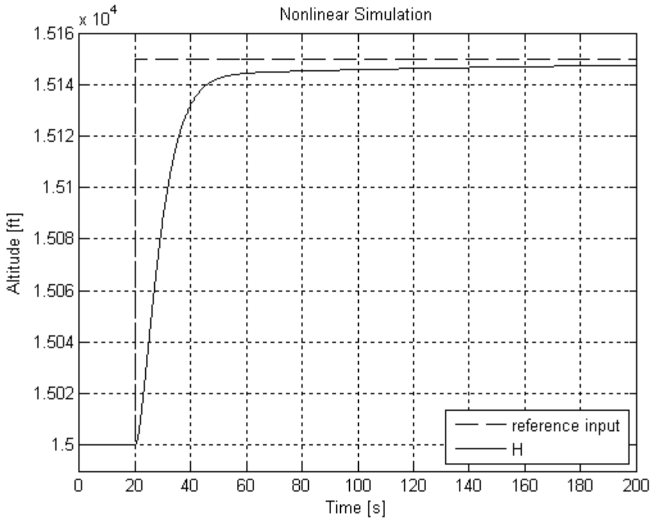

Figure 4: *The altitude as output of the nonlinear simulation environment in Figure 2*

The elevator displacements corresponding to the responses shown in Figure 3 and 4 can be seen in Figure 5 and 6, respectively.

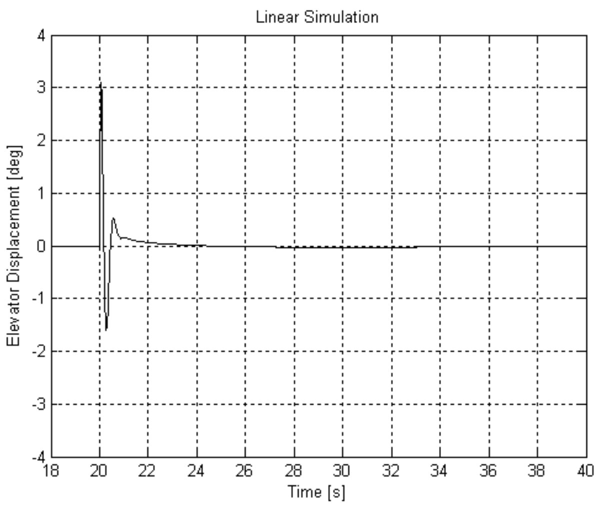

Figure 5: *The elevator deflection by the linear simulation*

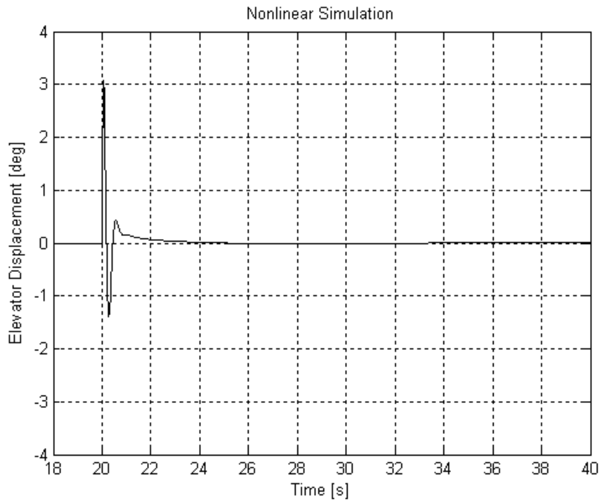

Figure 6: *The elevator deflection by the nonlinear simulation*

Then a sinusoidal reference with amplitude of 150 ft is input to both simulation environments at t=0 s. The altitude responses of the linear and nonlinear systems are given in Figure 7 and 8.

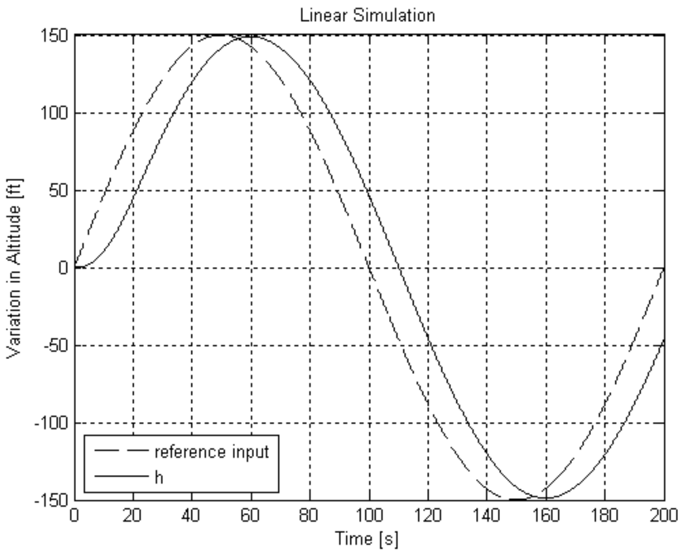

Figure 7: *The variation in altitude as response to sinusoidal input in linear simulation*

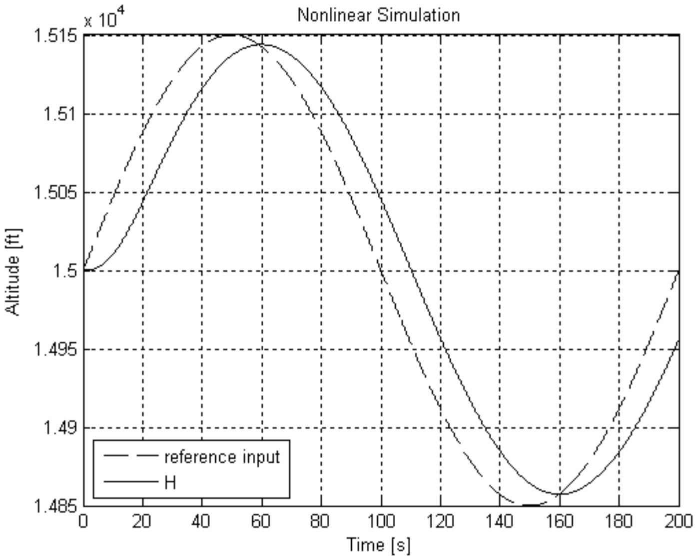

Figure 8: *The altitude variation as response to sinusoidal input in nonlinear simulation*

RESULTS AND CONCLUSION

The comparison of responses in Figure 3 and 4 leads to the conclusion that when employed to control the complete longitudinal flight dynamics, the altitude autopilot designed by root locus compensation converges to the step reference, but with a steady state error. The steady state is reached after approximately 1000 seconds. The variation in U is neglected in design however the nonlinear simulation results show that U varies. The reduced order dynamics obtained by short period approximation in linear design simplifies the design, however the controller based on the reduced-order dynamics cannot perform in nonlinear realistic environment as well as in linear design environment. The designed altitude hold system's control coefficients are valid only for the flight regime, at which the linearization is carried out. The farther the step reference is from the initial steady state value of the altitude, the higher the final steady state error of the altitude response gets.

If the reference is a sinusoidal variation, the altitude autopilot having the structure shown in Figure 1 cannot be designed to perform without a steady-state error as seen from Figure 7. The control system includes only proportional and derivative effects. To remove the phase lag in responses of both linear and nonlinear simulation, various different controllers may be replaced with the present controller, which is the topic of further studies. Figure 8 presents that the altitude hold system also exhibits unsatisfactory performance in reaching the reference amplitude, which is thought also to emerge from the effect of the neglected dynamics in deriving the short period transfer functions. For similar nonlinear performance evaluations of flight control systems, the authors recommend the book [Stevens and Lewis, 2003].

References

Blakelock. J.H. (1991) *Automatic Control of Aircraft and Missiles*, John Wiley and Sons, Inc., 1991

Sofyalı, A. and Caferov, E. (2012) *Performance Evaluation of Displacement Autopilot Designed by Root Locus Method In Nonlinear Simulation Environment*, IV. National Aerospace Conference (UHUK 2012), Sep 2012

Stevens, B.L. and Lewis, F.L. (2003) *Aircraft Control and Simulation*, John Wiley and Sons, Inc., 2003

APPENDIX

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function [dUdt,dQdt,dWdt,ALFA,GAMA] = fcn(U,Q,W,TETA,delta_e,ro)
% Flight regime parameters (F-94A):
S=239;                  % [ft^2]
Uinf=591;               % [ft/s]
U0=Uinf;                % [ft/s]
q_U0=261;               % [lb/ft^2 (psf)]
q_U=0.5*ro*(U^2+W^2);   % [lb/ft^2 (psf)]
c=6.4;                  % [ft]
W_g=13614;              % [lb]
g=32.2;                 % [ft/s^2]
m=W_g/g;                % [slug]
Iy=26543;               % [slug*ft^2]
CL=0.219;
CL_alpha=5.27;          % [1/rad]
CD=0.018;
CD_alpha=0.0844;        % [1/rad]
Cm_alha=-0.44;          % [1/rad]
Cm_delta_e=-0.934;      % [1/rad]
Cm_q0=-0.046;           % [s/rad]
Cm_q=Cm_q0*(U0/U);      % [s/rad]
%
ALPHA0=0/180*pi;                                    % [rad]
TETHA0=0/180*pi;                                    % [rad]
ALPHA_i=(W_g-CL*S*q_U0)/(CL_alpha*S*q_U0);          % [rad] (Incidence
Angle)
delta_e0=-(Cm_alpha/Cm_delta_e)*(ALPHA0+ALPHA_i);   % [rad]
delta_e=-delta_e+delta_e0;
L0=(CL+CL_alpha*(ALPHA0+ALPHA_i))*S*q_U0;
D0=(CD+CD_alpha*(ALPHA0+ALPHA_i))*S*q_U0;
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```
ALPHA=atan(W/U);
GAMMA=TETHA-ALPHA;
GAMMA=GAMMA/pi*180;
% Lift (L) and drag (D) forces dependent on angle of attack (ALPHA)
% variation and elevator deflection (delta_e):
L=(CL+CL_alpha*(ALPHA+ALPHA_i))*S*q_U;
D=(CD+CD_alpha*(ALPHA+ALPHA_i))*S*q_U;
T0=W_g*sin(TETHA0)+D0*cos(ALPHA0)-L0*sin(ALPHA0);
T=T0*(ro/0.001495383691938)^0.7;
% Pitching moment (M) dependent on angle of attack (ALPHA) variation and
% elevator deflection (delta_e):
M=(Cm_alpha*(ALPHA+ALPHA_i)+Cm_delta_e*delta_e+Cm_q*Q)*S*q_U*c;
% Equations of motion describing longitudinal flight dynamics:
dUdt=-W*Q+(-W_g*sin(TETA)-D*cos(ALFA)+L*sin(ALFA)+T)/m;
dQdt=M/Iy;
dWdt=U*Q+(W_g*cos(TETA)-D*sin(ALFA)-L*cos(ALFA))/m;
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