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VIBRATION ANALYSIS OF A ROTATING THIN-WALLED COMPOSITE BLADE

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ABSTRACT

In this paper, vibration analysis of a blade modeled as an anisotropic composite thin-walled beam is carried out. The analytical formulation of the beam is derived for the flapwise bending, torsion and flapwise transverse shear deformations. The derivation of both strain and kinetic energy expressions are made and the equations of motion are obtained by applying the Hamilton's principle. The equations of motion are solved by applying the extended Galerkin method (EGM) for symmetric lay-up configuration that is also referred as Circumferentially Asymmetric Stiffness (CAS). Consequently, the natural frequencies are validated by making comparisons with the results in literature and it is observed that there is a good agreement between the results. Effects of flap-twist coupling, transverse shear, fiber orientation, and rotational speed on the natural frequencies and the mode shapes of the rotating thin-walled composite beams are further investigated.

INTRODUCTION

Due to their high structural efficiency and several potential advantages, thin-walled structures made of anisotropic composite materials are likely to be widely used in the design of new advanced aeronautical or space vehicles, robot arms, helicopter/turbine rotor blades and high-altitude-long-endurance (HALE) aircraft and uninhabited aerial vehicles (UAVs). The directionality property of composite materials can provide a wide range of elastic couplings in this type of structures (Vo, Thuc Phuong and Lee, Jaehong (2008a,b); Haddadpour, H. and Zamani Z. (2012); Sina, S. A., Ashrafi, M. J., Haddadpour, H., and Shadmehri, F. (2011). It is well known that in order to avoid the occurrence of highly damaging aeroelastic instabilities, such as flutter and divergence, these coupling effects should be carefully addressed.

BEAM MODEL

A rotating thin-walled composite beam of length L which is fixed at $z = 0$ and free at $z = L$ is considered.

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The characteristic cross-sectional dimension of the beam and the maximum wall thickness are represented by d and h, respectively. Moreover, the rotational speed is denoted by Ω and the hub radius by R_0. The kinematic variables associated with the Cartesian coordinate system of the beam are denoted by the displacements and cross-sectional rotation which are u, v, w and ϕ. Here, s is tangent to the middle surface referred as circumferential coordinate and n is normal to the middle surface. The closed contour is defined by the coordinates $x = x(s)$ and $y = y(s)$. The front and side views of the blade geometry with the biconvex cross-section is demonstrated in Figure 1.

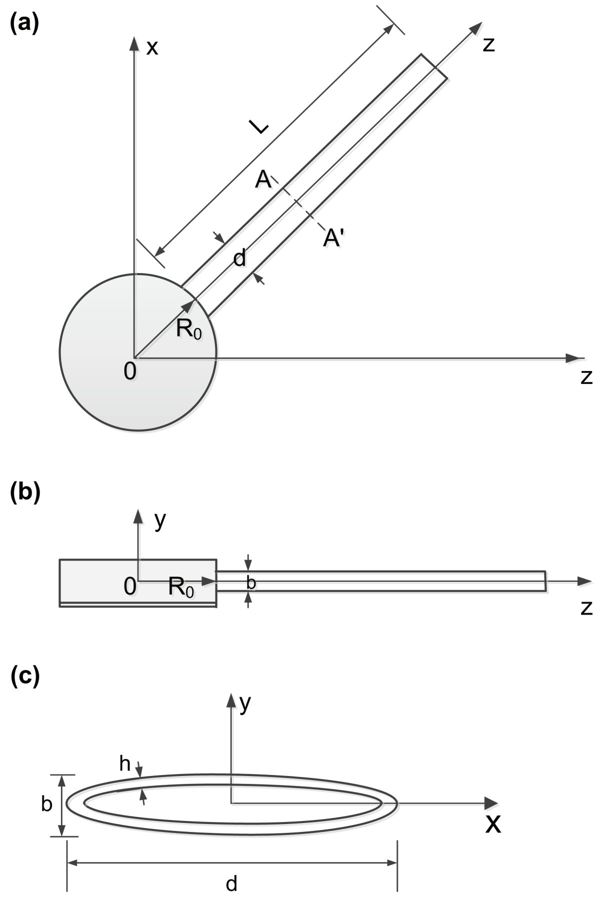


Figure 1: (a) Front-view, (b) Side-view of the blade, (c) cross-section of AA'

The position vector R measured from the centre of the hub, is expressed as

$$R = R_0 + R_v + \Delta \tag{1}$$

where $R_0 = R_0\mathbf{k}$, $R_v = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ and $\Delta = u\mathbf{i} + v\mathbf{j} + w\mathbf{k}$
The velocity and acceleration vectors are defined as

$$\dot{R} = [\dot{u}_0 + \Omega(R_0 + z + w_0)]\,\mathbf{i} + \dot{v}_0\mathbf{j} + [\dot{w}_0 - \Omega(x + u_0)]\,\mathbf{k} \tag{2}$$

$$\ddot{R} = \left[\ddot{u}_0 + 2\Omega\dot{w}_0 - \Omega^2(x + u_0)\right]\mathbf{i} + \ddot{v}_0\mathbf{j} + \left[\ddot{w}_0 - 2\Omega\dot{u}_0 - \Omega^2(R_0 + z + w_0)\right]\mathbf{k} \tag{3}$$

Displacement Field

In this section, the displacement field of a composite thin-walled beam that undergoes flapwise bending, flapwise transverse shear and torsion deflections is derived. Here, the Cartesian coordinate system is represented by (x,y,z) while the coordinates of the curvilinear system is denoted by (n,s,z_s). The in-plane translations of point $S(x,y)$ located at mid-contour, are described by u and v.

$$u(x,y,z,t) = u_0(z,t) - y\phi(z,t) \tag{4}$$

$$v(x,y,z,t) = v_0(z,t) + x\phi(z,t) \tag{5}$$

Here, t is the time, u_0 and v_0 are the displacements of pole point P, which is located at the origin $(x_P = y_P = 0)$ and $\phi(z,t)$ is the rotation of the cross-section. The tangential and normal displacement components associated with the curvilinear coordinate system are u_t and u_n, respectively.

The axial displacement accounting both for primary and secondary warping is given below,

$$\begin{aligned} w(s,z,t) = \quad & w_0(z,t) + \left[y(s) - n\tfrac{\mathrm{d}y}{\mathrm{d}s}\right]\theta_x(z,t) + \left[x(s) + n\tfrac{\mathrm{d}x}{\mathrm{d}s}\right]\theta_y(z,t) \\ & - \left[F_w(s) - nr_t(s)\right]\phi'(z,t) \end{aligned} \tag{6}$$

The primary warping function accompanied by the quantities off the mid-surface is updated as

$$F_w = \int_C \left[r_n(s) - \psi(s)\right]\mathrm{d}s \tag{7}$$

Note that the secondary warping function is equivalent to $nr_t(s)$ (Librescu, L. and Ohseop, S. (2006)). where

$$r_t(x,s) = x\frac{\mathrm{d}x}{\mathrm{d}s} + y\frac{\mathrm{d}y}{\mathrm{d}s} \tag{8}$$

$$r_n(x,s) = x\frac{\mathrm{d}x}{\mathrm{d}s} - y\frac{\mathrm{d}y}{\mathrm{d}s} \tag{9}$$

Here, r_n and r_t are the perpendicular distances from the pole point P to arbitrary points on the mid-contour S and off the mid-contour S' that are shown in Figure 2.

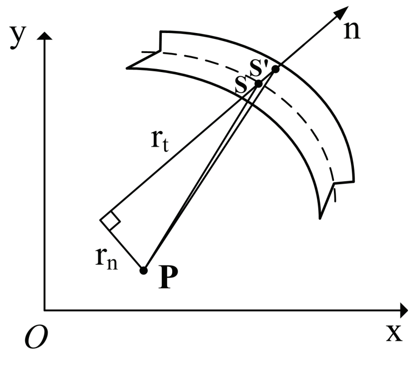


Figure 2: The perpendicular distances r_n and r_t

Strain Field

The second assumption implies that the cross-sections do not deform in its own plane (cross-section non-deformability) (Librescu, L. and Ohseop, S. (2006); Gjelsvik, A. (1981)). As a result of this, the strain components ε_{xx}, ε_{yy} and γ_{xy} becomes zero. Furthermore, the non-zero strain components ε_{zz}, γ_{xz} and γ_{yz} are re-expressed using Eqs. 4, 5 and 6

$$\varepsilon_{zz}(s,z,n,t) = \varepsilon_{zz}^{(0)}(s,z,n,t) + n\varepsilon_{zz}^{(1)}(s,z,t) \tag{10}$$

where

$$\begin{aligned} \varepsilon_{zz}^{(0)}(s,z,n,t) \quad = \quad & w_0'(z,t) + y(s)\theta_x'(z,t) + x(s)\theta_y'(z,t) \\ & -\phi''(z,t)\left[\int_0^s r_n(\lambda)\,\mathrm{d}\lambda - \int_0^s \frac{\oint r_n(s)\,\mathrm{d}s}{\oint \mathrm{d}s}\,\mathrm{d}\lambda\right] \end{aligned} \tag{11}$$

and

$$\varepsilon_{zz}^{(1)}(s,z,t) = \frac{\mathrm{d}y}{\mathrm{d}s}\theta_y'(z,t) - \frac{\mathrm{d}x}{\mathrm{d}s}\theta_x'(z,t) - \phi''(z,t)r_t(s) \tag{12}$$

Similarly, the shear strain components off the mid-line contour are described in terms of displacement quantities

$$\Gamma_{sz}(s,z,n,t) = \gamma_{sz}^{(0)}(s,z,n,t) + \gamma_{sz}^{(t)}(s,z,n,t) + n\gamma_{sz}^{(1)}(s,z,t) \tag{13}$$

$$\Gamma_{nz}(s,z,n,t) = \gamma_{nz}^{(0)}(s,z,n,t) \tag{14}$$

where

$$\gamma_{sz}^{(0)}(s,z,n,t) = \frac{\mathrm{d}x}{\mathrm{d}s}\left[u_0'(z,t) + \theta_y(z,t)\right] + \frac{\mathrm{d}y}{\mathrm{d}s}\left[v_0'(z,t) + \theta_x(z,t)\right] \tag{15}$$

$$\gamma_{sz}^{(t)}(s,z,t) = \psi(s)\phi'(z,t) \quad \text{where} \quad \psi(s) = \frac{\oint r_n(s)\,\mathrm{d}s}{\oint \mathrm{d}s} \tag{16}$$

$$\gamma_{sz}^{(1)}(s,z,t) = 2\phi'(z,t) \tag{17}$$

and

$$\gamma_{nz}^{(0)}(s,z,n,t) = \frac{\mathrm{d}y}{\mathrm{d}s}\left[u_0'(z,t) + \theta_y(z,t)\right] - \frac{\mathrm{d}x}{\mathrm{d}s}\left[v_0'(z,t) + \theta_x(z,t)\right] \tag{18}$$

The superscript $(.)^{(0)}$ indicates the strain components that are zero off the mid-line contour, while the stains that are different of zero off the mid-line contour are shown by superscripts $(.)^{(1)}$.

Energy Expressions

Before deriving the energy expressions of the beam, initially the application of Hamilton's principle will be elaborated. To do so, consider the beam having the potential energy of U, the kinetic energy of K and the work done by the external loads and body forces of W_e. The displacements denoted by $\Delta_i = \Delta_i(x,y,z,t)$ satisfy the boundary conditions $\Delta_i = \bar{\Delta}_i$ and the variations of the displacements also fulfills the condition $\delta\Delta_i = 0$ at two arbitrary times, t_0 and t_1. It is ensured by the Hamilton's principle that, the following variational is stationary for the actual path of motion from time t_0 to t_1 and given as

$$\delta J = \int_{t_0}^{t_1} \delta(U - K - W_e)\,\mathrm{d}t = 0 \tag{19}$$

Since the presented study only covers the free vibration problem, there are no external loads acting on the beam, $\delta W_e = 0$.

Strain Energy

Under the assumption of the cross-section deformability, the strain components ε_{ss}, γ_{nn} and γ_{sn} were zero. Thus, the beam cross-sections remain rigid in their own planes. With the non-zero strain components, the strain energy expression becomes

$$U = \frac{1}{2}\int\limits_{V} \sigma_{ij}\varepsilon_{ij}\,\mathrm{d}V \tag{20}$$

The variation of the strain energy is simply given as,

$$\delta U = \frac{1}{2}\int\limits_{t_0}^{t_1}\int\limits_{0}^{L}\oint\limits_{C}\int\limits_{h}\left[\sigma_{zz}\delta\varepsilon_{zz} + \sigma_{sz}\delta\Gamma_{sz} + \sigma_{nz}\delta\Gamma_{nz}\right]\mathrm{d}n\mathrm{d}s\mathrm{d}z \tag{21}$$

Kinetic Energy

The kinetic energy of the beam, and its variational form can be expressed as follows

$$K = \frac{1}{2}\int_{V}\rho\left(\dot{R}^2\right)\,\mathrm{d}V \tag{22}$$

$$\delta K = -\int\limits_{t_0}^{t_1}\mathrm{d}t\int_{V}\rho\left(\ddot{R}\delta R\right)\,\mathrm{d}V \tag{23}$$

The derivation of energy expressions will be elaborated in the full paper.

STRUCTURAL COUPLING CONFIGURATION

Symmetric configuration also referred as *circumferentially uniform stiffness*(CAS), is adopted to the thin-walled beam presented here, and as a result various coupled vibration modes are exhibited. This type of beam features two sets of independent couplings: *i*) extension-chordwise bending-chordwise transverse shear coupling, *ii*) flapwise bending-flapwise transverse shear-twist coupling. This study presents the flexural-torsional vibration of rotating thin-walled composite beams with CAS configuration. For this purpose the second set of coupling is employed to the beam model and the solution of first set of coupling is not included here.

THE GOVERNING SYSTEM OF EQUATIONS

The governing equations of motion for a rotating thin-walled composite beam with CAS configuration of flapwise bending (v_0)-twist (ϕ)-flapwise transverse shear (θ_x) are given as (Sina, S. A., Ashrafi, M. J., Haddadpour, H., and Shadmehri, F. (2011)):

$$\delta v_0: \qquad a_{55}\left(v_0'' + \theta_x'\right) - a_{56}\phi''' + T_z\left(v_0'\right)' = b_1\ddot{v}_0 \tag{24}$$

$$\begin{aligned}\delta\phi: \qquad & a_{56}\left(v_0''' + \theta_x''\right) - a_{66}\phi'''' + a_{37}\theta_x'' + a_{77}\phi'' + T_r\left(\phi'\right)' \\ & + \left(b_4 - b_5\right)\Omega^2\phi = \left(b_4 + b_5\right)\ddot{\phi} - \left(b_{10} + b_{18}\right)\left(\ddot{\phi}'' - \Omega^2\phi''\right)\end{aligned} \tag{25}$$

$$\begin{aligned}\delta\theta_x: \qquad & a_{33}\theta_x'' + a_{37}\phi'' - a_{55}\left(v_0' + \theta_x\right) + a_{56}\phi'' \\ & = \left(b_4 + b_{14}\right)\left(\ddot{\theta}_x - \Omega^2\theta_x\right)\end{aligned} \tag{26}$$

The static and dynamic boundary conditions are given on the right and the left sides, respectively, as follows:

$$\delta v_0: \qquad v_0 = 0 \quad \text{and} \qquad a_{55}\left(v_0' + \theta_x\right) - a_{56}\phi'' + b_1\Omega^2 R = 0 \tag{27}$$

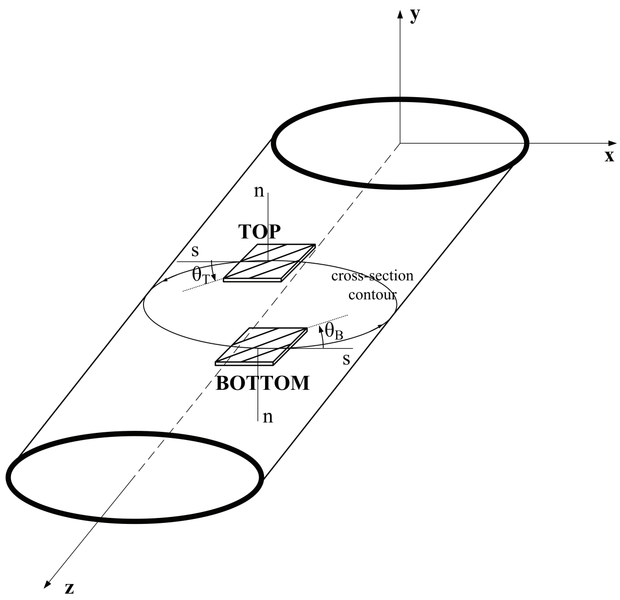


Figure 3: Lay-ups in circumferentially asymmetric stiffness configuration (CAS).

$$\begin{aligned} \delta\phi: \qquad \phi = 0 \quad \text{and} \qquad & a_{56}\left(v_0'' + \theta_x'\right) - a_{66}\phi''' + a_{37}\theta_x' + a_{77}\phi' \\ & = -b_1\Omega^2 R I_p \phi' - \left(b_{10} + b_{18}\right)\left(\ddot{\phi}' - \Omega^2\phi'\right) \end{aligned} \tag{28}$$

$$\delta\theta_x: \qquad \theta_x = 0 \quad \text{and} \qquad a_{33}\theta_x' + a_{37}\phi' = 0 \tag{29}$$

$$\delta\phi': \qquad \phi' = 0 \quad \text{and} \qquad a_{56}\left(v_0' + \theta_x\right) - a_{66}\phi'' = 0 \tag{30}$$

$$\delta\theta_x: \qquad \theta_x = 0 \quad \text{and} \qquad a_{33}\theta_x' + a_{37}\phi' = 0 \tag{31}$$

Discarding the axial force one can have $T_z(z,t) = b_1\Omega^2 R(z)$ and $R(z) = R_0(L-z) + \frac{1}{2}\left(L^2 - z^2\right)$.

SOLUTION

For the rotating thin-walled composite beam model investigated here, the governing equations involve several elastic couplings and the associated boundary conditions are quite complicated. As a result it is not easy to get the exact solution. Therefore, Extended Galerkin Method (EGM) will be used to obtain the dynamic characteristics of the beam model. This method suggests selecting weighting functions that only need to fulfill the geometric boundary conditions Meitrovich, L. (1997). To discretize the eigenvalue problem, by using the extended Galerkin method the displacements v_0, ϕ and θ_x are assumed to be in the following form:

$$v_0(z,t) = N_v^T(z)\, q_v(t) \tag{32}$$
$$\phi(z,t) = N_\phi^T(z)\, q_\phi(t) \tag{33}$$
$$\theta_x(z,t) = N_x^T(z)\, q_x(t) \tag{34}$$

Here, the trial functions are represented by N_v, N_ϕ and N_x, which are also called the shape functions [Meitrovich, L. (1997)] with the dimension $N \times 1$, while q_v, q_ϕ and q_x are the vectors of the generalized coordinates. The results in the following section are accurately calculated by assuming the shapes functions in polynomial form η_i, (order $\approx 7-9$).

The free vibration problem is specifically described by defining the mass and the stiffness matrices with the generalized coordinate vectors for each of the corresponding models. By the help of Hamilton's principle, the discretized equation of motion is obtained as follows

$$\mathbf{M}\ddot{q}(t) + \mathbf{K}q(t) = 0 \tag{35}$$

To carry out free vibration analysis, $q = X\mathrm{e}^{\mathrm{i}\omega t}$ is assumed and the eigenvalue problem is defined as follows

$$(\lambda \mathbf{I} - \mathbf{M}^{-1}\mathbf{K})X = 0 \tag{36}$$

where $\lambda = \omega^2$. Here, the eigenvectors and eigenvalues are represented by X and λ, respectively, while the natural frequencies of the system are denoted by ω. The mass and the stiffness matrices with the vectors of the generalized coordinates are defined in the following forms:

$$M = \int_0^1 \begin{bmatrix} N_v N_v^T & 0 & 0 \\ 0 & \mu_2 N_\phi N_\phi^T + \mu_3 N'_\phi N'^T_\phi & 0 \\ 0 & 0 & \mu_4 N_x N_x^T \end{bmatrix} \mathrm{d}\eta$$

$$K = \int_0^1 \begin{bmatrix} \frac{1+T_z}{AR^2} N'_v N'^T_v & -\frac{a_{56}}{a_{55}L^2 AR} N'_v N''^T_\phi & \frac{1}{AR} N'_v N_x^T \\ -\frac{a_{56}}{a_{55}L^2 AR} N''_\phi N'^T_v & \begin{matrix} \frac{a_{77}+T_r}{a_{55}L^2} N'_\phi N'^T_\phi + \frac{a_{66}}{a_{55}L^4} N''_\phi N''^T_\phi \\ -(b_{10}+b_{18})\,\Omega^2 N'_\phi N'^T_\phi \\ -(b_4-b_5)\,\Omega^2 N_\phi N_\phi^T \end{matrix} & \frac{a_{37}}{a_{55}L^2} N'_\phi N'^T_x - \frac{a_{56}}{a_{55}L^2} N''_\phi N_x^T \\ \frac{1}{AR} N_x N'^T_v & \frac{a_{37}}{a_{55}L} N'_x N'^T_\phi - \frac{a_{56}}{a_{55}L^2} N_x N''^T_\phi & \begin{matrix} N_x N_x^T + \frac{a_{33}}{a_{55}L^2} N'_x N'^T_x \\ -(b_4+b_{14})\,\Omega^2 N_x N_x^T \end{matrix} \end{bmatrix} \mathrm{d}\eta$$

$$q = \left\{ q_w \quad q_u \quad q_x \right\}^T$$

The variables and quantities that are defined to obtain the non-dimensional equations of motion are given by,

$$\begin{aligned} \eta &= \frac{z}{L} \qquad \frac{\mathrm{d}(.)}{\mathrm{d}\eta} = \frac{1}{L}\frac{\mathrm{d}(.)}{\mathrm{d}z} \qquad AR = \frac{b}{L} \qquad v_0(\eta,t) = \frac{v_0}{b} \qquad \mu_1 = (b_5+b_{15})/b_1 \\ \mu_2 &= (b_4+b_5)/b_1 \qquad \mu_3 = (b_{10}+b_{18})/b_1 \qquad \mu_4 = (b_4+b_{14})/b_1 \end{aligned}$$

RESULTS AND DISCUSSION

The results of the dynamic analysis of rotating thin-walled composite beams are presented in this section. Firstly, the dynamic analysis are conducted for the non-rotating beam thin-walled composite beam (Ω=0). The validation of the natural frequencies is made for the beam of Ref. (Librescu, L. and Ohseop, S. (2006)) and the results are tabulated for selected ply angles in Table 1. Note that the hub radius is zero ($R_0 = 0$).

The variation of the stiffness quantities are plotted with respect to ply-angle orientation and given in Figure 4. Next, for zero rotational speed Figure 5 shows the first, the second and the third eigenfrequencies as a function of ply-angle. Here solid lines with and without circles represent the unshearable (US) and shearable beam theories, respectively.

Table 1: The first three mode eigenfrequencies for the selected ply-angles.

Ply-angle θ^o	ω_i (rad/s)	Present study	Librescu & Song (2006)
	ω_1	38.6	40.3
0	ω_2	240.3	250.6
	ω_3	614.7	625.6
	ω_1	42.3	43.5
30	ω_2	262.3	270.1
	ω_3	725.2	746.7
	ω_1	51.3	52.4
45	ω_2	317.7	324.4
	ω_3	873.3	892.0
	ω_1	76.0	77.5
60	ω_2	464.0	472.7
	ω_3	1238.9	1270.0
	ω_1	134.3	136.0
75	ω_2	765.9	773.0
	ω_3	1308.3	1333.0
	ω_1	236.8	239.0
90	ω_2	630.8	640.0
	ω_3	1269.7	1280.0

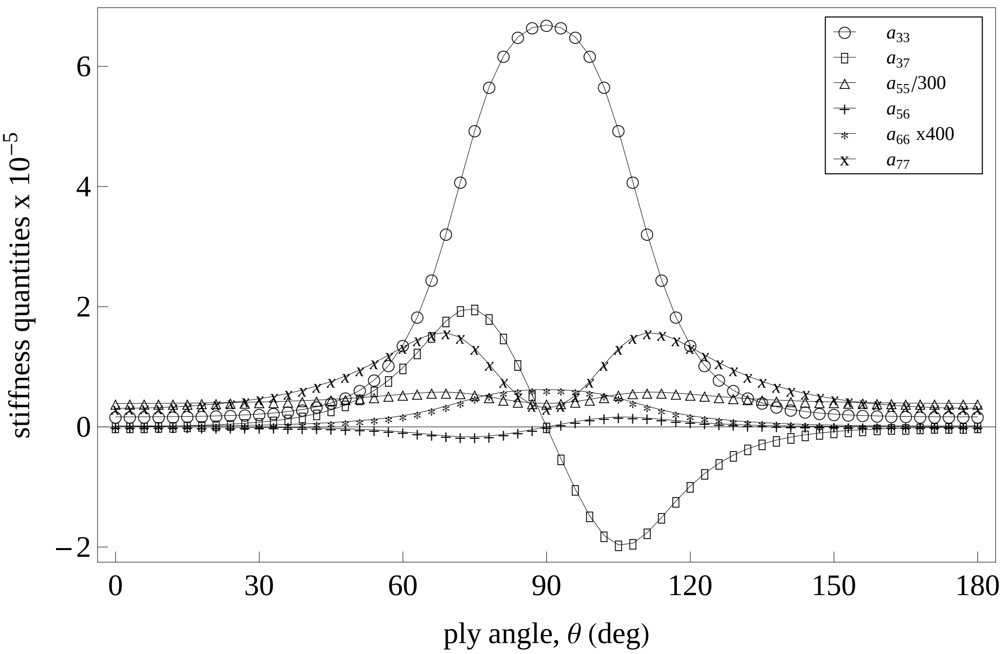


Figure 4: Stiffness quantities vs ply-angle

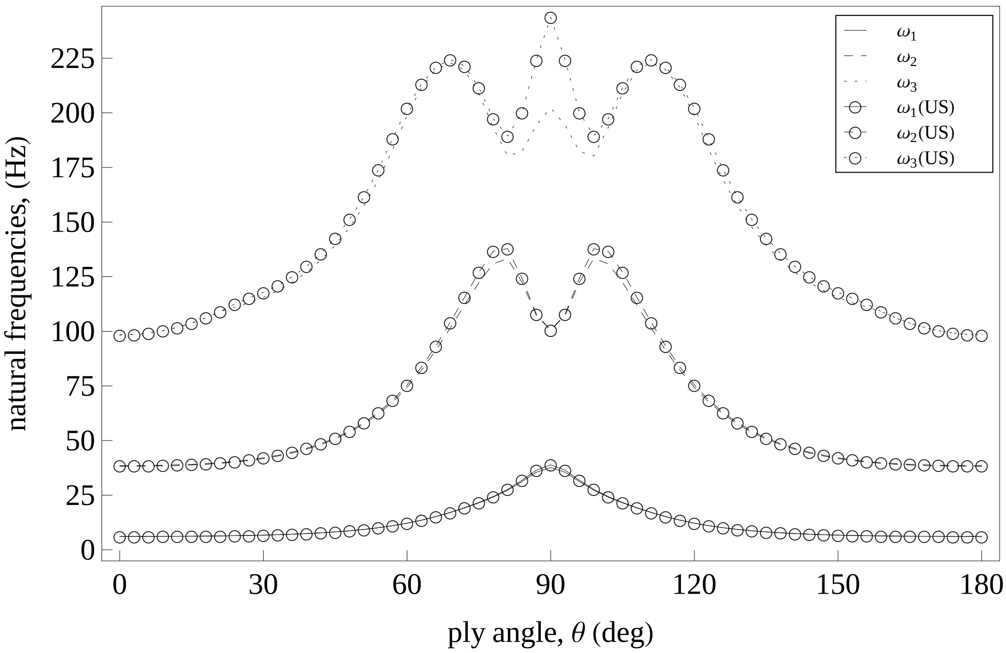
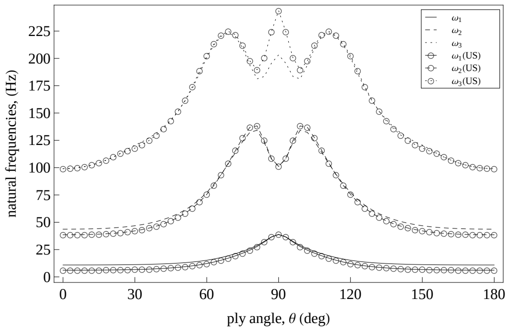
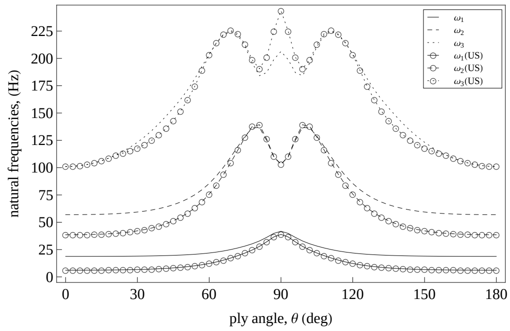


Figure 5: Natural frequencies vs ply-angle $(\Omega = 0)$

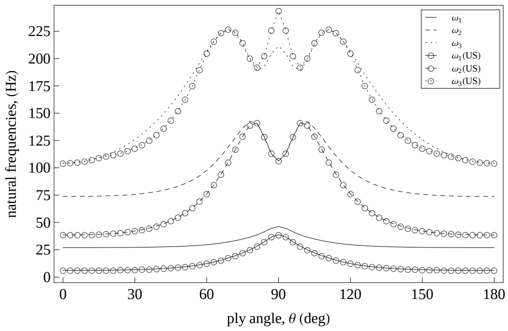
Moreover, Figures 6(a)-6(d) display the variation of the first three natural frequencies with respect to ply-angle for nonzero rotational speeds. As seen from both figures, discarding transverse shear effect underestimates the natural frequencies and this underestimation becomes more drastic for higher rotational speeds.



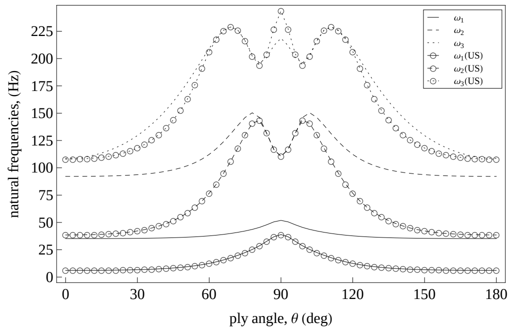
(a) Ω=500 rpm



(b) Ω=1000 rpm



(c) Ω=1500 rpm



(d) Ω=2000 rpm

Figure 6: Variation of natural frequencies for different rotational speeds.

APPENDIX

The elements of the stiffness matrix that appear in Eqs. 24-26 are given as

$$a_{33} = \oint \left[K_{11}y^2 - 2K_{14}y\tfrac{\mathrm{d}x}{\mathrm{d}s} + K_{44}\tfrac{\mathrm{d}x}{\mathrm{d}s}\right] \mathrm{d}s$$

$$a_{37} = \oint \left[K_{13}y - K43\tfrac{\mathrm{d}x}{\mathrm{d}s}\right] \mathrm{d}s$$

$$a_{55} = \oint \left[\bar{K}_s\left(\tfrac{\mathrm{d}x}{\mathrm{d}s}\right)^2 + K_{22}\left(\tfrac{\mathrm{d}y}{\mathrm{d}s}\right)^2\right] \mathrm{d}s$$

$$a_{56} = \oint \left[K_{12}F_w\tfrac{\mathrm{d}y}{\mathrm{d}s} - K_{24}r_t\tfrac{\mathrm{d}y}{\mathrm{d}s}\right] \mathrm{d}s$$

$$a_{77} = \oint \left[2K_{53} + K_{23}\psi\right] \mathrm{d}s$$

Modified stiffness quantities are given by,

$$K_{11} = A_{22} - \tfrac{A_{12}^2}{A_{11}}$$

$$K_{12} = K_{21} = A_{26} - \tfrac{A_{12}A_{16}}{A_{11}}$$

$$K_{13} = 2\left(B_{26} - \tfrac{A_{12}B_{16}}{A_{11}}\right) + \psi\left(A_{26} - \tfrac{A_{12}A_{16}}{A_{11}}\right)$$

$$K_{14} = K_{41} = B_{22} - \tfrac{A_{12}B_{12}}{A_{11}}$$

$$K_{22} = A_{66} - \tfrac{A_{16}^2}{A_{11}}$$

$$K_{23} = 2\left(B_{66} - \tfrac{A_{16}B_{16}}{A_{11}}\right) + \psi\left(A_{66} - \tfrac{A_{16}^2}{A_{11}}\right)$$

$K_{24} = K_{42} = B_{26} - \frac{A_{16}B_{12}}{A_{11}}$
$K_{43} = 2\left(D_{26} - \frac{B_{12}B_{16}}{A_{11}}\right) + \psi\left(B_{26} - \frac{A_{16}B_{12}}{A_{11}}\right)$
$K_{44} = D_{22} - \frac{B_{12}^2}{A_{11}}$
$K_{51} = B_{26} - \frac{A_{12}B_{16}}{A_{11}}$
$K_{52} = B_{66} - \frac{A_{16}B_{16}}{A_{11}}$
$K_{53} = 2\left(D_{66} - \frac{B_{16}^2}{A_{11}}\right) + \psi\left(B_{66} - \frac{A_{16}B_{16}}{A_{11}}\right)$
$K_{54} = D_{26} - \frac{B_{12}B_{16}}{A_{11}}$

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