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FIN MIXING OPTIMIZATION TO MINIMIZE CONTROL COUPLING EFFECTS

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ABSTRACT

One of the problems that flight mechanics deals with is control allocation, which is basically distributing a set of control commands (virtual controls) among a redundant set of controllers. The simplest form of control allocation problem is encountered in tail controlled flight vehicles and is given a special name as fin mixing problem. In the fin mixing problem, there are 3 virtual controls (roll – δ_a, pitch – δ_e and yaw – δ_r) and 4 actual controls (control fins); i.e. infinitely many solutions. Different solutions to this problem have been proposed in the literature depending on the application. In this paper, a method for obtaining optimal fin mixing strategy to minimize cross couplings between virtual controls is explained and demonstrated on a flight vehicle(FV) which has cruciform tail fins.

NOMENCLATURE

AFCS	= automatic flight control system
C_l	= rolling moment coefficient
C_m	= pitching moment coefficient
C_n	= yawing moment coefficient
FV	= flight vehicle
M	= Mach number
p	= rolling rate
q	= pitching rate
r	= yawing rate
α	= angle of attack
β	= angle of side slip
δ	= deflection
$_{ac}$	= actual control
$_{vc}$	= virtual control
$_{e}$	= elevator
$_{a}$	= aileron
$_{r}$	= rudder
$_{SM}$	= squeeze mode

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PROBLEM DEFINITION

Fin mixing problem can be defined as finding a set of fin deflections in response to control commands. For the motion of an automated flight vehicle (FV), onboard automatic flight control system (AFCS) calculates three control commands (sometimes called the virtual control commands or virtual controls), namely aileron deflection (δ_a) to control rolling motion about x axis, elevator deflection (δ_e) to control pitching motion about y axis and rudder deflection (δ_r) to control yawing motion about z axis. These virtual commands must be converted to actuator commands using an appropriate conversion strategy. Depending on the type of actuators used for control, there are different strategies to tackle this problem. If the FV under consideration has more than one means of control, such as thrust vector control and control fins, or control canards and reaction jets, etc., then a control allocation problem must be solved to distribute the control commands among the set of control actuators. However, for a FV with a unique set of control actuators (such as tail fins or canards, etc.), the problem becomes a fin mixing problem that of solely deciding on the strategy of distributing control commands to control fins.

The control fins might be positioned in a number of ways on the FV. If there are two fins, they are positioned along the y axis to provide roll and pitch authority, while the yaw authority might be relaxed or maintained via utilization of complex mechanisms such as “splitting” some or all of the control fins. If there are three fins, they might be positioned along the y and z axis of the FV as “T” or “inverted T”, or one along the z axis and two at an angle as ”Y” or “inverted Y”. When the number of fins is more than three, some or all of the fins may be positioned off-axis on the FV.

For the case of four control fins, a number of different fin mixing strategies can be found in the literature. The basic strategy for a flight vehicle with control fins in “+” configuration (with positive deflections shown in Figure 1) is given by [Nielsen, 1960] as Eq.1, where $\delta_{1\text{-}4}$ are fin deflections:

$$\begin{aligned} \delta_a &= \frac{\delta_1-\delta_2}{2}, \frac{\delta_3-\delta_4}{2} \\ \delta_e &= \frac{\delta_1+\delta_2}{2} \\ \delta_r &= \frac{\delta_3+\delta_4}{2} \end{aligned} \quad \text{(Eq. 1)}$$

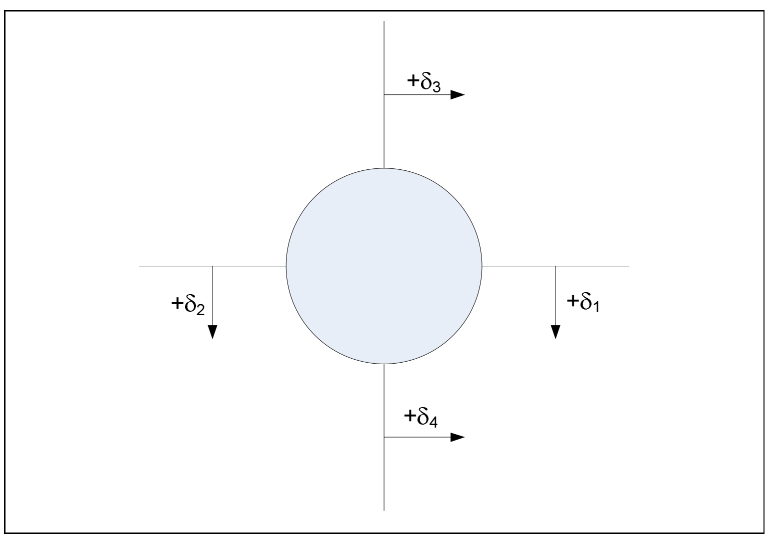

Figure 1: ***Nielsen’s convention for positive fin deflections***

[Cronvich, 1986] states the indeterminate nature of the fin mixing problem and defines a “squeeze mode” condition to overcome this situation as Eq. 2. Cronvich’s positive fin deflection convention is given in Figure 2. Apparently, the squeeze mode is chosen such that the axial force caused by the deflection of the fins is minimized (δ_{SM} = 0).

$$\delta_{SM} = \frac{\delta_1 - \delta_2 + \delta_3 - \delta_4}{4} \quad \text{(Eq. 2)}$$

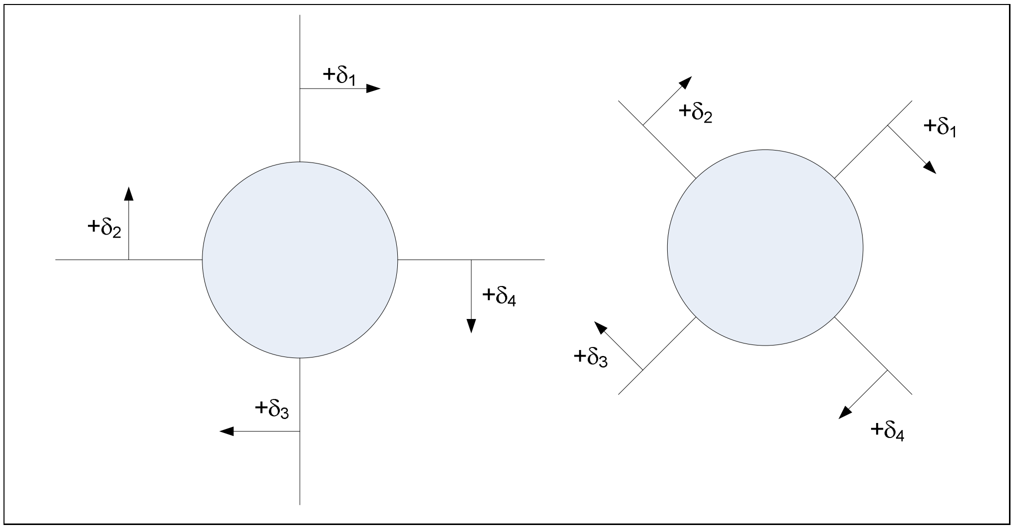

Figure 2: ***Cronvich's convention for positive fin deflections***

Although utilization of the squeeze mode condition turns the indeterminate fin mixing problem into a determinate one, it need not be satisfied at all times; some solutions to the fin mixing problem gives one of the fins as fixed at zero degrees, regardless of the orientation of the fins on the FV.

Following Cronvich's approach [Cronvich, 1986], but defining the positive fin deflections as given in Figure 3, the fin mixing equations for "+" configuration can be written as Eq.3a and for "x" configuration as Eq.3b :

$$\begin{aligned} \delta_a &= \frac{\delta_1 + \delta_2 + \delta_3 + \delta_4}{4} \\ \delta_e &= \frac{\delta_2 - \delta_4}{2} \\ \delta_r &= \frac{\delta_1 - \delta_3}{2} \\ \delta_{SM} &= \frac{\delta_1 - \delta_2 + \delta_3 - \delta_4}{4} \end{aligned} \quad \text{(Eq. 3a)}$$

$$\begin{aligned} \delta_a &= \frac{\delta_1 + \delta_2 + \delta_3 + \delta_4}{4} \\ \delta_e &= \frac{\delta_1 + \delta_2 - \delta_3 - \delta_4}{4} \\ \delta_r &= \frac{\delta_1 - \delta_2 - \delta_3 + \delta_4}{4} \\ \delta_{SM} &= \frac{\delta_1 - \delta_2 + \delta_3 - \delta_4}{4} \end{aligned} \quad \text{(Eq. 3b)}$$

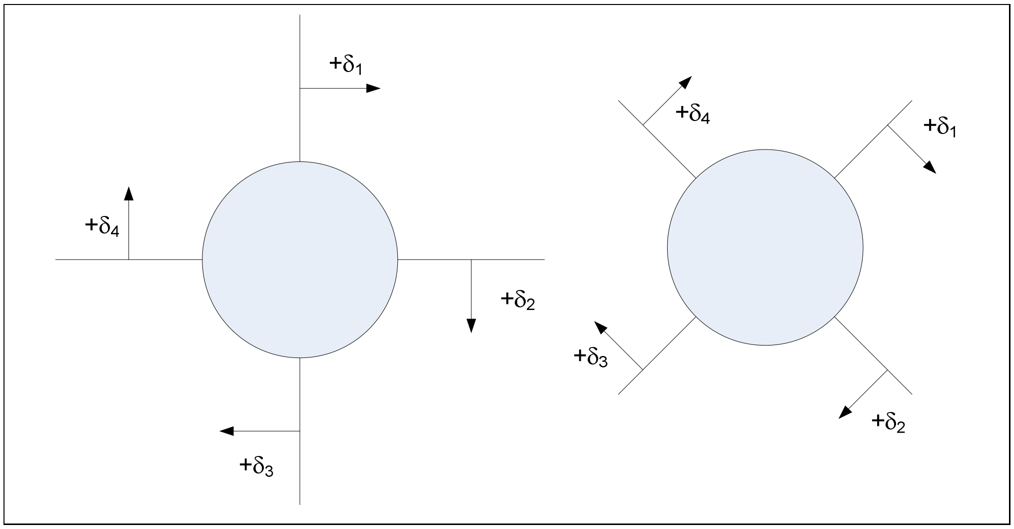

Figure 3: ***Selected convention for positive fin deflections***

In fact, above equations are the relations for obtaining virtual controls, given the actual controls. In application, the reverse of these equations are needed. For this purpose Eq.3 can be rewritten in matrix form as $\delta_{vc} = K^{-1}\delta_{ac}$ (Eq. 4), where K denotes the "fin mixing matrix", δ_{vc} stands for virtual controls and δ_{ac} stands for actual controls:

$$\begin{bmatrix} \delta_a \\ \delta_e \\ \delta_r \\ \delta_{SM} \end{bmatrix} = \begin{bmatrix} 1/4 & 1/4 & 1/4 & 1/4 \\ 0 & 1/2 & 0 & -1/2 \\ 1/2 & 0 & -1/2 & 0 \\ 1/4 & -1/4 & 1/4 & -1/4 \end{bmatrix} \begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \end{bmatrix} \quad \text{(Eq. 4a)}$$

$$\begin{bmatrix} \delta_a \\ \delta_e \\ \delta_r \\ \delta_{SM} \end{bmatrix} = \begin{bmatrix} 1/4 & 1/4 & 1/4 & 1/4 \\ 1/4 & 1/4 & -1/4 & -1/4 \\ 1/4 & -1/4 & -1/4 & 1/4 \\ 1/4 & -1/4 & 1/4 & -1/4 \end{bmatrix} \begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \end{bmatrix} \quad \text{(Eq. 4b)}$$

Then the fin mixing equation for actual controls is obtained as Eq. 5 ($\delta_{ac} = K\delta_{vc}$):

$$\begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & -1 \\ 1 & 0 & -1 & 1 \\ 1 & -1 & 0 & -1 \end{bmatrix} \begin{bmatrix} \delta_a \\ \delta_e \\ \delta_r \\ \delta_{SM} \end{bmatrix} \quad \text{(Eq. 5a)}$$

$$\begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix} \begin{bmatrix} \delta_a \\ \delta_e \\ \delta_r \\ \delta_{SM} \end{bmatrix} \quad \text{(Eq. 5b)}$$

If squeeze mode is selected to be equal to zero; Eq.5 reduces to:

$$\begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} \delta_a \\ \delta_e \\ \delta_r \end{bmatrix} \quad \text{(Eq. 6a)}$$

$$\begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 1 & -1 & -1 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} \delta_a \\ \delta_e \\ \delta_r \end{bmatrix} \quad \text{(Eq. 6b)}$$

Various derivations of fin mixing equation are available in the literature. A common practice is to relate the fin mixing matrix, K, to the external geometry of the flight vehicle through the use of fin positioning angles from the z axis [Ridgely et.al., 2006 and Fleeman, 2009].
Nevertheless, this approach relies on one basic assumption: in order for Eq.1 - 6 to be valid, the flow field around the fins must be identical, so that when deflected, the effect of each fin is identical throughout the flight envelope. This is an assumption that seldom holds in practice. In actual applications, utilization of the above approach results in cross control couplings between virtual controls. The extent of the cross coupling depends on the external geometry of the flight vehicle as well as the flight conditions and under some circumstances it might lead to performance degradation or even failure of mission or vehicle.

METHODOLOGY

In an ideal case, application of the virtual control about one axis does not create additional moments about the other axes. That is, for an aileron input, only additional rolling moment is created while the pitching and rolling moments do not change. Similarly an elevator deflection creates no additional moments but pitching moment and a rudder input creates no additional moments but yawing moment. This is shown in Eq.7, where “0” subscripts indicate the reference values of the moment coefficients for a given flight condition with no control input and Δ indicate the total control moments. However, most of the time, virtual controls not only changes the intended control moment but also causes extra moment about at least one other axis.

$$\delta_a \rightarrow \begin{cases} C_l = C_{l_0} + \Delta C_l \\ C_m = C_{m_0} \\ C_n = C_{n_0} \end{cases}, \delta_e \rightarrow \begin{cases} C_l = C_{l_0} \\ C_m = C_{m_0} + \Delta C_m \\ C_n = C_{n_0} \end{cases}, \delta_r \rightarrow \begin{cases} C_l = C_{l_0} \\ C_m = C_{m_0} \\ C_n = C_{n_0} + \Delta C_n \end{cases} \quad \text{(Eq. 7)}$$

Although this coupling is acceptable to some extent (see [Fleeman, 2006] for guidelines), it is still desirable to have a coupling free control strategy from a systems engineering point of view for reasons including robustness and control power usage.
To eliminate the coupling problem, a fin mixing strategy can be devised, which provides solution for the entire flight regime. To do this, an optimization problem can be defined as:

> “Find the actual controls (fin deflections) that cause minimum increase in drag force of the flight vehicle while providing coupling free control moments for a given set of virtual controls”

Although the definition of the problem is straightforward, the solution is not! It is not possible to find control moments for given virtual controls without knowing the fin mixing matrix first. However, fin mixing matrix is not known; it is the solution of the problem. So the approach to the problem must be revised.
The nondimensional aerodynamic moment coefficients of the flight vehicle can be modeled as functions of flight parameters such as Mach number, angle of attack, angle of sideslip, angular rates and fin deflections. Then, any of the aerodynamic coefficient can be separated into static part, damping part and control part as given in Eq.8.

$$C_{moment}(\alpha,\beta,M,p,q,r,\delta_{ac}) = C_{static}(\alpha,\beta,M) + C_{damping}(\alpha,\beta,M,p,q,r) + C_{control}(\alpha,\beta,M,\delta_{ac})$$

(Eq. 8)

For a given range of flight parameters α, β and Mach, minimum and maximum possible control moments can be found by solving a minimization/maximization problem as:

"Find the minimum/maximum value of the function $C_{control}(\alpha,\beta,M,\delta_{ac})$ for a given flight condition (α, β and Mach) with $|\delta_{ac}| \le \delta_{\max}$"

where δ_{max} is the physical deflection limit of the control fin. Once this minimization/maximization problem is solved for entire flight envelope (sweeping flight parameters for entire range), the maximum and minimum values of control moments ($C_{control_{\min}}$, $C_{control_{\max}}$), as well as actual controls ($\delta_{ac_{\min}}$, $\delta_{ac_{\max}}$) that result in maximum moments are obtained separately. Next, the original optimization problem is revised as:

"Find the actual controls (fin deflections) that cause minimum increase in drag force of the flight vehicle for a set of given control moments corresponding to unknown virtual controls"

Or mathematically speaking

$$\begin{aligned} &\min(\|\delta_{ac}\|) \text{ such that} \\ &C_l(\alpha_{given},\beta_{given},M_{given},\delta_{ac}) = C_{l_{given}} \\ &C_m(\alpha_{given},\beta_{given},M_{given},\delta_{ac}) = C_{m_{given}} \\ &C_n(\alpha_{given},\beta_{given},M_{given},\delta_{ac}) = C_{n_{given}} \end{aligned}$$

(Eq. 9)

where

$$\begin{aligned} C_{l_{\min}} &\le C_{l_{given}} \le C_{l_{\max}} \\ C_{m_{\min}} &\le C_{m_{given}} \le C_{m_{\max}} \\ C_{n_{\min}} &\le C_{n_{given}} \le C_{n_{\max}} \end{aligned}$$

(Eq.10)

all denote control moments with the upper and lower bounds obtained from the minimization/maximization mentioned above. Notice that, Eq.9 and Eq. 10 must be solved for each flight condition over the range of possible attainable control moments. This means that, the optimization algorithm must sweep the flight envelope as well as control moments.

Since there are no spatial or temporal relations between flight conditions, i.e. the fin mixing strategy is not sought on-line during a trajectory simulation but off-line, a static optimization can be carried out using Optimization Toolbox of Matlab®. Recall that, Eq.7 provides the relations of desired values of control moments for virtual controls about different axes.

Once the optimization is completed, coupling free actual controls corresponding to attainable control moments throughout the flight envelope is obtained. However, for autopilot design and operation, relation of virtual controls with control moments is still needed.

Without loss of generality, δ_{vc} can be assumed to vary linearly with control moment. This is possible, since the required control moment for a given trajectory is constant (Eq. 11), i.e. the change in amount of virtual control will be balanced by a change in the control moment derivative with respect to virtual control and amount of change will be decided by fin mixing strategy selected.

$$C_{control} = C_{control_{\delta_{vc}}} \delta_{vc}$$

(Eq. 11)

That is

$$\delta_{vc_{\min}} = \delta_{ac_{\min}}$$
$$\delta_{vc_{\max}} = \delta_{ac_{\max}}$$
$$\delta_{vc_0} = \delta_{ac_0}$$
$$\delta_{vc} = \begin{cases} \delta_{vc_{\min}} + i \cdot \Delta\delta_{vc}, C_{control_{\min}} \le C_{control} \le C_{control_0} (\delta_{vc_{\min}} \le \delta_{vc} \le \delta_{vc_0}) \\ \delta_{vc_0} + i \cdot \Delta\delta_{vc}, C_{control_0} \le C_{control} \le C_{control_{\max}} (\delta_{vc_0} \le \delta_{vc} \le \delta_{vc_{\max}}) \end{cases} \quad \text{(Eq. 12)}$$

where i is the step number of optimization during control moment sweep for a given flight condition and $\Delta\delta_{vc}$ is the step size, both of which are the parameters of the optimization routine.

RESULTS

Devised methodology was tested on a FV which is aerodynamically controlled by a set of “x” configuration tail fins.The results for a selected number of flight conditions and virtual controls are provided in Table 1. For the classical fin mixing approach (as devised by [Cronvich, 1986]) FV experiences some degree of coupling between virtual controls as seen in Table 1.Here, control coupling is defined as ratio of induced control moment to ratio of desired control moment. With the optimal fin mixing approach derived in this paper, the results show that control coupling is almost completely eliminated.

Table 1**: *Application example for an FV with cruciform tail fins.***

Flight Condition			Virtual Control	Classical Fin Mixing – Control Couplings			Optimal Fin Mixing – Control Couplings		
Mach	AoA [°]	AoSS [°]		Roll	Pitch	Yaw	Roll	Pitch	Yaw
Low Subsonic Mach #	minus α	minus β	δa	1.00	0.05	0.06	1.00	0.00	0.00
			δe	0.19	1.00	0.18	0.00	1.00	0.00
			δr	-0.10	0.01	1.00	0.00	0.00	1.00
			δa+δe+δr	0.87	1.01	0.83	0.90	1.00	0.82
	0	0	δa	1.00	0.08	0.03	1.00	0.00	0.00
			δe	0.01	1.00	0.09	0.00	1.00	0.00
			δr	0.03	0.04	1.00	0.00	0.00	1.00
			δa+δe+δr	0.91	0.98	0.82	0.96	0.95	0.81
	plus α	plus β	δa	1.00	0.09	-0.01	1.00	0.00	0.00
			δe	0.01	1.00	0.08	0.00	1.00	0.00
			δr	-0.05	0.08	1.00	0.00	0.00	1.00
			δa+δe+δr	0.94	1.00	0.90	1.04	0.93	0.91
Moderate Subsonic Mach #	minus α	minus β	δa	1.00	-0.06	0.00	1.00	0.00	0.00
			δe	0.10	1.00	0.13	0.00	1.00	0.00
			δr	-0.03	0.05	1.00	0.00	0.00	1.00
			δa+δe+δr	0.89	1.02	0.81	0.94	1.06	0.91
	0	0	δa	1.00	-0.04	-0.02	1.00	0.00	0.00
			δe	0.01	1.00	0.03	0.00	1.00	0.00
			δr	0.04	0.07	1.00	0.00	0.00	1.00
			δa+δe+δr	0.91	0.94	0.77	0.95	0.98	0.87
	plus α	plus β	δa	1.00	-0.03	-0.05	1.00	0.00	0.00
			δe	-0.01	1.00	0.04	0.00	1.00	0.00
			δr	-0.02	0.09	1.00	0.00	0.00	1.00
			δa+δe+δr	0.92	0.93	0.86	1.01	0.93	0.93
Transonic Mach #	minus α	minus β	δa	1.00	-0.03	0.04	1.00	0.00	0.00
			δe	0.05	1.00	0.11	0.00	1.00	0.00
			δr	-0.02	-0.05	1.00	0.00	0.00	1.00
			δa+δe+δr	0.86	0.88	0.88	0.94	0.99	0.93
	0	0	δa	1.00	-0.02	-0.04	1.00	0.00	0.00
			δe	0.01	1.00	0.04	0.00	1.00	0.00
			δr	0.02	0.07	1.00	0.00	0.00	1.00
			δa+δe+δr	0.90	0.88	0.82	0.95	0.92	0.92
	plus α	plus β	δa	1.00	-0.02	-0.10	1.00	0.00	0.00
			δe	0.01	1.00	0.06	0.00	1.00	0.00
			δr	0.00	0.12	1.00	0.00	0.00	1.00
			δa+δe+δr	0.95	0.87	0.86	1.01	0.89	0.95

In practical applications, it is assumed that the control commands can be superposed. However, Table 1 shows that, validity of this assumption depends on flight conditions; for some cases, attained control moments might be 20% lower than desired moments when classical fin mixing is used. It should be noted that, utilization of optimal fin mixing strategy also improves this situation.

CONCLUSION

The devised methodology provides an efficient way to eliminate control couplings associated with fin mixing strategy. Although, a closed form solution like a fin mixing matrix is not obtained, a function can be fit to the resulting look-up table through the utilization of regression methods.
As a future work, studies will be concentrated on designing a limiter to prioritize virtual commands depending on the selected maneuver type.

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