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MODELING, SIMULATION AND SYSTEM IDENTIFICATION OF A QUADROTOR HELICOPTER

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ABSTRACT

The aim of this study is to get an accurate dynamic and simulation model of a quadrotor helicopter for both hover and vertical conditions. The main factors which determine the movement of the quadrotor are aerodynamic effects and body forces and torques created by the four rotors in three different axes. Hence, the calculations of the rotor forces and moments are essential that establish an exact simulation of the model in forward flights. For this reason, a special study was planned to experimentally calculate forces and torques created by the rotors and aerodynamic effects on the quadrotor in a wind tunnel. First, forces and torques in three axes measured in various angular speeds of the rotor for both hover and forward flight conditions. Measurements then used to identify the parameters in a mathematical formulation for aerodynamic forces based on the momentum and blade element theories. This study aims to better model the quadrotor movement and thus better simulate the quadrotor in the virtual environment. Most studies in the literature only consider rotor thrust forces acting on the quadrotor and use a quadratic relation between rotor speed and thrust force, ignoring the influences of aerodynamic flows due to vehicle motion on body and rotor forces. In this paper, effects of vehicle motion on rotor forces were modeled using a combination of momentum and blade element theories. Forces and moments acting on the body are taken into account as well and the parameters of the developed model will be identified through wind tunnel tests. The developed model allowed us to accurately predict the behavior of quadrotor in various rotor speeds and flight conditions.

Key Words: Quadrotor, Modeling and Simulation, Parameter Estimation, System Identification, Rotor Aerodynamics

LIST OF SYMBOLS

u,v, w	Linear speeds of the quadrotor in x, y, z body axes, respectively
p,q, r	Angular speeds of the quadrotor in x, y, z body axes, respectively
x_e,y_e, z_e	Linear speeds of the quadrotor in Earth frame x, y, z axes, respectively
ϕ,θ, ψ	Euler Angles
b	Thrust coefficient
d	Drag moment coefficient
X,Y, Z	Forces acting on the quadrotor in x, y, z body axes, respectively
L, M, N	Moments acting on the quadrotor in x, y, z body axes, respectively
I_{xx}, I_{yy}, I_{zz}	Inertia moments

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J_r	Rotor inertia
T	Thrust force
H	Hub force
Q	Drag moment (Rotor torque)
P	Pitching moment
$\mathcal{Y}$	Side force
R	Rolling moment
C_T	Thrust coefficient
C_H	Hub force coefficient
C_R	Rolling moment coefficient
C_P	Pitching moment coefficient
$C_{\mathcal{Y}}$	Side force coefficient
C_l	Lift coefficient
C_D	Drag coefficient
ρ	Air density
A	Area captured by rotor
Ω	Angular speed of the rotor
R	Radius of the rotor
μ	Rotor advanced ratio
θ_{tw}	Twist angle
λ	Inflow ratio
N	Number of blades
σ	Solidity ratio
a	Lift curve slope
c	Chord
$\dot{x}$	inflow velocity component in x axis
$\dot{y}$	inflow velocity component in y axis
$\dot{z}$	inflow velocity component rotor z axis
V	Sideway velocity
V_c	Climb velocity

v_i	Induced velocity
U_T	Horizontal force of blade
U_P	Vertical force of blade
Ψ	Azimuth angle
y	Radius of blade section
α_r	Rotor a.o.a
h	Distance from rotor center to C.o.G
l	Distance between rotor axis and C.o.G
m	*Mass of the quadrotor*

INTRODUCTION

A quadrotor, which is known as quadrocopter or quadrotor helicopter, is an aircraft that is lifted by four rotors. In recent years, due to their broad field of applications, such as near field surveillance, exploration and survey, Quadrotor Unmanned Aerial Vehicles (UAVs), have been studied both in civil and military applications. It can be used both indoors and outdoors. Quadrotors are among the most preferred types of UAVs because of their basic structure, ability to move omnidirectionally and hover, and reduced gyroscopic effect which is caused by the reverse turning of rotors.

The difference between the conventional helicopters with a main and tail rotor and quadrotors is frequently questioned. On a conventional one, torque resulted in main rotor is balanced by a tail rotor. The balance of hovering is affected adversely when a helicopter tries to move sideways while hovering. Unlike helicopters, quadrotors perform better hovering, which makes them more practical. However, besides the advantages mentioned above, quadrotors have some disadvantages like low useful-load capacity and high energy consumption. It is significant that some improvements should be done so that quadrotors can be used widely in practical areas.

In literature, mostly, the hover situation of a quadrotor has been studied where the rotors are assumed to produce thrust force only. Therefore, in these studies, thrust force only depends on angular speeds of rotors. However, in forward flights, it depends on also sideway velocity, induced velocity and climbing velocity.

It is proposed that in order to get an accurate model of quadrotor it is necessary to add forces and moments created by the rotors in other axes such as hub force (rotor forces in the horizontal plane), rolling moment and rotor torque (drag moment) [Bouabdallah S. and Siegwart R., 2007].

The writers proposed that performance and dynamic response of a micro UAV is affected by the aerodynamic influences. They studied ground and ceiling effects for the vehicle. The aerodynamics of the forward flight and vertical flight are also studied in this article. The change of the thrust force with the rotor angular speed in various a.o.a is observed [Powers C., Mellinger D., Kushleyev A., Kothmann B., Kumar V.,]

It is studied that total thrust variation in translational flight by considering induced velocity [Huang H., Hoffmann G.M., Waslander S. L., Tomlin C. J., 2009]. Dynamic behavior of the quad-rotor in various flight conditions by using aerodynamic derivatives of the quadrotor was studied [Sudiyanto T., Muljowidodo, Budiyono A., 2009].

Aerodynamics of the rotor is an important factor in quadrotor’s flight, especially in forward flights. It should be studied and applied to quadrotor simulation and control. In order to have an exact model of the quadrotor, the aerodynamic factors act on it should be studied properly.

Thrust force, hub force, rotor torque and pitching moment derivations are made by using momentum and blade element theory. Induced velocity is taken into account not only in hover but also in forward flights. Based on this study, forces and torques are depended on the total inflow velocity , induced velocity , component of inflow velocity in z axis , and angular speed of each rotor [Fay G., 2001].

Powers C., Kumar V. mentioned about the importance of rotor aerodynamics for micro UAVs. The relation between thrust force and angular speed of rotor, inflow and induced velocity was presented. The coefficients between mentioned factors are found experimentally. Surface effects are also presented in this study.

Thrust force, hub force and moment coefficients were calculated by using the blade element and the momentum method [Orsag M., Bogdan S., 2012]. In this study, thrust force is assumed to be proportional to the square of angular speed of rotor. By means of measuring voltage, rpm, induced velocity and thrust force, rotor angle is found. They also presented a nonlinear mathematical model by taking into account of aerodynamic effects.

METHOD

This study focuses on the modeling, simulation, and system identification of a quadrotor helicopter. The Newton-Euler Method was used to model the dynamics of quadrotor. The use of MATLAB-Simulink facilitated the simulation of the vehicle.

Dynamics of the quadrotor:

To develop the model of the quadrotor following assumptions have been made:

1. The quadrotor’s structure and propellers are assumed to be rigid.
2. The CoG and the origin of body-fixed frame are supposed to coincide.
3. Aerodynamic moments are assumed to be negligible.
4. The blades of rotors have a constant chord.
5. The lift coefficient varies linearly with the angle of attack.
6. The rotor blade has a constant chord.
7. The twist changes linearly with radial position.
8. Lift acting on a blade is about an order of magnitude higher than drag.
9. The local inflow angle is assumed to be very small.

The coordinate systems used in the derivation of equations of motion are the body-fixed frame and the earth-fixed frame.

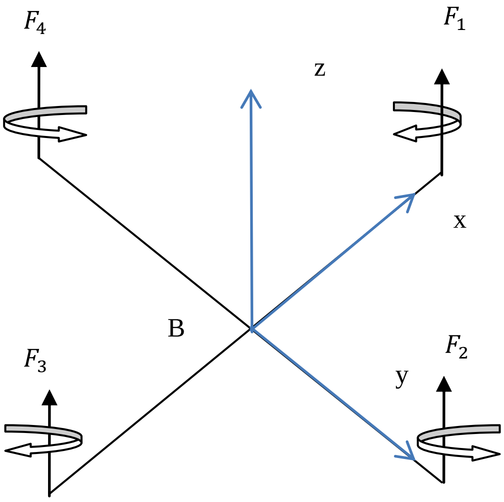

Figure 1: Body-Fixed Frame

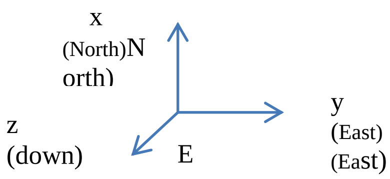

Figure 2: Earth-Fixed Frame

The use of the body-fixed frame, Figure 1, is practical because the inertia matrix is time-invariant and rotor and body aerodynamic forces can be expressed with respect to the body. Besides it is easy to convert the body-fixed frame to the earth frame, Figure 2. General equations to represent the rigid body motion of a body are summarized below.

Transformation between the body-fixed frame and the earth frame:

$$\begin{bmatrix} x_e \\ y_e \\ z_e \end{bmatrix} = \begin{bmatrix} cos\psi cos\theta & -sin\psi cos\phi + cos\psi sin\theta sin\phi & sin\psi sin\phi + cos\psi sin\theta cos\phi \\ sin\psi cos\theta & cos\psi cos\phi + sin\psi sin\theta sin\phi & -cos\psi sin\phi + sin\psi sin\theta cos\phi \\ -sin\theta & cos\theta sin\phi & cos\theta cos\phi \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

Rotation between the body-fixed frame and the earth frame:

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & sin\phi tan\theta & cos\phi tan\theta \\ 0 & cos\phi & -sin\phi \\ 0 & sin\phi sec\theta & cos\phi sec\theta \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

Translational equations of the motion in body-fixed frame:

$$\dot{u} = \frac{X}{m} - qsin\theta - qw + rv$$

$$\dot{v} = \frac{Y}{m} + gsin\varphi cos\theta - ru + pw$$

$$\dot{w} = \frac{Z}{m} + gcos\varphi cos\theta - pv + qu$$

Rotational equations of the motion in body-fixed frame:

$$\dot{p} = \frac{L}{I_x} + qr\frac{(I_y - I_x)}{I_x}$$

$$\dot{q} = \frac{M}{I_y} + pr\frac{(I_z - I_x)}{I_y}$$

$$\dot{r} = \frac{N}{I_z} + pq\frac{(I_x - I_y)}{I_z}$$

Aerodynamics of the quadrotor:

The aerodynamics forces and moments acting on the rotors are obtained from the combination of Momentum Theory and Blade Element Theory. Momentum Theory is used to derive the induce velocity. The aim of the Blade Element Theory is to calculate the total forces and moments acting on the rotor shaft.

Based on the sources [3, 13]:

Thrust force is the resultant vertical forces acting on the blade elements.

$$\mathrm{T} = \mathrm{C_T} \rho \mathrm{A} (\Omega\, \mathrm{R})^2$$

$$\frac{C_T}{\sigma a} = \left(\frac{1}{6} + \frac{1}{4}\mu^2\right)\theta_0 - (1+\mu^2)\frac{\theta_{tw}}{8} - \frac{1}{4}\lambda$$

$$\mathrm{v}_i = \sqrt{-\frac{\mathrm{V}^2}{2} + \sqrt{\left(\frac{\mathrm{V}^2}{2}\right)^2 + \frac{W}{2\rho \mathrm{A}}^2}}$$: Induced velocity in forward flights

At hover, it is assumed that thrust force is proportional to the square of the angular speed of rotor.

$$\mathrm{T} = \mathrm{b}\,\mathrm{x}\,\Omega^2$$

In forward flights:

$$T = \Omega^2 x\left(\frac{1}{6}\rho \mathrm{A}\sigma a\theta_0 \mathrm{R}^2 - \frac{1}{8}\sigma a\rho \mathrm{A}\theta_{tw}\mathrm{R}^2\right) + \mathrm{V}^2\, x\, \left(\frac{1}{4}\rho \mathrm{A}\sigma a\theta_0 - \frac{1}{8}\sigma a\rho \mathrm{A}\theta_{tw}\right) - (\Omega x \mathrm{v}) x\, \left(-\frac{1}{4}\sigma a\rho \mathrm{A}R\right) + (\Omega x\dot{z}) x\, \left(\frac{1}{4}\rho \mathrm{A}\sigma aR\right)$$

$$T = t_1 x\Omega^2 + t_2 x\mathrm{V}^2 + t_3\, x\, (\Omega x \mathrm{v}) + t_4\, x\, (\Omega x\dot{z})$$

Where t_1, t_2, t_3 and t_4 are constants which depend on rotor geometry alone.

Hub force is the resultant horizontal forces on the blade element.

$$\mathrm{H} = \mathrm{C_H}\rho \mathrm{A}(\Omega \mathrm{R})^2$$

$$\frac{C_H}{\sigma a} = \frac{1}{4a}\mu C_d + \frac{1}{4}\lambda\mu\left(\theta_0 - \frac{\theta_{tw}}{2}\right)$$

$$\mathrm{H} = (\Omega xV) x\left(\frac{1}{4}\rho \mathrm{A}\sigma C_d R\right) + (\mathrm{V}x\mathrm{v}) x\left[\frac{1}{4}\rho \mathrm{A}\sigma a\left(\theta_0 - \frac{\theta_{tw}}{2}\right)\right] + (\dot{z}xV) x\left[-\frac{1}{4}\sigma a\rho \mathrm{A}\left(\theta_0 - \frac{\theta_{tw}}{2}\right)\right]$$

$$H = h_1 x(\Omega xV) + h_2 x(\mathrm{V}x\mathrm{v}) + h_3\, x(\dot{z}xV)$$

Where h_1, h_2 and h_3 are constants which depend on rotor geometry alone.

Side force $y = 0$.

Drag moment, which can be called as rotor torque moment, is the torque about the rotor shaft. It is found by integrating the horizontal aerodynamic forces on each rotor section with a moment arm equal to the radius of that section.

$$\mathrm{Q} = \mathrm{C_Q}\rho \mathrm{A}(\Omega\, \mathrm{x}\, \mathrm{R})^2 \mathrm{R}$$

$$\frac{C_Q}{\sigma a} = \frac{1}{8a}(1+\mu^2)C_d + \lambda\left(\frac{1}{6}\theta_0 - \frac{1}{8}\theta_{tw} - \frac{1}{4}\lambda\right)$$

At hover, it can be assumed that rotor torque is proportional to the square of the angular speed of rotor.

$$\mathrm{Q} = \mathrm{d}\ \mathrm{x}\ \Omega^{\,2}$$

In forward flights:

$$Q = \Omega^2 x\left(\frac{1}{8}\rho A\sigma C_d R^3\right) + V^2\ x\ \left(\frac{1}{8}\rho A\sigma R C_d\right) + (\Omega x \mathrm{v}) x\ \left(\frac{1}{6}\sigma a\rho A\theta_0 R^2 - \frac{1}{8}\rho A\sigma a\theta_{tw} R^2\right) + (\Omega x\dot{z})x\ \left(-\frac{1}{6}\sigma a\rho A\theta_0 R^2 + \frac{1}{8}\rho A\sigma a\theta_{tw} R^2\right) + (\mathrm{v} - \dot{z})^2 x(-\frac{1}{4}\sigma a\rho A R)$$

$$Q = d_1 x\Omega^2 + d_2 x V^2 + d_3\, x\,(\Omega x \mathrm{v}) + d_4\, x\,(\Omega x\dot{z}) + d_5\, x(\mathrm{v} - \dot{z})^2$$

Where $d_1,\ d_2, d_3\ d_4$ and d_5 are constants which depend on rotor geometry alone.

Rolling moment is the integral over the entire rotor the moments caused by the lift.

$$\mathrm{R} = -\mathrm{N}\rho a\bar{c}(\Omega \mathrm{R})^2 R^2\mu\left(\frac{1}{6}\theta_0 - \frac{1}{8}\theta_{tw} - \frac{1}{8}\lambda\right)$$

Rolling moment coefficient:

$$\frac{C_R}{\sigma a} = -\mu\left(\frac{1}{6}\theta_0 - \frac{1}{8}\theta_{tw} - \frac{1}{8}\lambda\right)$$

R=-[$(\Omega x V)x\ \left(-\frac{1}{6}\sigma a\rho A\theta_0 R^2 + \frac{1}{8}\rho A\sigma a\theta_{tw} R^2\right) + (V x \mathrm{v})x\left(\frac{1}{8}\sigma a\rho A R\right) + (V x\dot{z})\left(-\frac{1}{8}\sigma a\rho A R\right)$]

$$R = r_1 x(\Omega x V) + r_2 x(V x \mathrm{v}) + r_3\, x\,(V x\dot{z})$$

Where $r_1,\ r_2$ and r_3 are constants which depend on rotor geometry alone.

Pitching moment $P = 0$

Experiments:

The difficult part to simulate above equations is to find X, Y, Z and L, M, N accurately. These are the forces and moments acting on the quadrotor. They depend on different physical and aerodynamic effects. In this study, the behavior of forces and moments according to rotor changing effects and flight conditions was planned to be found. Hence, we had an advanced model for hover. To measure mentioned factors the quadrotor UAV, Figure 3, was mounted on a balance system that can measure forces and moments in all three axes in a wind tunnel shown in Figure 4. It is a low speed suction type wind tunnel, which includes a 2D contraction section with an area ratio of 1:5 and is powered by a 45 kW speed-controlled electrical motor. The wind tunnel also includes inlet guide vanes at the entrance of the contraction, a honeycomb and a screen are installed upstream of the test section to maintain appropriate flow quality.

According to measured forces and moments in different speeds of the quadrotor, the behavior of the quadrotor is estimated.

Figure 3: Quadrotor UAV

Figure 4: Wind Tunnel

Thrust force, hub force and rotor torque and Rolling moment are depended on the total inflow velocity V, induced velocity v_i, angular speed of each rotor, and the component of the inflow velocity in z axis, $\dot{z}$, where $\dot{z} = -V_{climb}$.
Forces and moments are depended on different factors. Therefore, to get a faithful model of the vehicle, parameters mentioned above were needed to be determined empirically.

Figure 5: Quadrotor with load cell and RPM Sensor

Set up used in the experiments is presented in Figure 5. It consists of a velocity sensor which has a 0-30 m\s ranges and measures the inflow velocity and a load cell which is for to measure forces and moments on the quadrotor, and an optical rpm sensor for measuring the angular velocity of the rotor. The mechanical part was produced for this study. The vehicle and the load cell were inserted on this mechanical part. Then the whole systems were mounted into the wind tunnel [Figure 4] in order to measure inflow velocity, and the forces and moments simultaneously. The velocity sensor and the load cell were connected to the data acquisition system which transfers the information taken to the load cell and velocity sensor to the computer. Single rotor and single motor were run during the experiments.

RESULTS

Table 1: Omega, Thrust, and Rotor Torque Values at Zero Incline

Omega(RPM)	Thrust Force(N)	Rotor Torque(Nm)
1764	0.182	0.00346
1985	0.252	0.00422
2375	0.384	0.00526
2675	0.496	0.0069
2902	0.575	0.00817
3575	0.921	0.01358
3690	0.974	0.01428
3905	1.079	0.01483
4290	1.319	0.01903
4596	1.513	0.02082

Table 2: Parameter Estimation Results for Hover

Relation between thrust force and omega,	$\mathrm{T} = \mathrm{b} \times \Omega^2$
Relation between rotor torque and omega,	$\mathrm{Q} = \mathrm{d} \times \Omega^2$
Thrust Coefficient	$b = 7.1103x10^{-8}\ (N\backslash(RPM^2))$
Rotor torque coefficient	$d = 1.0088x10^{-9}\ (N\backslash(RPM^2))$

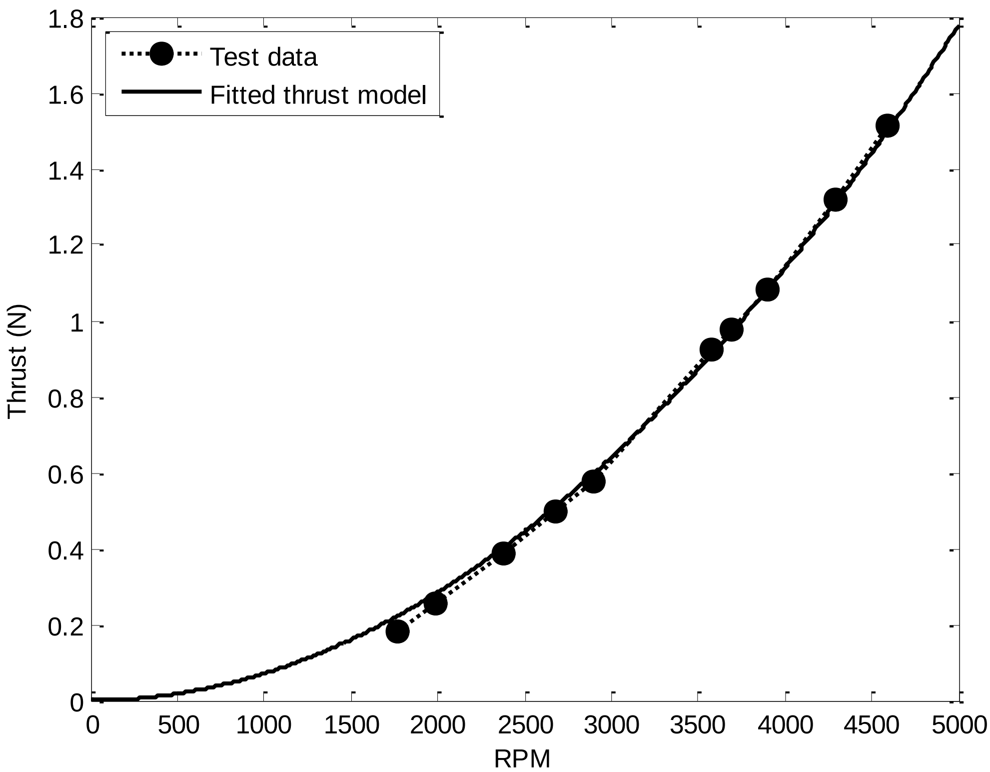

Figure 3: Thrust change w.r.t. the angular velocity of the rotor

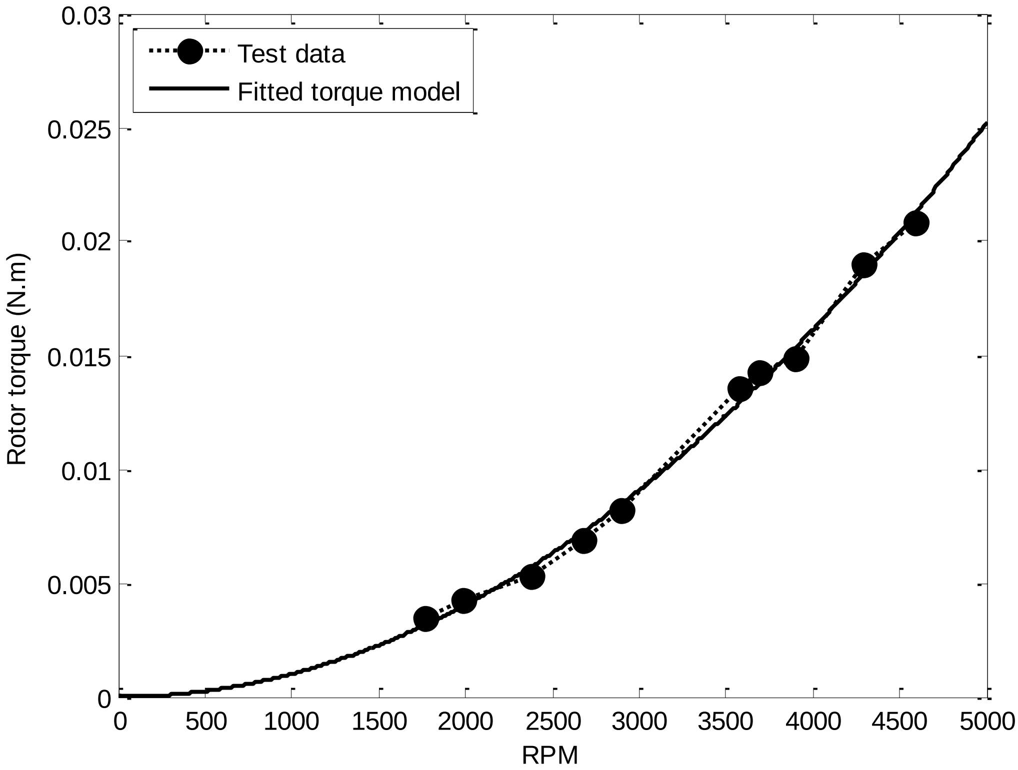

Figure 4: Rotor Torque (Drag Moment) change w.r.t. the angular velocity of the rotor

Table 3: Vertical Climb Results

Wind speed	Thrust force (@4348 RPM)	Thrust force(@4631 RPM)
1.120335	1.147	1.312
2.342962	0.901	1.070
3.761440	0.474	0.673
4.809472	-0.059	0.144
6.266468	-0.692	-0.533
7.684782	-1.675	-1.528
9.141776	-2.766	-2.641

Table 4: Parameter Estimation Results for Vertical Flight

Relation between thrust force and omega,induced velocity, climb velocity	$T = t_1 x\Omega^2 + t_2 x\mathrm{V}^2 + t_3\, x\ (\Omega x \mathrm{v}) + t_4\, x\ (\Omega x\dot{z})$ V=0 (for vertical climb) $T\ (omega = 4348) = 7.1103x10^{-8}x\Omega^2 + 1.1640x10^{-6}x\ (\Omega x \mathrm{v}) + -5.1876x10^{-5}x\ (\Omega x\dot{z})$ $T(omega = 4631) = 7.1103x10^{-8}x\Omega^2 + 7.8303x10^{-7}x\ (\Omega x \mathrm{v}) + -4.8122x10^{-5}x\ (\Omega x\dot{z})$
Thrust Coefficients for omega=4348	$t_1 = 7.1103x10^{-8}\ (N\backslash(RPM^2))$ $t_3 = 1.1640x10^{-6}\ (N\backslash(RPM^2))$ $t_4 = -5.1876x10^{-5}\ (N\backslash(RPM^2))$

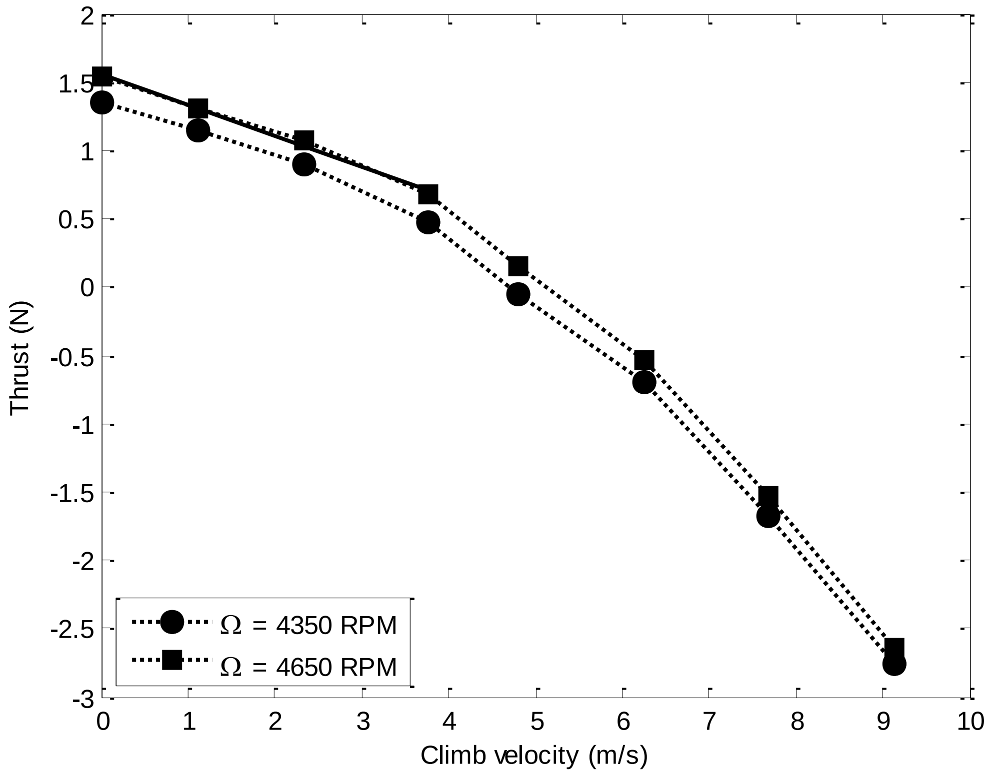

Figure 5: Thrust change w.r.t. climb velocity at constant RPMs

CONCLUSION&FUTUREWORKS

In this study, hover condition and vertical flight of a quadrotor helicopter was studied by taking into consideration of rotor aerodynamics. Firstly, a theoretical approach was conducted by using blade element theory and momentum theory. Thrust force, hub force, side force and rotor torque, rolling moment and pitching moment were derived by using blade element theory. Secondly, by use of momentum theory, induced velocity was found for hover, vertical flight, and forward flight. The quadrotor UAV was placed in the wind tunnel. Inflow velocity, rotor rpm, forces and moments were measured by using a hot wire, load cell and a rpm sensor. Through the relations between them parameters were found by use of MATLAB.
In conclusion, an empirical approach enables to get a more faithful modeling and simulation. It estimates the behavior of the vehicle precisely in accordance with a numerical study

Futurework will focus on finding parameters and doing simulation for forward flight by using the experimental values, and verification of the simulation by flight tests.

APPENDIX

Constants of quadrotor which depend on the rotor geometry and the vehicle:

Table 4: Properties of the quadrotor used in the experiments

Name	Symbol	Value	Unit
Distance between rotor axis and C.o.G	l	0.17	m
Mass of the quadrotor	m	0.48	kg
Radius of the rotor	R	0.1	m
Inertia of the rotor in x axis	$\mathrm{I_{xx}}$	5.6 x 10^{-3}	$kg\ x\ m^2$
Inertia of the rotor in y axis	$\mathrm{I_{yy}}$	5.6 x 10^{-3}	$kg\ x\ m^2$
Inertia of the rotor in z axis	$\mathrm{I_{zz}}$	8.1 x 10^{-3}	$kg\ x\ m^2$

Properties of the quadrotor used in the study:

Table 5: Type of the quadrotor

Manufacturer	Ascending Technologies GmbH
Model	Asc Tec Hummingbird
Motor type	Asc Tec X-BL 52s
Motor controller	AscTec X-BLDC
Flight control and IMU board	AscTec AutoPilot

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