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THE SIMULATION OF ATTITUDE CONTROLLER DESIGN FOR A QUADROTOR MODEL VIA SEVERAL METHODS FROM LITERATURE

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ABSTRACT

This paper includes attitude controller design strategies for a quadrotor platform. Possessing highly non-linear and unstable characteristics in flight dynamics, the quadrotor does not lose its popularity as a powerful tool among enthusiastic researchers who target at having trials with various controller ideas. The procedure is initialized with the construction of the system model relevant to the characteristics of the test bed. After the justification of the modeling phase, with the aid of sensors on the off-the-shelf platform, the controllers are designed to achieve tracking of the reference commands that contain the desired trajectories and attitudes. The research contains a general discussion over controller options, namely nonlinear dynamic inversion, model reference adaptive control and integral back-stepping control and the investigation of the trade-off between performance and robustness. Any tenacity of purpose in the state controller paves the way for more complex algorithm structures such as autonomous flight phases, obstacle avoidance and way-point targeting.

The perception of robustness contemplates the impacts on model owing to the existence of uncertain parameters or disturbances defined within some set for decisions. It is therefore another expectation that the system operates as desired when exposed to parametric uncertainties or unexpected disturbances from the exterior. This paper also includes an overview of the cases with parametric uncertainty and the existence of noise while grading the controller options.

NOMENCLATURE

p, q, r	$The\ rotational\ velocity\ components\ of\ the\ Body\ Fixed\ Frame$
u, v, w	$The\ translational\ velocity\ components\ of\ the\ Body\ Fixed\ Frame$
X, Y, Z_{BFF}	$Body\ Fixed\ Frame\ axes$
O_{BFF}:	$Origin\ of\ Body\ Fixed\ Frame$
F_i, n_i	$Thrust\ and\ rotational\ speed\ generated\ by\ the\ indicated\ propeller$
X, Y, Z_{EFF}	$Earth\ Fixed\ Frame\ axes$
ϕ, θ, ψ	$Roll, pitch\ and\ yaw\ angle$

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M_{prop}	*Propeller torque*
$M_x = L,\ M_y = M,$ $M_z = N$	*Body-fixed Moments*
r_i	*Distance from each propeller axis to center of mass*
ω,H	*angular velocity and momentum*
k_m, k_n	*Momentum and rotational speed multiplier*
L_f, L_g	*Lie derivatives*
ζ, ω_0	*Damping ratio and natural frequency*
v	*Virtual control*
I_{xx}, I_{yy}, I_{zz}	*Moments of inertia*
k_d, k_p, k_i	*Error controller gains*
$c_1, c_2, c_3, c_4, c_5, c_6,$ a_1, a_2, b_1, b_2, b_3	*IB gains and parameters*

INTRODUCTION

Burying the past for the first generation of quadrotors interfering in manned flight, this work perceives the recent generation as a key for the academic purpose of acquiring experience over unmanned flight tasks and controller algorithms. Other significant aspects to this concept are the minimal maintenance requirement of the platform and the convenience in the verification of the effectiveness of the enhanced ideas through real-time experiments.

With non-linear, but relatively simple dynamics, the quadrotor has enough characteristics to pose a meaningful challenge to controller design methods even including non-linear schemes. Besides, being substantially susceptible to wind gust disturbances, testing the disturbance related capabilities of controllers is possible as well. Therefore, this control problem presents interesting control challenges and an excellent opportunity for developing and testing new control design methodologies.

The realistic description of the system model is vital due to the fact that the reactions to given inputs designated by changes in the states should be predicted. This can be achieved by utilizing a non-linear model or, in case that the controller requires linearity, a linear model that is coherent with the non-linear one. The structure of the quadrotor points out a rigid body, the states of which form the state variables. Although the quadrotor has 6 degrees of freedom (DOF), it is equipped just with 4 control inputs represented by four propeller-motor combinations thus making it improbable to reach a desired set-point for all the DOF. Nevertheless, thanks to its unique structure, a controller may show adequate performance to allow the quadrotor to reach certain height and attitude, by conceiving the four best controllable input variables that in turn forms four fundamental movements that are throttle (standing for descent or ascent), pitch, roll and yaw.

The rotors on a quadrotor should be categorized into two groups with each group rotating in the opposite direction of the other group to form a controllable vehicle in various directions. Besides, since quadrotor is a symmetric structure in 2-axis, pitch and roll axes indicate exactly the twin of each other. Another important point about the dynamics of the quadrotor appears as couplings such as pitch angle & x-direction coupling, roll angle & y-direction coupling. The following figure shows the axis and motion definitions for an off-the-shelf quadrotor:

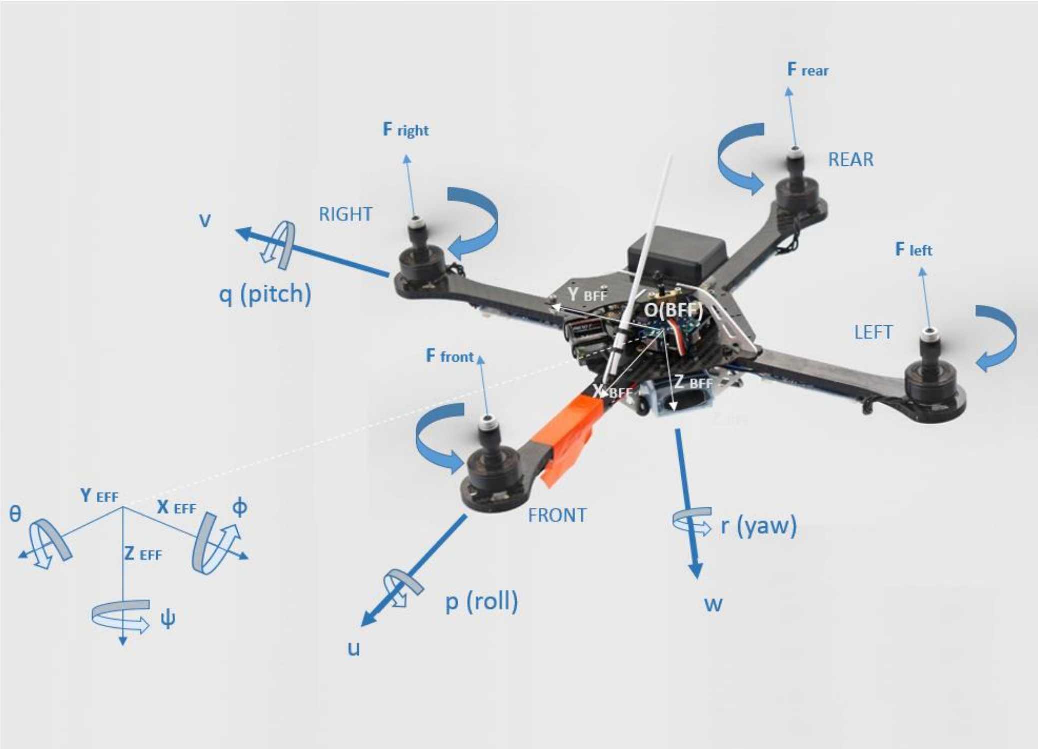


Figure 1: *The Axis & Movement Definitions for a Quadrotor*

Table 1 indicates an overview for the advantages and drawbacks of the control methods to be implemented as branched by Kendoul for rotorcrafts from the literature [Kendoul, 2010]:

Control Method	Advantages	Disadvantages
Learning-Based Control [Montgomery, 1998] [Sugeno et al., 1995]	• Model-free—flexible for implementation • Fast & reactive behavior---Allows direct mapping btw data & actuator changes	• Difficult to analyze stability and robustness • No extensive experiments over a wide range of scenarios when compared to others
Linear Flight Control [How et al.,2008] [La Civita et al., 2002]	• Most widely accepted • Straightforward design and implementation procedures. • Many tools for gain scheduling and analyzing the performance and robustness. • Successful past use in aerospace systems to achieve wide range of tasks	• Suffers from performance degradation when the rotorcraft leaves the nominal conditions or performs aggressive maneuvers. • Difficult to prove the asymptotic stability of the complete closed-loop system theoretically. • Saturations not considered • Full bandwidth and dynamics of the system not available for use

Model-Based Control [Achtelik et. al, 2010]	• An alternative for advanced flight control • Well-documented & experimented researches on nonlinear techniques with successful results • Nonlinear controllers outperform linear techniques in terms of robustness to unmodeled dynamics and disturbances, tracking accuracy over a wider flight envelope.	• The experimental results have not shown a significant progress in flying capabilities when compared to standard linear controllers. • With unknown model parameters, this imprecision also adds even more complexity to the controller. • The lack of experimental work in rigorously implementing and extensively flight testing developed algorithms.

Table 1: *Advantages and Drawbacks of Various Control Method Branches*

Table 2 includes the linear control methods exploited for quadrotor test beds in literature:

Control Method	Content of The Controller Utilized And Deductions From The Research
[Mistler et al., 2001]	• Exact Linearization and non-interacting control via dynamic feedback
[Pounds et al., 2002]	• Use of linear control for system stabilization • Double Lead Compensator and Feedback Loop Design for Pilot Augmentation Control System
[Bouabdallah et al., 2004]	• Comparison Study for LQ and PID based controllers
[Mokhtari et al., 2004]	• A dynamic feedback controller for closed loop linearization • The wind parameters estimation via Lyapunov functions
[Mokhtari et al., 2005]	• Robust Feedback Linearization and GH_{∞} linear controller
[Pounds et al., 2006]	• PID controller with full flapping model and linearization around hover state
[Benallegue et al., 2006]	• Feedback Linearization Controller with a High Order Sliding mode Observer
[Hoffman et. al, 2007]	• PID control on attitudes and feedback control on vertical acceleration to make variation in thrust diminish proportionally • Failure due to wind disturbances
[Hoffman et al., 2008]	• Attitude PID Control extended with angular acceleration feedback • Along tracking control by PI and cross tracking control by PID

Table 2: *Options of Linear Controller Methods for Quadrotors*

The demand on reaching higher performance achievement in operations such as aggressive maneuvers leads to the application of non-linear controllers on quadrotor models. Some of these non-linear controller options are sorted out of the literature as depicted in Table 3:

Control Method	Content Of The Controller Utilized And Deductions From The Research
[Altug et al., 2002]	• Feedback Linearization and Back-stepping Controller Options • Visual Feedback
[Bouabdallah et al., 2004]	• Inner/Outer Loop Control Approaches verified via use of Lyapunov theorem
[Bouabdallah et al., 2005]	• Modification for Back-stepping controller by Sliding Mode for a cascaded variable to introduce robustness
[Madani et al., 2006]	• Back-stepping Sliding Mode Controller
[Castillo et al., 2006]	• Backstepping Control with Lyapunov analysis for convergence • Comparison Study with a PD controller for aggressive perturbation response
[Morel et al., 2006]	• Adaptive algorithm for trajectory tracking with its law derived from dynamic surface control and back-stepping procedure
[Nicol et al., 2008]	• Adaptive neural network control for stabilization against uncertainties and modeling errors • Comparison Study with adaptive techniques: e-modification and dead-zone
[Raffo et al., 2009]	• Integral Predictive and Nonlinear Robust Control strategy for path following problem
[Lee et al., 2010]	• Geometric Tracking Control by Specially defined Euclidean Group
[Achtelik et al., 2010]	• The comparison of attitude controller design approaches: nonlinear dynamic inversion and model reference adaptive control (also with or without pseudo-control hedging) • Investigation of the effect of the presence of uncertain parameters
[Diao et al., 2011]	• A continuous time varying attitude controller with a Lyapunov based approach against uncertainties
[Fernando et al., 2011]	• Robust adaptive tracking control of the attitude dynamics defined with a special orthogonal group to avoid complexities and ambiguities
[Mellinger et al., 2011]	• Optimal Trajectory Generation and Nonlinear Tracking Controller
[Lee et al., 2013]	• Non-linear Controllers Introduced for Attitude and Position almost globally stable and robust to mode switching • Special Euclidean Definition for Model
[Satici et al., 2013]	• L1-optimal control of a quadrotor

Table 3: *Options of Non-Linear Controller Methods for Quadrotors*

MODELLING

System Dynamics

Every controller design process commences with analyzing the dynamics of the system that is desired to be controlled. Therefore, this section summarizes the ideology behind the construction of the system dynamics part embedded into the simulation of the quadrotor model. For a quadrotor, the dynamic model features high nonlinearities and strong couplings between its subparts. To design a controller, the specified quadrotor is mainly considered as a rigid-body evolving in 3D space generating force and torque vectors. A high-fidelity model can be divided into four subgroups:

1) *Force and torque generation:* is a process for calculating the resultant force and torque vectors experienced by the rigid-body. These force and moment vectors depend mainly on the thrust & torque generated by each rotor, propeller and motor dynamics, geometrical parameters, and the orientation of each produced thrust. Moreover, the thrust generation that keeps its significance for height and position control strategies occurs here.

2) *Rigid-body dynamics:* Generally described by the Newton-Euler equations of motion, or the energy-oriented approaches such as the Lagrange formulation, the rigid-body equations of motion can be expressed in the body frame or in the inertial frame, and can have different model structures and parameterizations. For instance, if the researcher is to deal with the attitude control, the initial point to be aware of is that the generated moments give rise to angular accelerations described by angular momentum dynamics. Eventually, the realization of angular acceleration changes in the model acts as inputs to orientaiton dynamics and cause angular attitude changes.

3) *Rotor aerodynamics and dynamics:* Includes augmentation of rigid body model augmented with simplified rotor dynamics and aerodynamics, using a combination of momentum and blade element theory for more accuracy. The produced aerodynamic forces and torques depend on operating conditions and vehicle motion. This part is excluded in this paper.

4) *Actuator dynamics:* For small-scale, it is necessary to model the dynamics of actuators to improve model fidelity, especially for aims related to simulation.

The combination of these headlines for system analysis is depicted on figure 2:

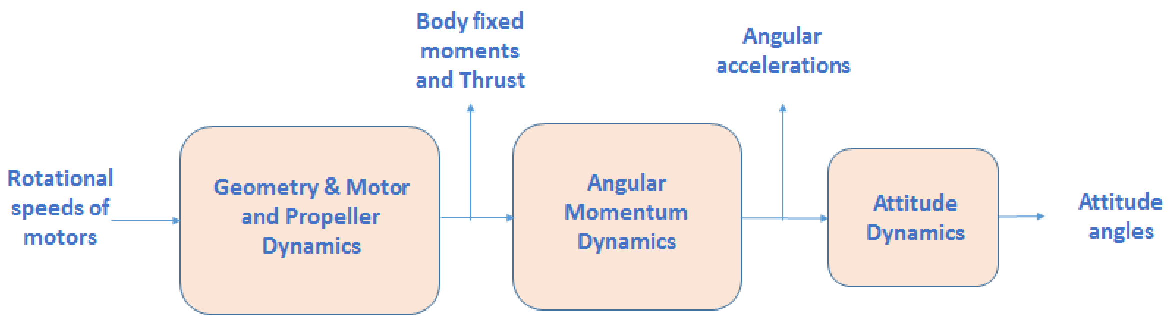


Figure 2: *Modeled Dynamics Blocks for Quadrotor*

The relations and parameters having influence on the blocks on Figure 2 can be constructed and summarized as explained and chosen in [Achtelik, 2010].

Geometry and Propeller dynamics: A static relation between propeller and motor dynamics exist. The force equation which is based upon the square root of rotation speed and propeller type is deduced from Schenk's work [Schenk, 2010]. Furthermore, assuming the relation between the thrust and torque linear by a momentum multiplier, the propeller torque can be decided.

$$F = \mathrm{k_n n^2} \ (1)$$

$$M_{prop} = k_m F \ (2)$$

Using motor thrusts, it is trivial to calculate the body-fixed moments and total vertical force throughout a geometrical organization and remembering the axis definitions on Figure 1:

$$M_i = r_i x F_i \ (3)$$

$$M = \left[M_x\ M_y\ M_z\right]^T = [\overrightarrow{r_1}x\overrightarrow{e_3}\ \overrightarrow{r_2}x\overrightarrow{e_3}\ \overrightarrow{r_3}x\overrightarrow{e_3}\ \overrightarrow{r_4}x\overrightarrow{e_3}\ ]^T[F_1F_2F_3F_4]^T$$

$$\begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix} = \left(\begin{pmatrix} r \\ 0 \\ 0 \end{pmatrix} x \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}\ \begin{pmatrix} 0 \\ r \\ 0 \end{pmatrix} x \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}\ \begin{pmatrix} -r \\ 0 \\ 0 \end{pmatrix} x \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}\ \begin{pmatrix} 0 \\ -r \\ 0 \end{pmatrix} x \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}\right) \begin{pmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{pmatrix} (4)$$

Also note that the moment originated in z-axis can be stated by propeller moment equation for each individual rotor. Including total vertical thrust to the relations is the last step of this block:

$$T_c = F_1 + F_2 + F_3 + F_4\ (5)$$

$$\begin{bmatrix} M_x \\ M_x \\ M_x \\ T_c \end{bmatrix} = \begin{bmatrix} L \\ M \\ N \\ T_c \end{bmatrix} = \begin{bmatrix} 0 & -r & 0 & r \\ r & 0 & -r & 0 \\ -k_m & k_m & -k_m & k_m \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix} = [F\ to\ M] \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix} (6)$$

Angular Momentum dynamics: Nelson states that the derivative of the angular momentum of an aircraft referred to a rotating body frame having an angular velocity ω can be represented by the following vector identity which brings the angular momentum equations out [Nelson, 1997]:

$$\left(\frac{dH}{dt}\right)_I = \left(\frac{dH}{dt}\right)_B + \omega \mathrm{x} H\ (7) \text{ where } H = \sum r\delta m \mathrm{\ x\ } V_c + \sum[r \mathrm{\ x\ } (r \mathrm{\ x\ } \omega)]\delta m = [I_B][p\ q\ r]^T\ (8)$$

$$\dot{\omega} = [\dot{p}\ \dot{q}\ \dot{r}]^T = \mathrm{I_B^{-1}}[\ M_B - \omega \mathrm{\ x\ } I_B\omega](9)$$

Attitude dynamics: Attitude propagation equations used for aerial vehicles are designated in Nelson's Flight Stability and Automatic Control using Euler angles and body fixed rotational rates as follows [Nelson, 1997]:

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & S_\Phi\ tan\theta & C_\Phi\ tan\theta \\ 0 & C_\Phi & -S_\Phi \\ 0 & S_\Phi sec\theta & C_\Phi sec\theta \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} = P(\theta, \Phi) \begin{bmatrix} p \\ q \\ r \end{bmatrix} (10)$$

$$\begin{bmatrix} \ddot{\phi} \\ \ddot{\theta} \\ \ddot{\psi} \end{bmatrix} = P(\theta, \Phi) \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} + \frac{dP(\theta, \Phi)}{\mathrm{d}\theta} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \dot{\theta} + \frac{dP(\theta, \Phi)}{\mathrm{d}\Phi} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \dot{\phi}\ (11)$$

METHODS

As mentioned in previous parts, the main focus of this paper is the discussion of various controller probabilities that may be applied to the orientation dynamics of quadrotor. This section presents the necessary knowledge to comprehend the methodologies.

Nonlinear Dynamic Inversion: To use the best of the linear and nonlinear approaches, designing attitude controller by nonlinear dynamic inversion which is the base controller of this work is an option available for selection. The nonlinear dynamic inversion can be comprehended as a tool to control a nonlinear system as if it is linear. Consider a SISO system with x being state vector and u control input as follows where f(x) may be a nonlinear function:

$$\dot{x} = \mathrm{f(x)} + \mathrm{g(x)u}\ (12)$$

Transforming into companion form where all the nonlinear terms interact only with the n^{th} state:

$$\begin{bmatrix} \dot{x}_1 \\ \vdots \\ \dot{x}_{n-1} \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} x_2 \\ \vdots \\ x_n \\ b(x) \end{bmatrix} + \begin{bmatrix} 0 \\ \vdots \\ 0 \\ a(x) \end{bmatrix} u \ (13)$$

Next define virtual control input to control the entire system in a simple linear way :

$$v = b(x) + a(x)u \Leftrightarrow u = a^{-1}(x)[v - b(x)] \ (14.a\&b)$$

State feedback is often used to set this virtual control input turning the whole system into a linear closed loop system of form:

$$v = -k_0 x - k_1 \frac{dx}{dt} - k_2 \frac{d^2x}{dt^2} - \cdots - k_{n-1} \frac{d^{n-1}x}{dt^{n-1}} = \frac{d^n x}{dt^n} \ (15)$$

$$k_0 x + k_1 \frac{dx}{dt} + k_2 \frac{d^2x}{dt^2} + \cdots + k_{n-1} \frac{d^{n-1}x}{dt^{n-1}} + \frac{d^n x}{dt^n} = 0 \ (16)$$

Closed system properties can be set by choosing right parameter values and the procedure for finding the virtual control constitutes the outer loop of Nonlinear Dynamic Inversion whereras the procedure for u and applying it to the real system is the inner loop.

Now if the next step is to give an error definition for tracking error problem as $e = x - x_d$ (17), an evident control law can be deduced:

$$v = -k_0 e - k_1 \frac{de}{dt} - k_2 \frac{d^2e}{dt^2} - \cdots - k_{n-1} \frac{d^{n-1}e}{dt^{n-1}} \ (18)$$

$$k_0 e + k_1 \frac{de}{dt} + k_2 \frac{d^2e}{dt^2} + \cdots + k_{n-1} \frac{d^{n-1}e}{dt^{n-1}} + \frac{d^n e}{dt^n} = 0 \ (19)$$

Putting a nonlinear system into companion form is another difficulty that should be solved. Input-output linearization is benefited such that until input y appears in the derivative of the output, the derivation continues. The system can be seen then as a linear system as well.

The amount of times we need to differentiate the output is called the relative degree r of the system. The order of the system is denoted by n. We always have $r \leq n$. For a condition where the relative degree is less than the order, the internal dynamics which is frankly the unobservable part of the system should be investigated carefully in terms of stability. For a linear system, internal dynamics is stable if it is minimum phase.

Lie Derivative, State Transformation: Consider a system of form where h is a scalar and f and g are vector functions: $\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{g}(\boldsymbol{x})\boldsymbol{u}; \ \boldsymbol{y} = \boldsymbol{h}(\boldsymbol{x})(\mathbf{20})$. The Lie Derivative is the gradient of a certain function of scalar origin such as h(x) as projected along a certain vector function f(x).

$$L_f h(x) = \nabla h(x) f(x) = \sum_{i=1}^{n} \frac{\partial h(x)}{\partial x_i} f_i(x) (21)$$

Applying Lie Derivative multiple times:

$$L_f^k h(x) = \mathrm{L_f}\left(\mathrm{L_f^{k-1} h(x)}\right) = \nabla\left(\mathrm{L_f^{k-1}(x) h(x)}\right) \mathrm{f(x)} \ (22) v \ with \ L_f^0 h(x) = h(x) (22)$$

Lie Derivative is applied for state transformation going from old state x to new state z. Define z:

$$z_i = \phi_i(x) = L_f^{i-1} h(x) \ (23) \ swith \ i \ \ such \ that \ 1 \leq i \leq r$$

$$L_g \phi_i(x) = 0 \ if \ 1 \leq i < r \ (24)$$

$$L_g \phi_i(x) \neq 0 \ if \ i = r \ \ (25)$$

The whole state transformation is then: $z = \phi(x) \Leftrightarrow x = \phi^{-1}(z)$ $(26.a\&b)$

- For $1 \leq i < r$, the new coordiantes satisfy:

$\dot{z}_1 = z_2, \dots, \dot{z}_{r-1} = z_r, \dot{z}_r = a(z) + b(z)u$ where $a(z) = L_g L_f^{r-1} h(x) = L_g z_r$; $b(z) = L_f^r h(x)$ (27)

- For $r + 1 \leq i < n$, the new coordiantes satisfy:

$$\dot{z}_i = L_f \phi_i(x) + L_g \phi_i(x) u = L_f \phi_i(x) = L_f z_i \text{ where } L_g \phi_i(x) = 0 \quad (28)$$

$$a(z) + b(z)u = \frac{dz_r}{dt} = \frac{d^2 z_{r-1}}{dt^2} = \cdots = \frac{d^r z_1}{dt^r} = \frac{d^r y}{dt^r} \quad (29)$$

The system is now in canonical form and functions a and b are the same as in input-output linearization. The remaining states from z_{r+1} to z_n do not have influence on the output. Such an approach clearly seperates the observable and unobservable parts. Next step here is to generalize ideas to MIMO systems in the form:$\dot{x} = f(x) + g(x)u; y = h(x)$ (30)

Note that state derivatives affinely depends on the input u. The input vector u has size m while the output y has size p. In MIMO systems, there are individual relative degrees (i.e. $r_1 \dots r_p$). Besides, the following relations hold true for all i such that $1 \leq i \leq p$:

$$r = \mathrm{r}_1 + \cdots + r_p \leq n \quad (31)$$

$$\phi_j^i(x) = L_f^{j-1} h_i(x) \quad (32)$$

$$\dot{\phi}_1^i(x) = \phi_2^i(x); \dots; \dot{\phi}_{r_i-1}^i(x) = \phi_{r_i}^i(x) \quad (33)$$

$$\dot{\phi}_{r_i}^i(x) = L_f^{r_i} h_i(x) + \sum_{j=1}^{m} L_{g_j} L_f^{r_i-1} h_i(x) u_j \quad (34)$$

The eventual step of this method is to derive the expression for virtual input which is now of size p:

$$v = b(x) + A(x)u = \begin{bmatrix} L_f^{r_1} h_1(x) \\ L_f^{r_2} h_2(x) \\ \vdots \\ L_f^{r_p} h_p(x) \end{bmatrix} + \begin{bmatrix} L_{g1} L_f^{r_1-1} h_1(x) & L_{g2} L_f^{r_1-1} h_1(x) & \dots & L_{gm} L_f^{r_1-1} h_1(x) \\ L_{g1} L_f^{r_2-1} h_2(x) & L_{g2} L_f^{r_2-1} h_2(x) & \cdots & L_{gm} L_f^{r_2-1} h_2(x) \\ \vdots & \vdots & \ddots & \vdots \\ L_{g1} L_f^{r_p-1} h_p(x) & L_{g2} L_f^{r_p-1} h_p(x) & \dots & L_{gm} L_f^{r_p-1} h_p(x) \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_m \end{bmatrix} \quad (35)$$

Solving for u: $$u = A^{-1}(x)[v - b(x)] \ (36)$$

The next task to be completed is defining the states to control $x = (\phi\ \dot{\phi}\ \theta\ \dot{\theta}\ \psi\ \dot{\psi})$ and the alternative controls as $(v_1 v_2 v_3) = (\dot{p}_{cmd} \dot{q}_{cmd} \dot{r}_{cmd})$ due to the fact that there is no directimpact of motor commands on attitude dynamics. Applying this procedure to the given model, we have relative degree 2 for 3 states which sums up to 6, exactly the number of states chosen. Therefore, it is concluded that the sytem has no internal dynamics. With these definitions, the following equation is now attained and the system is ready to be inverted: $\ddot{y} = F(x, v) = b(x) + A(x)v$ (37). Before starting inversion, define pseudo controls as:

$$\left[\ddot{\phi}_{ps} \ddot{\theta}_{ps} \ddot{\psi}_{ps}\right]^T = [\ddot{y}_1\ \ddot{y}_2\ \ddot{y}_3]^T = [v_1 v_2 v_3]^T = v^T \quad (38)$$

The alternative control is now $v = A^{-1}(x)[v - b(x)]$ as required by the method. Then the inversion of momentum dynamics and geometry/propeller dynamics are achieved as indicated below:

$$M_B = I_B v + \omega \mathrm{x} I_B \omega - M_{0_B} (39)$$

$$[F_1 F_2 F_3 F_4] = [F\ to\ M]^{-1} [L\ M\ N\ T_c]^T \quad (40)$$

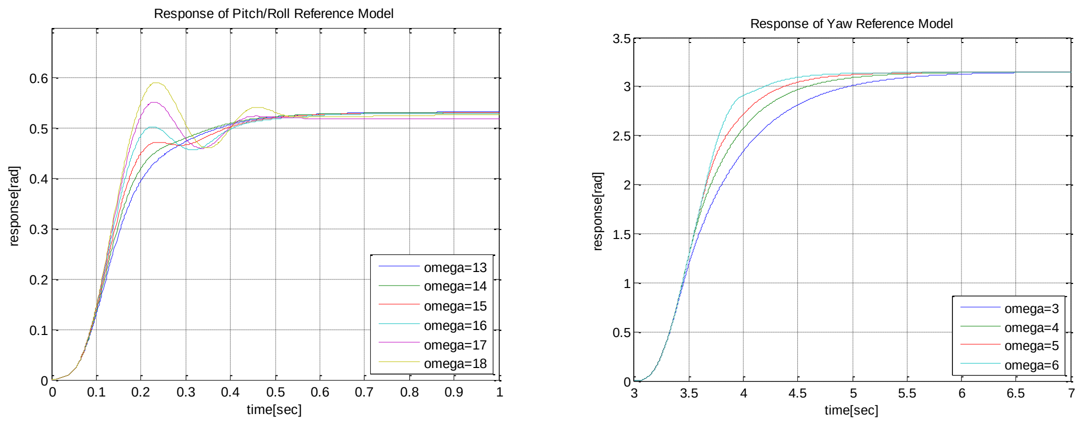
$$n_i = \frac{\sqrt{F_i}}{k} \quad (41)$$

Simple second order linear models are used so as to generate reference trackings of control variables. The following equations consititute these pseudo control commands:

$$\begin{bmatrix}\ddot{\phi}_{ref}\\ \ddot{\theta}_{ref}\\ \ddot{\psi}_{ref}\end{bmatrix} = \begin{bmatrix}\omega_{0_\Phi}^{\,2} & 0 & 0\\ 0 & \omega_{0_\theta}^{\,2} & 0\\ 0 & 0 & \omega_{0_\psi}^{\,2}\end{bmatrix}\left[\begin{bmatrix}\phi\\ \theta\\ \psi\end{bmatrix}_{cmd} - \begin{bmatrix}\phi\\ \theta\\ \psi\end{bmatrix}_{ref}\right] - \begin{bmatrix}2\omega_{0_\Phi}\zeta_\phi & 0 & 0\\ 0 & 2\omega_{0_\theta}\zeta_\theta & 0\\ 0 & 0 & 2\omega_{0_\psi}\zeta_\psi\end{bmatrix}\begin{bmatrix}\dot{\phi}\\ \dot{\theta}\\ \dot{\psi}\end{bmatrix}_{ref} \quad (42)$$

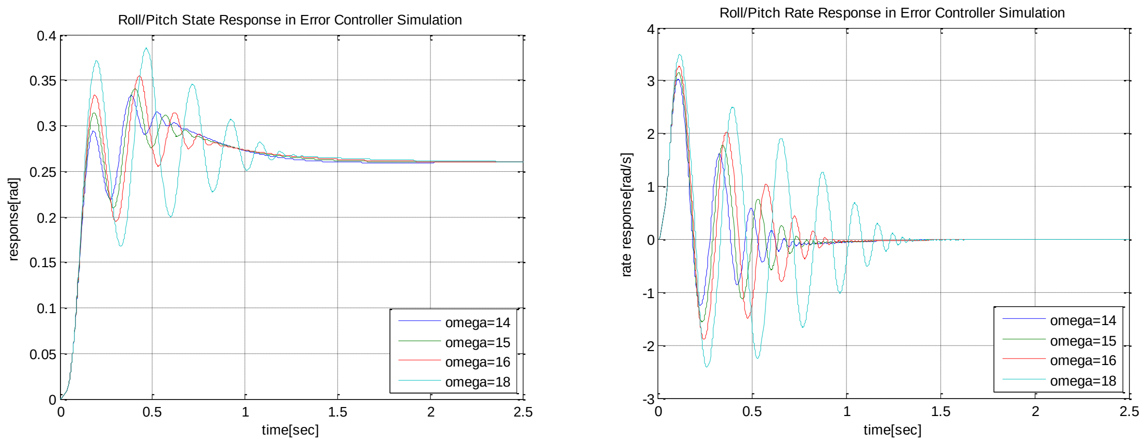
A stabilizing error controller is a must for such an unstable system with two integrators at output level that create a deviation between the fed reference value and the result in addition to the difference between pseudo controls and second derivative of outputs caused by uncertainties, modeling errors or disturbances. Th control error is described as the subtraction of sensed value from the reference value for outputs and their time derivatives. Eventually, the modeling error is $\Delta_i(x) = \ddot{y}_i - \nu_i$ (43) and the pseudo control for error is: $\nu_i = \ddot{y}_{i,ref} + k_d\dot{e}_i + k_p e_i + \frac{k_i e_i}{s}$ (44) where $k_p = \omega_0^2$; $k_d = 2\zeta\omega_0$ (45).

In order to determine how to choose natural frequencies, various trials are conducted for an assumed critically damped system. Examining responses of various attempts with natural frequency change, the natural frequencies of the state reference models are chosen based on the figure shown below for pitch, roll and yaw:

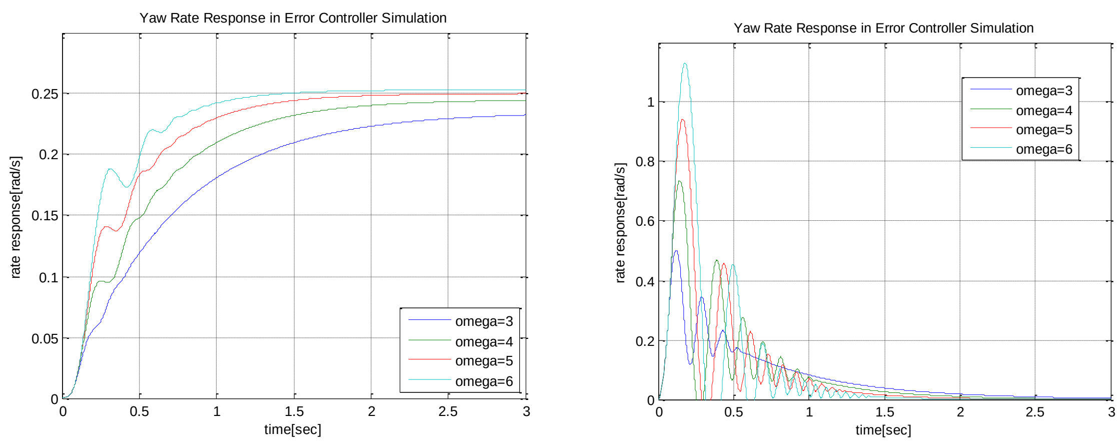


Figures 3.a and 3.b: *Reference Model Responses for Various Natural Frequency Values for: a) Pitch & Roll b) Yaw*

Additionally, the responses of error controllers are significant in determining reference model natural frequencies due to the fact that the gains of error controllers are directly affected by the selection for natural frequency and damping ratio values. The following figures include this consideration for roll, yaw and pitch state with their rates simulated for several values of natural frequencies. Note that these figures include the state error simulated as step input with a specified error value:



Figures 4.a and 4.b: *Responses of States in Error Controllers with Various Natural Frequency Values: a) Pitch & Roll b) Rates of Pitch & Roll*



Figures 5.a and 5.b: *Responses of States in Error Controllers with Various Natural Frequency Values: a) Yaw b) Yaw Rate*

Pseudo Control Hedging: Actuator dynamics are utilized in this work to compute parameters for reference model and error controller. Nevertheless, although the dynamic inversion and control allocation parts of the work have nonchanging actuator and propeller dynamics, saturations cannot be neglected. The deviation between reference and plant outputs stems from the actuator dynamics as well. This deviation can be added to the modeling error in error dynamics and represented as

$$v_{hedge} = v(x,u) - \hat{v}(x,u_c) \text{ and } v_i = \ddot{y}_{i,ref} + v_{i,hedge} \quad (46)$$

The main reason for utilizing pseudo control hedging approach is that in the presence of an integrator in error control, it can prevent the integrator wind-ups by hiding actuator dynamics from the integrators. On the other hand, the disadvantage is the fact that stability of reference model cannot be guaranteed by appropriate parameter choices since the dynamic behaviour of reference model is ot only affected by reference model but also by hedging signal. All these reasons drive the researcher to extend the work with this method developed by Johnson [Johnson, 2000].

Model Reference Adaptive Control (MRAC): Geometry or system based parameters of quadrotor like moment of inertia, motor and propeller related values require exhausting identification and experimentation work. Hence, a controller with the competence to identify the unknown parameters or resist parameter changes may provide robustness by adding adaptive elements using system inputs and the controller error. The direct approach for MRACs adapts the controller to compensate for the unknown plant parameters and the derivation of these adaptations and the stability of closed loop are achieved through Lyapunov analysis. The general trend in the application to quadrotor is that the plant dynamics is allocated into well-known nonlinear and unknown parameters which are often quite expensive to obtain through experiments for small scale UAVs and thus matched by adaptive methods. For this specific research, using the analogy conducted by Achtelik, the controller is separated into two parts, namely, Adaptive Rate and Attitude Controller loops. Figure 6 indicates this cascaded controller structure [Achtelik, 2010]:

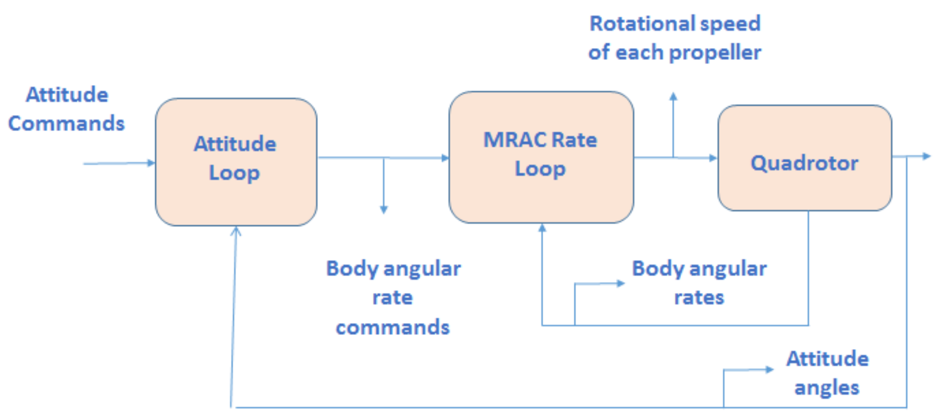


Figure 6: *Modified Controller Structure for Adaptive Laws*

The parameters including uncertainties are a part of angular momentum dynamics whereas the attitude loop is composed only of trigonometric operations. The plant dynamics can be treated as $\dot{x}_p = A_p x_p + f^*(x_p, u)$ (47)where u is the system inputs, A_p is the linear part of uncertain dynamics and $f^*(x_p, u)$ includes the geometric relations, moments of inertia and nonlinear rotational forces. This methodology brings us to the relation $f^*(x_p, u) = B_p' u + f^{**}(x_p)$ where $B_p' = B_p \Lambda$ (48). The nonlinear part denoted by $f^{**}(x_p)$ can be simplified as $\propto_p f(x_p)$where $\propto$is the matrix including inertia multiplication terms and $f(x_p)$ is the cross multiplication vector of body angular rates. Besides, defining a constant disturbance d such that $d = D * i$ (49) where i is defined as a direction vector with the same identity magnitude in all directions, the state equation becomes $\dot{x}_p = A_p x_p + B_p \Lambda u + \propto_p f(x_p) + d$ (50). The uncertain parameters in this analogy are $A_p, \propto_p$ *and* d which should be compensated or identified by an adaptive control law so as to finally make plant behavior match reference dynamics. As an additional note, such an approach requires including slower reference dynamics than that of plant apparently due to the applicability of the analogy.

The next step to leap within the scopes of this second control approach is to define the adaptive gains in the following statement: $u = \Theta_x x_p + \Theta_r r + \Theta_\alpha f(x_p) + \Theta_d i$ (51) where $\boldsymbol{r}$ are the commands of body angular rates and $\boldsymbol{\Theta_{x,r,d,\alpha}}$are adaptive gains. The closed loop of MRAC structure can immediately be stated as$\dot{x}_p = (A_p + B_p \Lambda \Theta_x) x_p + B_p \Lambda \Theta_r r + (B_p \Lambda \Theta_\alpha + \propto_p) f(x_p) + (B_p \Lambda \Theta_d + D) i$ (52). The extended rate adaptive controller structure can be investigated in the describing figure of number 4 as visualized in [Achtelik, 2010]:

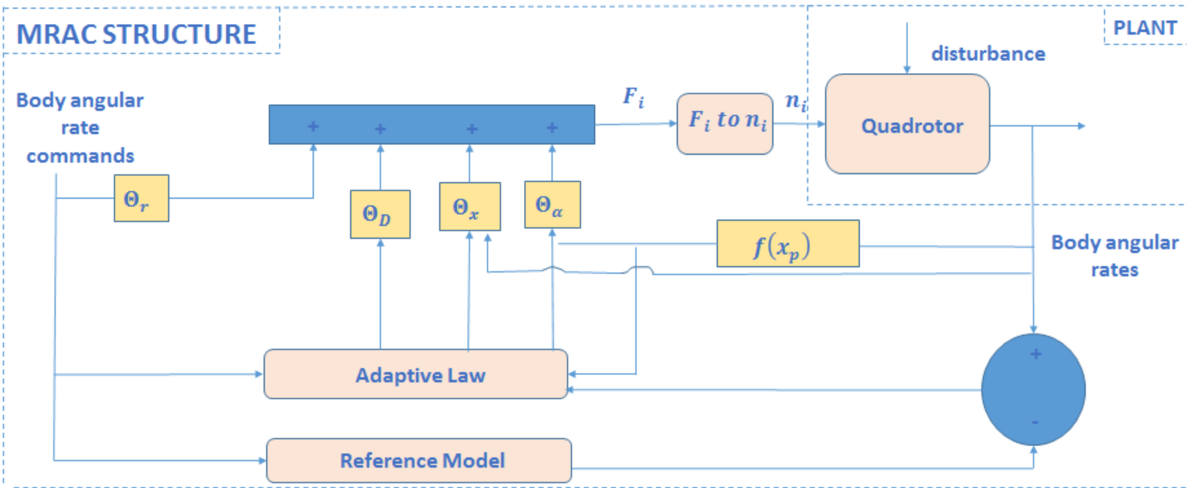


Figure 7: *Model Reference Adaptive Controller Structure with Adaptive Gains*

As observed in the content of MRAC structure, the reference model is in the form $\dot{x}_M = A_M x_M + B_M r$ (53) . Based on the matching condition, the error dynamics can be summarized such that $\dot{e}_c = A_m e_c + B_P \Lambda \tilde{\Theta}_x x_p + B_p \Lambda \tilde{\Theta}_r r + B_p \Lambda \tilde{\Theta}_\propto f(x_p) + B_p \Lambda \tilde{\Theta}_d i$ (54) where $\tilde{\Theta}$ is the distance to the ideal parameters and e_c is the control error.

Remaining as a vital question to be responded to as in all the control problems, the stability issue needs to be examined. Lyapunov's second method [Lyapunov, 1892] known as Direct Method resides in the control engineer's hand as a powerful tool to complete the stability analysis of nonlinear and adaptive control techniques explained in [Slotine & Li, 1991]. This method takes the advantage of using a function V(x), so called "Lyapunov function". In conformity with the theorem, notice an autonomous nonlinear dynamical system expressed as $\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}(\boldsymbol{t}))$, $\boldsymbol{x} = \boldsymbol{0}$ being an equilibrium point. $\boldsymbol{V}(\boldsymbol{x}): \mathbb{R}^n \to \mathbb{R}\ \boldsymbol{s.t.}$

$$\mathbf{1)}\ \boldsymbol{V}(\boldsymbol{x}) \geq \mathbf{0}\ \boldsymbol{iff}\ \boldsymbol{x} = \mathbf{0}\ (\boldsymbol{positive\ definiteness})\ (55)$$

$$\mathbf{2.1)}\dot{\boldsymbol{V}}(\boldsymbol{x}) = \frac{\boldsymbol{dV}(\boldsymbol{x})}{\boldsymbol{dt}} \leq \mathbf{0}\ \boldsymbol{iff}\ \boldsymbol{x} = \mathbf{0}\ (\boldsymbol{negative\ semidefiniteness\ and\ stable\ case})\ \ (\mathbf{56})\ \boldsymbol{or}$$

$$\mathbf{2.2)}\dot{\boldsymbol{V}}(\boldsymbol{x}) = \frac{\boldsymbol{dV}(\boldsymbol{x})}{\boldsymbol{dt}} < \mathbf{0}\ \boldsymbol{iff}\ \boldsymbol{x} = \mathbf{0}\ (\boldsymbol{negative\ definiteness\ and\ asym.stable\ case})\ (\mathbf{57})$$

Every function satisfying the 1st and one option of the 2nd statement is a Lyapunov function. (1892) Applying the methodology to the study case, choose a Lyapunov function of type $V =$

$\frac{1}{2}x_1^T A x_1 + \ldots + \frac{1}{2}Tr[x_2^T B x_2]$ (58) used in adaptive systems. Utilizing the same Lyapunov function chosen by Achtelik based on Narendra and TUM DI course, the following parameter update laws are attained with the assumption of constant ideal parameters [Narendra, 1989]:

$$\dot{\Theta}_x = -\Gamma_x B_p^T P e_c x_p^T \ (59)$$
$$\dot{\Theta}_r = -\Gamma_r B_p^T P e_c r^T \ (60)$$
$$\dot{\Theta}_\propto = -\Gamma_\propto B_p^T P e_c f(x_p)^T \ (61)$$
$$\dot{\Theta}_d = -\Gamma_d B_p^T P e_c i \ (62)$$

where $\Gamma_\propto, \Gamma_x, \Gamma_d, \Gamma_r$ are positive definite adaptation rate matrices and P is a symmetric positive definite matrix. Also note that the Lyapunov equation states that $A_M^T P + P A_M = -Q_0$ where Q_0 is positive definite and symmetric. Experimenting with $Q_0, P\ and\ \Gamma_\propto, \Gamma_x, \Gamma_d, \Gamma_r$, the adaptive law can be adjusted as desired. Using Barbalat's Lemma [Slotine & Li,1991], it can be seen that the convergence of error control to zero is satisfied, therefore the error dynamics are asymptotically stable; however, the adaptive parameters do not converge to their true values.

When it comes to the attitude loop part, the nonliner dynamic inversion method is again chosen. Defining pseudo control, ν, angular rate commands, u_c and using attitude dynamics, state feedback linearization is achieved:

$$[p_c\ q_c\ r_c]^T = P^{-1}(\theta, \Phi)\left[\dot{\phi}_{ps}\dot{\theta}_{ps}\dot{\psi}_{ps}\right]^T \Rightarrow u_c = P^{-1}(\theta, \Phi)\nu \ \ (63)$$

Hereby, the system states and outputs are the Euler angles and their first derivatives are in influence of inputs, meaning relative degree of one for three states which is just equal to the number of the states. Therefore, there is no internal dynamics. Although the relative degree of attitude loop is one, the quadrotor has a relative degree of two as can be remembered resulting in the decision to choose a second order reference model for reasoning. The error defined in this part is the difference between the outputs, $\dot{y}$ and the pseudo controls, ν. The fact that propogating this error through one integrator results in a deviation between the command reference and output requires stabilization of error dynamics. This is achieved by following controller: $\nu_i = \dot{y}_{i,ref} + k_p e_i + k_i e_i / s$ (64). Finally, the pseudo control hedging is implemented into this controller as well by adding the hedge control term to the first derivative of the output signal.

Integral Back-stepping: Integral backstepping design approach was first proposed by [Kanellakopoulos, 1993] and put into application by [Tan et. al, 2000]. The attitude controller designed with this manner in this study stems from the interpretation of Bouabdallah. In this option, asymptotic stability is guaranteed and some robustness to uncertainties is achieved. Moreover, the integral action deals with the steady state error accumulation. Firstly, error dynamics can be represented for sample roll state, $\emptyset$ as : $\boldsymbol{e_1} = \boldsymbol{\emptyset_d} - \emptyset$ and $\frac{\boldsymbol{de_1}}{\boldsymbol{dt}} = \dot{\boldsymbol{\emptyset}}_{\boldsymbol{d}} - \boldsymbol{\omega_x}$ (65) where $\boldsymbol{\omega_x}$is angular speed in inertia x direction. Therefore, virtual control can be designed for desired behaviour: $\boldsymbol{\omega_{x_d}} = \boldsymbol{c_1 e_1} + \dot{\boldsymbol{\emptyset}}_{\boldsymbol{d}} + \boldsymbol{\lambda_1 \kappa_1}$ (66) with $\boldsymbol{c_1}, \boldsymbol{\lambda_1}$positive constants and the integral of state error, $\boldsymbol{\kappa_1} = \int_0^t \boldsymbol{e_1}(\boldsymbol{\tau})\boldsymbol{d\tau}$ $(\mathbf{67})$. Here, $\boldsymbol{\omega_{x_d}}$ is another term possessing the error dynamics of angular velocity tracking as well which can be defined as:

$$e_2 = \omega_{x_d} - \omega_x \ (68)$$
$$\frac{de_2}{dt} = c_1(\dot{\emptyset}_d - \omega_x) + \ddot{\emptyset}_d + \lambda_1 e_1 - \ddot{\emptyset} \ (69)$$

Using this knowledge of error definitions, above dynamics can be expressed with these:

$$\frac{de_1}{dt} = -c_1 e_1 - \lambda_1 \kappa_1 + e_2 (70)$$

$$\frac{de_2}{dt} = c_1(\dot{\emptyset}_d - \omega_x) + \ddot{\emptyset}_d + \lambda_1 e_1 - \dot{\theta}\dot{\psi}a_1 - \dot{\theta}\Omega a_2 - b_1 U_{roll}$$
$$= c_1(-c_1 e_1 - \lambda_1 \kappa_1 + e_2)\ddot{\emptyset}_d + \lambda_1 e_1 - \dot{\theta}\dot{\psi}a_1 - \dot{\theta}\Omega a_2 - b_1 U_{roll} (71)$$
$$\frac{de_2}{dt}_{desired} = -c_2 e_2 - e_1 \; (72)$$

If this equation is equalized to the desired dynamics for roll tracking; the control input is:

$$c_1(-c_1 e_1 - \lambda_1 \kappa_1 + e_2)\ddot{\emptyset}_d + \lambda_1 e_1 - \dot{\theta}\dot{\psi}a_1 - \dot{\theta}\Omega a_2 - b_1 = -c_2 e_2 - e_1$$

$$U_{roll} = 1/b_1\left[(1 - c_1^2 + \lambda_1)e_1 + (c_1 + c_2)e_2 - c_1\lambda_1\kappa_1 + \ddot{\emptyset}_d - \dot{\theta}\dot{\psi}a_1 - \dot{\theta}\Omega a_2\right] (73)$$

$$where\ c_2 angular\ speed\ loop\ convergence\ constant$$
$$a_1 = \left(I_{yy} - I_{zz}\right)/I_{xx}$$
$$a_2 = J_r/I_{xx}\ ;\ b_1 = 1/I_{xx}$$
$$\Omega\text{: propeller angular speed}$$

Using the same analogy, pitch and yaw inputs can be generated:

$$U_{pitch} = 1/b_2\left[(1 - c_3^2 + \lambda_2)e_3 + (c_3 + c_4)e_4 - c_3\lambda_2\kappa_2 + \ddot{\theta}_d - \dot{\emptyset}\dot{\psi}a_3 + \dot{\emptyset}\Omega a_4\right] (74)$$
$$U_{yaw} = 1/b_3[(1 - c_5^2 + \lambda_3)e_5 + (c_5 + c_6)e_6 - c_5\lambda_3\kappa_3] (75)$$

$$where\ c_3, c_4, c_5, c_6, \lambda_2, \lambda_3\ are\ positive\ constants\ and\ \kappa_2, \kappa_3\ are\ the\ integral\ track\ errors\ \ of\ other\ states$$

In similarity with the concept summarized in the first controller method, this method requires a pre-work for gain adjustments. As a sample of this pre-work, following graphs highlight the state responses for various gain values:

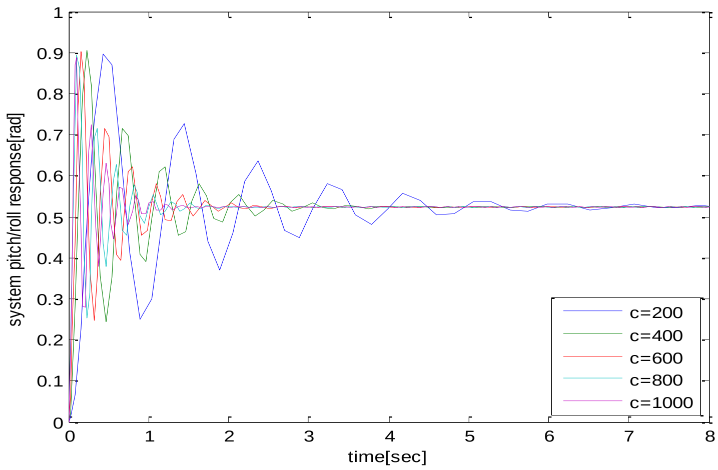


Figure 8: *Pitch & Roll Reference Model Responses for Various IB Gain Values*

SIMULATIONS & DISCUSSION

This section provides the results of the simulations for the attitude control design procedure on AscTec's Hummingbird Quadrotor [AscTec Autopilot Manual,2009] which is the main focus of this work including the methods, Nonlinear Dynamic Inversion Controller (NDI), Model Reference Adaptive Controller (MRAC) and Integral Backstepping Controller (IB) mainly constructed by the explanations stemming from the research, respectively, first two conducted by [Achtelik, 2010] and the last by [Bouabdallah, 2004, 2005] as mentioned before. This part includes the discussion for the presented controllers with constructed noisy measurements which are essential for reflecting the effect of real sensors as well. The properties of AscTec's Hummingbird are given as in the manual [AscTec Autopilot Manual, 2009]:

$$I_{xx} = I_{yy} = 5.6 * 10^{-3}$$
$$I_{zz} = 8.1 * 10^{-3}$$
$$m = 0.48\ kg$$

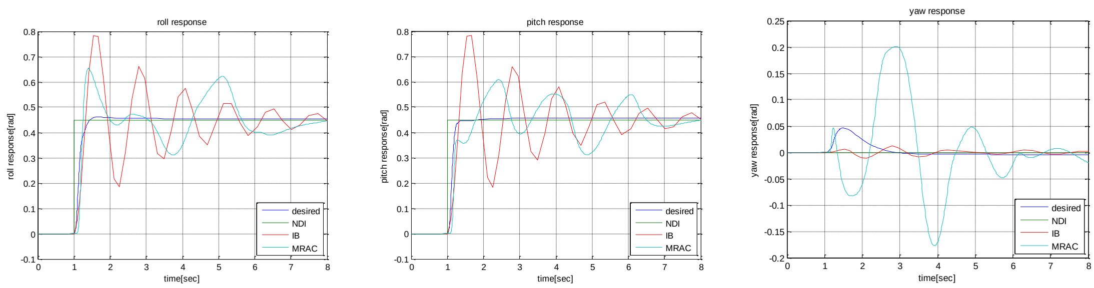
In the simulations, noise is modeled as Gaussian acting on angular readings additively. The statistics of the additive Gaussian white noise used to disrupt the measurement signals are at level of 10^{-6} for attitude angles and 10^{-2} for Euler and body angular rates.

Two command types are defined as base for the simulations. First, a desired constant orientation command type and then a sinusoidal tracking command are given to the system. Step inputs of pi/7 for both roll and pitch commands and zero for yaw command are fed to the code in the constant angular attitude control part. Eventually, the sinusoidal tracking part includes a track command of sine type in the roll state. Due to the fact that quadrotor suffers uncertainties in parameters and measurement noise, the system is simulated not only with the additive noise but also with parameter changes. Two scenarios are performed for the parameter uncertainty. While Case 1 contains a slight change in moments of inertia and propeller distances from center of mass, case 2 includes the insertion of the parameters of an entirely different quadrotor model into the code.

The results of simulations for the case when there are no measurement noises are given in the following figures for each controller. The first three graphs indicate the states and the latter three graphs show the error accumulation.

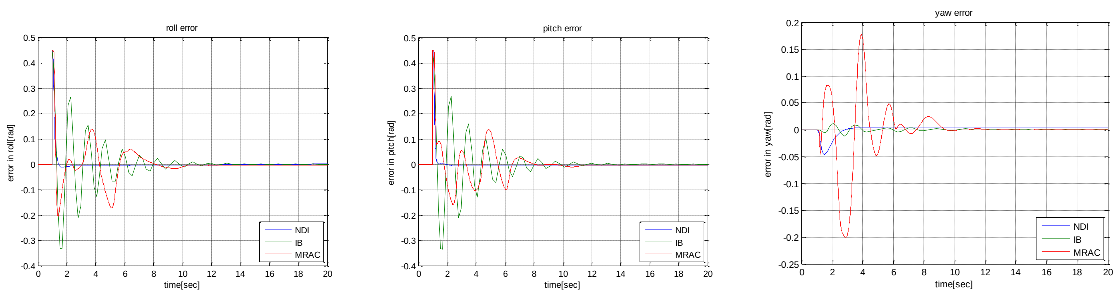
Fixed Attitude Motion Control on Roll and Pitch

This subsection consists of the results attained with first command type:



(a) (b) (c)

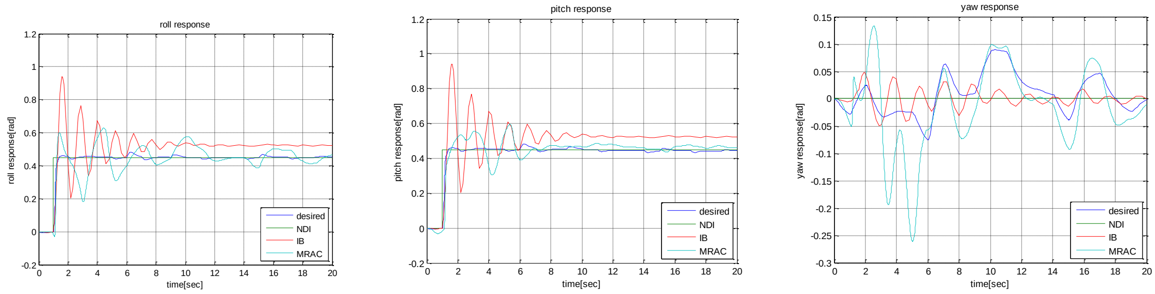
Figures 9.a, 9.b and 9.c: *Simulation of fixed angular orientation control with no noise: Roll State Response (a), Pitch State Response (b) and Yaw State Response (c)*



(d) (e) (f)

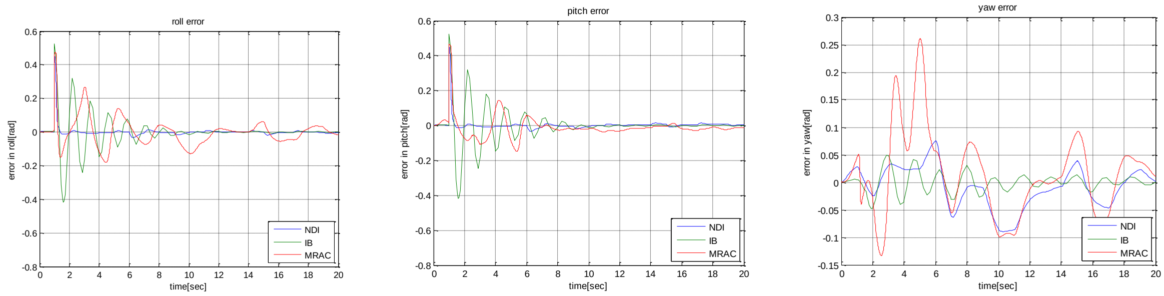
Figures 9.d, 9.e and 9.f: *Simulation of fixed angular orientation control with no noise: Roll Error (d), Pitch Error (e) and Yaw Error (f)*

Investigating the cases 9d, 9e and 9f, it is clear that the mean values of the errors reach a constant value that is slightly offset from zero. The figures from 10a to 10f indicate the results with the addition of Gaussian white noise, the variances of which are defined above. When noise is added, the results follow the trends below:



(a) (b) (c)

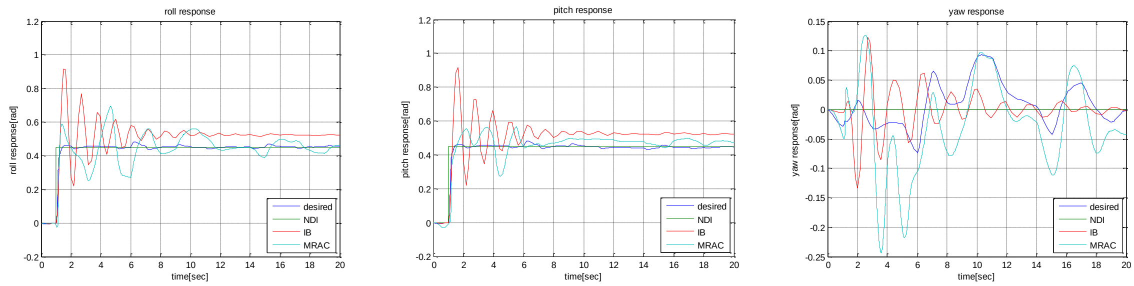
Figures 10.a, 10.b and 10.c: *Simulation of fixed angular orientation control with noise addition Roll State Response (a), Pitch State Response (b) and Yaw State Response (c)*



(d) (e) (f)

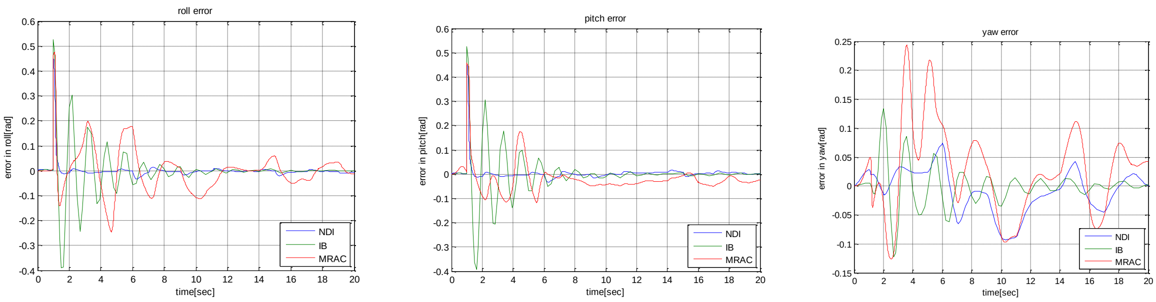
Figures 10.d, 10.e and 10.f: *Simulation of fixed angular orientation control with noise addition Roll Error (d), Pitch Error (e) and Yaw Error (f)*

NDI performs the best for initial analysis above; however, both IB and MRAC serves quite acceptable as well. As another trial, it is a wise decision to apply a change of inertia values to the system which means a slightly wrong modeling with noisy measurements from sensors. The following graphs show these slight uncertainty responses:



(a) (b) (c)

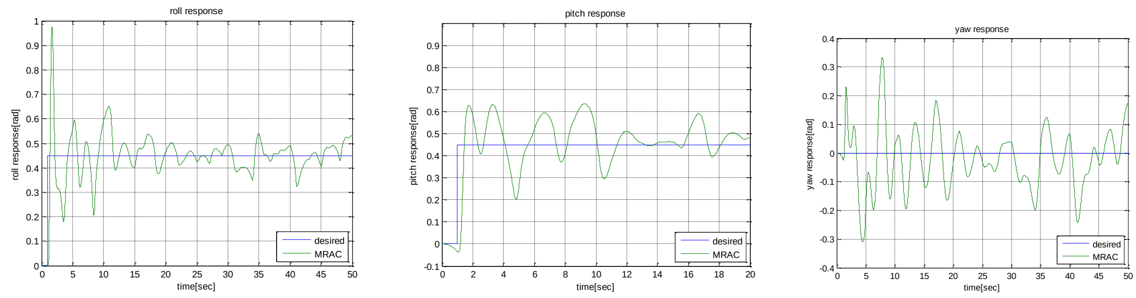
Figures 11.a, 11.b and 11.c: *Simulation of fixed angular orientation control with slight parameter uncertainty*: *Roll State Response (a), Pitch State Response (b) and Yaw State Response (c)*



(d) (e) (f)

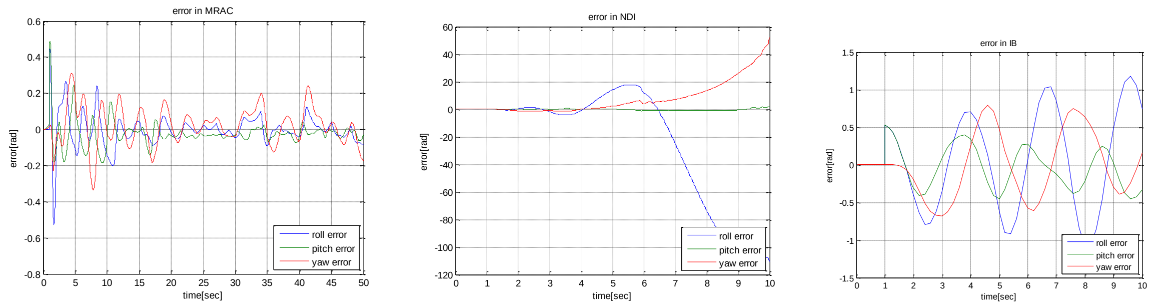
Figures 11.d, 11.e and 11.f: *Simulation of fixed angular orientation control with slight parameter uncertainty*: *Roll Error (d), Pitch Error (e) and Yaw Error (f)*

All three options do not lose their functionality in the presence of slight modeling errors. NDI appears to be the fastest method to converge again. Eventually, for the set point command type, the properties of the quadrotor model block in simulation are filled with that of a quadrotor model other than Humming Bird [Satıcı et al., 2013]. The new model consists of symmetric moments of inertia with values nearly 6 times the previous model for x & y and 4 times for z. The results clearly point out that in the existence of serious uncertainties, IB and NDI controllers fail while, as expected, MRAC serves the purpose of adaptation as requested.



(a) (b) (c)

Figures 11.a, 11.b and 11.c: *Simulation of fixed angular orientation control with a different quadrotor model*: *Roll State Response (a) Pitch State Response (b) Yaw State Response (c) in MRAC*



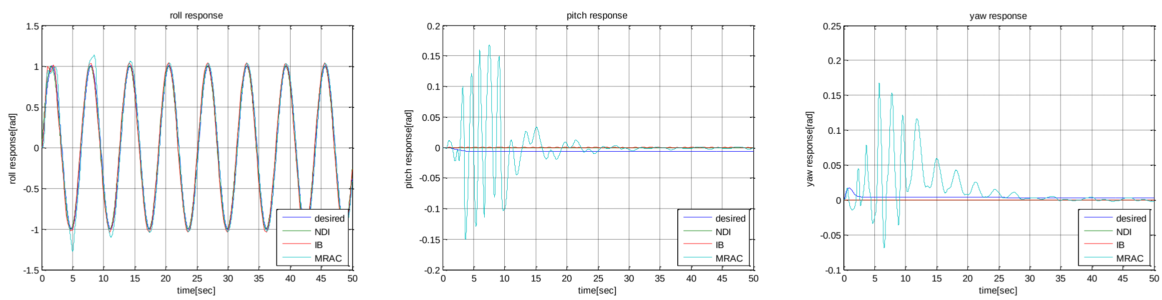
(d) (e) (f)

Figures 11.d, 11.e and 11.f: *Simulation of fixed angular orientation control with a different quadrotor model*: *State Errors in MRAC (d), NDI (e) and IB (f)*

As figures 11d to 11f introduce, solely MRAC does catch the dynamics of real system and keeps it under control whereas both NDI and IB controllers lose their applicability. The attitude angle responses of system with MRAC can be seen on figures 11a, 11b and 11c. This result is confirmation of the validity of adaptive strategy presented. Moreover, although initial roll and pitch errors in system nearly exceeds 0.4 radians, it then remains within an error corridor of magnitude 0.2 radians.

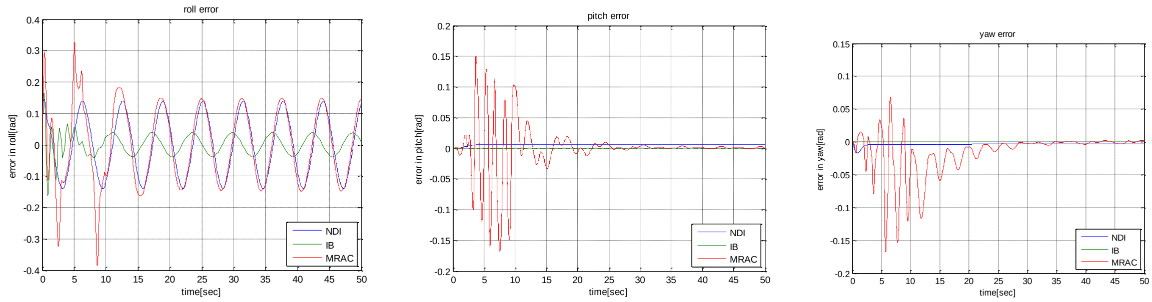
Sinusoidal Motion on Roll

The second input type is a desired sine wave motion in roll state. The second command type analyses are achieved under this subsection:



(a) (b) (c)

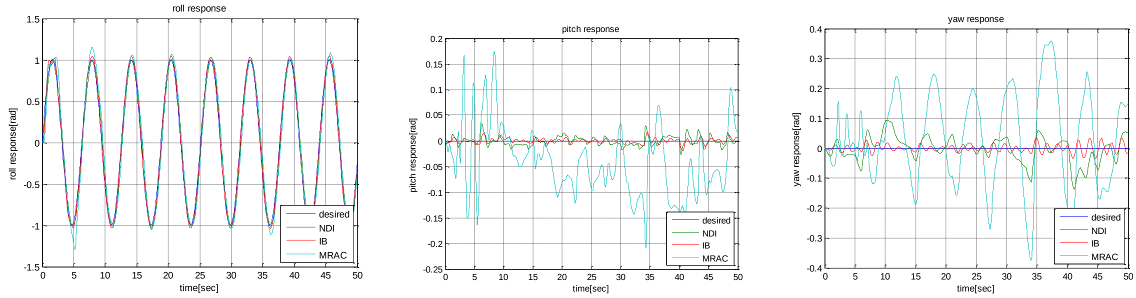
Figures 12.a, 12.b and 12.c: *Simulation of angular control for sinusoidal tracking with no noise: Roll State Response (a) Pitch State Response (b) Yaw State Response (c)*



(d) (e) (f)

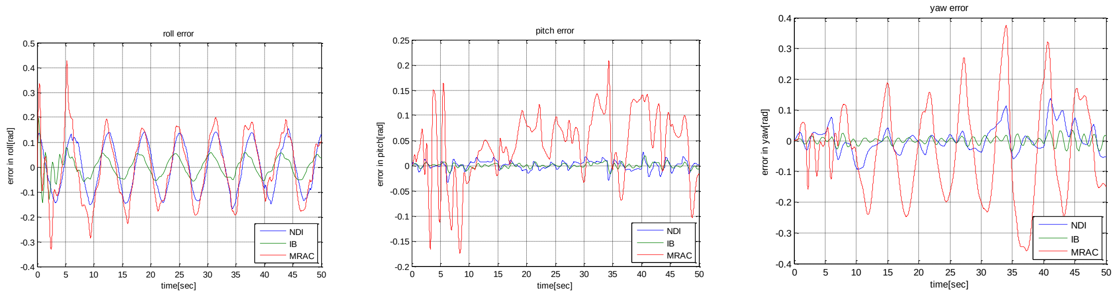
Figures 12.d, 12.e and 12.f: *Simulation of angular control for sinusoidal tracking with no noise: Roll Error (d), Pitch Error (e) and Yaw Error (f)*

Desired roll is tracked by all controllers in the absence of noise and parameter uncertainty as can be investigated on figure 12a. Figures 12b and 12c indicate the states excited by the alteration in roll. It is obvious that these states reach the offset zero.



(a) (b) (c)

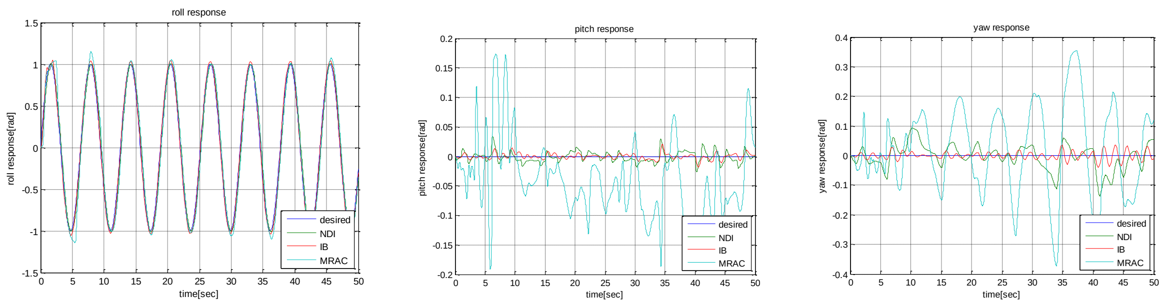
Figures 13.a, 13.b and 13.c: *Simulation of angular control for sinusoidal tracking with noise addition: Roll State Response (a) Pitch State Response (b) Yaw State Response (c)*



(d) (e) (f)

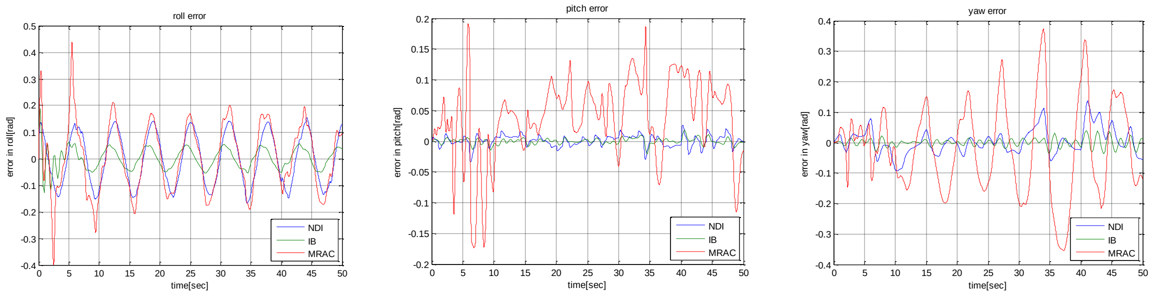
Figures 13.d, 13.e and 13.f: *Simulation of angular control for sinusoidal tracking with noise addition: Roll Error (d), Pitch Error (e) and Yaw Error (f)*

The addition of noise does not affect the general tracking behavior in roll response; however, the other states are excited, comparably more in MRAC. The following figures labeled as 14 below contain the influence of slightly altered parameters as in the set point control:



(a) (b) (c)

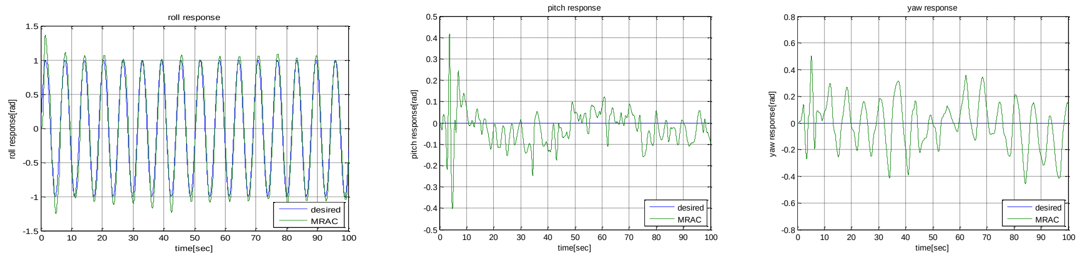
Figures 14.a, 14.b and 14.c: *Simulation of angular control for sinusoidal tracking with slight parameter uncertainty*: *Roll State Response (a), Pitch State Response (b) and Yaw State Response (c)*



(d) (e) (f)

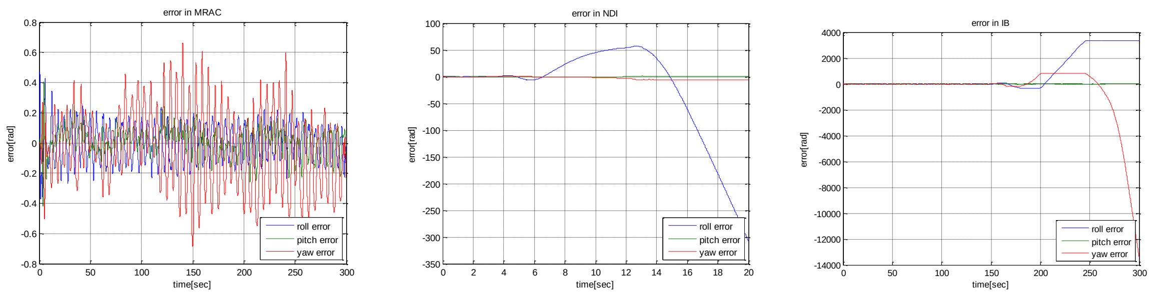
Figures 14.d, 14.e and 14.f: *Simulation of angular control for sinusoidal tracking with slight parameter uncertainty*: *Roll Error (d), Pitch Error (e) and Yaw Error (f)*

Still, all the controllers operate well in tracking when prone to cases in figures 14. Finally, when the new quadrotor model is extended to the simulation, the only working controller again remained thee adaptive one.



(a) (b) (c)

Figures 15.a, 15.b and 15.c: *Simulation of angular control for sinusoidal tracking with a different quadrotor model*: *Roll State Response (a), Pitch State Response (b) and Yaw State Response (c) in MRAC*



(d) (e) (f)

Figures 15.d, 15.e and 15.f: *Simulation of angular control for sinusoidal tracking with a different quadrotor model*: *State Errors in MRAC (d), NDI (e) and IB (f)*

Generally, adding an adaptive algorithm advances the robustness in the presence of approximation errors or external disturbances similar to this simulation. Therefore, as in the case with fixed state control, the results given here support the significance of having an adaptive algorithm for operation success.

The mean values and variances of the errors in orientation are summarized in the following table for the surveyed cases on the displayed graphs. Let $\sigma_{e_{roll}}$,$\sigma_{e_{pitch}}$,$\sigma_{e_{yaw}}$,$\mu_{e_{roll}}$, $\mu_{e_{pitch}}$ and $\mu_{e_{yaw}}$ be defined respectively as the variances and means of roll, pitch and yaw angles. The elements on the left of table stands for the following cases:

No noise: the simulation with no noise
Noise: the simulation with noise generation
Noise + u.p. : the simulation with noise and parameter uncertainty (i.e. slightly wrong modeling)
Diff.quad.: the simulation with an entire different quadrotor model

Table 4 includes variances and means of error for the first case and table 5 for the latter case.

Control Case & Method		$\sigma_{e_{roll}}$	$\sigma_{e_{pitch}}$	$\sigma_{e_{yaw}}$	$\mu_{e_{roll}}$	$\mu_{e_{pitch}}$	$\mu_{e_{yaw}}$
No noise	NDI	0.0200	0.0198	0.0001	0.0678	0.0618	-0.0011
No noise	IB	0.0160	0.0162	0.0000	0.0123	0.0121	-0.0000
No noise	MRAC	0.0089	0.0082	0.0024	0.0067	0.0126	-0.0017
Noise	NDI	0.0263	0.0260	0.0010	0.1024	0.1034	0.0039
Noise	IB	0.0292	0.0293	0.0004	-0.0213	-0.0212	-0.0029
Noise	MRAC	0.0115	0.0090	0.0055	0.0079	0.0034	0.0156
n+u.p.	NDI	0.0259	0.0255	0.0012	0.1021	0.1041	0.0039
n+u.p.	IB	0.0192	0.0190	0.0014	0.0097	0.0091	0.0012
n+u.p.	MRAC	0.0142	0.0100	0.0056	0.0011	0.0021	0.0307
Diff. quad	NDI	1.7481e+03	0.4779	0.1317e+03	-0.0418e+03	-0.4215	0.0112e+03
Diff. quad	IB	1.4521	0.0747	0.1597	-0.0428	-0.0056	0.0012
Diff. quad	MRAC	0.0097	0.0081	0.0109	0.0015	-0.0158	0.0219

Table 4: *Mean Values and Variances of Controllers in Attitude Holding Mission*

Control Case & Method		$\sigma_{e_{roll}}$	$\sigma_{e_{pitch}}$	$\sigma_{e_{yaw}}$	$\mu_{e_{roll}}$	$\mu_{e_{pitch}}$	$\mu_{e_{yaw}}$
No noise	NDI	0.0085	0.0023e-03	0.0053e-03	-0.0079	0.0062	-0.0039
No noise	IB	0.0017	0.0000	0.0000	0.0857e-03	0.0000	0.0000

No noise	MRAC	0.0157	0.0016	0.0010	-0.0004	0.0007	-0.0150
Noise	NDI	0.0085	0.0769e-03	0.0019	0.6574e-03	0.8534e-03	0.0034
Noise	IB	0.0021	0.0245e-03	0.1803e-03	0.4742e-03	0.1456e-03	-0.0635e-03
Noise	MRAC	0.0174	0.0045	0.0239	-0.0035	0.0417	-0.0329
n+u.p	NDI	0.0091	0.0811e-03	0.0018	0.0054	0.2377e-03	0.0040
n+u.p	IB	0.0019	0.0256e-03	0.1789e-03	-0.7854e-03	0.0118e-03	-0.0104e-03
n+u.p	MRAC	0.0128	0.0026	0.0203	-0.0033	0.0415	-0.0261
Diff. quad	NDI	0.0128e +06	0.1809	3.8398	-0.1107e+03	0.8470	-5.1323
Diff. quad	IB	1.9047e +06	0.5629	8.8060e+0 6	1.7883e+03	0.2573	-0.6658e+03
Diff. quad	MRAC	0.0150	0.0076	0.0654	-0.0012	0.0139	-0.0439

Table 5: *Mean Values and Variances of Controllers in Sinusoidal Tracking Mission*

CONCLUSION

This paper presents controller options for the quadrotor dynamical system simulated with excellent sensors, noisy measurements, and parameter uncertainty. The controller options chosen for this work include nonlinear dynamic inversion, model adaptive reference and integral backstepping techniques. Consequently, the model constructed was used to perform simulation studies for both attitude-hold and sinusoidal angular trajectory tracking missions. Consequently, the results were discussed with an overview of performance and robustness of controllers. As expected, all controllers were operating well except the existence of high-level parameter uncertainty. In coherence with its purpose of integration into the controller, only adaptive algorithm survived such a case and outperformed the others. Nevertheless, for other controllers, the exact knowledge of important system parameters like the moment of inertia, motor arm length or weight needed to be known in order to operate as desired with correct controller gains. That supports the ideology under the vitality of adaptation laws for the cases including the existence of disturbances and uncertainties. The experimentation and optimization of current design of attitude controllers and the design of position controllers may be taken into account as successive steps to this research for future aims like target tracking, obstacle avoidance and formation flight

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