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CONTINUOUS FIBER PATH OPTIMIZATION IN COMPOSITE STRUCTURES

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ABSTRACT

Fiber orientation angle stands out as one of the most effective design variables in the design optimization of composite structures. Fiber placement machines can place different width tows in curvilinear paths resulting in continuous change of the fiber orientation angle in a layer of the composite structure. By allowing the fibers to follow curvilinear paths in the composite structure, modification of load paths within the laminate can be obtained. Thus, more favorable stress distributions and improved laminate performance can be achieved. Such structures are called as variable stiffness composites structure. This study presents a fundamental study on the continuous fiber path optimization of composite structures, which can be produced with fiber placement machines. Optimized fiber paths are determined for different case studies. Continuous fiber path optimization is performed by means of an interface code that is developed. It is hard to find the global optimum for complex optimization problems with hundreds of design variables. In order to find the global optimum solution for such complex optimization problems, a gradient based optimization algorithm is not appropriate because there will be a lot of local minima for the problem and may be stuck at the local minimums. Therefore, an evolutionary algorithm is a better solver for such kind of complex optimization problems. In this study, genetic algorithm in MATLAB Optimization Toolbox is used for the optimizer and commercial finite element program Nastran is used for the structural solver. For the continuous fiber path optimizations these two programs are integrated with the interface code that is developed. Manufacturing constraints of a typical fiber placement machine is also included in the constraint definition of continuous fiber path optimization. The buckling load of a variable stiffness composite plate is increased %22 with respect to a composite plate with zero degree fiber orientation.

INTRODUCTION

In theory, discrete fiber orientation optimization gives the optimum orientation for the composite structures. However, in practical applications manufacturing of composite structures based on discrete fiber orientation angles, determined in an optimization framework, for the composite structures is not easy. There would be discontinuities in the fiber orientation angles in the composite structure, and this would make it almost impossible to manufacture the composite structure. If the design of the laminate is done according to the manufacturing constraints at the very beginning of the design process, then it would be possible to manufacture the composite structure. In this work the final goal is to optimize continuous fiber paths with genetic algorithm such that composite laminate can be manufactured. Variable stiffness concept is used to determine the fiber paths along the composite plate [Gürdal, Z. and Tatting, B. F,1998; Waldhart, C, 1996; Langley, P. T., 1996]. In the variable stiffness concept studied within this study, 3 variables, $T_0, T_1, \emptyset$, are considered to determine the fiber path T_0 is the fiber orientation angle of the reference fiber path measured at the center of the plate, T_1 is the fiber orientation angle measured at $x' = a/2$, where a is the reference dimension of the composite laminate and, ∅ is the rotation angle of the tow placement axis. Different case studies are conducted to

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minimize the total strain energy and to maximize the buckling load for single layered and multilayered composite plates.

METHOD

The variable stiffness concept and the genetic algorithm are explained in this part. The algorithm for coupling the MATLAB Genetic Algorithm Tool and Nastran Finite Element solver, with the manufacturing constraints embedded, is explained in detail.

Genetic Algorithm

It should be noted that genetic algorithm emulates the nature. Figure 1 shows the basic structure of the genetic algorithm. As in the science of genetics, the population, sub-population, organism, chromosomes and the genes are present in the basic structure. The genes are the optimization variables in the genetic algorithm. Differing from the real life, the genes are represented as binary codes [Soremekun, Grant A. E., 1997]. These binary codes are processed to have best solution in the design domain. In Figure 1, the cross-over and mutation operators, which process on the binary codes are also shown.

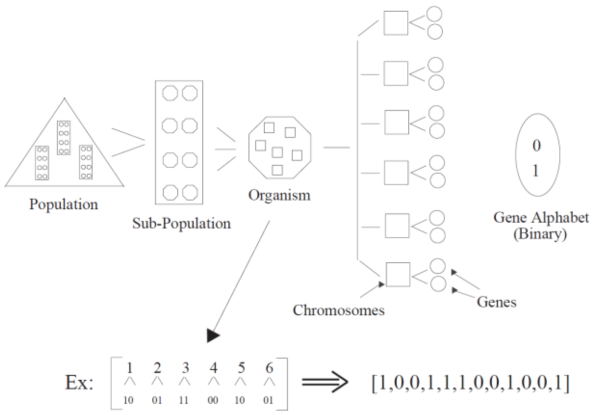


Figure 1: *The simple genetic algorithm structure taken from Reference [Soremekun, Grant A. E., 1997].*

In the genetic algorithm, at the beginning, random initial population is created in the whole design domain. The objective function is evaluated for every design alternative within the population. The evaluated objective function values are compared with each other. The best individuals are selected among the others. In a way, best individuals survive in the optimization process. The genes that generate the best values for the objective functions are processed to have new pairs of children. Cross-over operator affects the performance of the genetic algorithm. Cross over is simply combining a pair of parents [MATLAB, 2004].After that, the mutation operation is employed. Mutation operator makes random changes in the genes. Mutation avoids losing whole information coming from the heritage, and it is used to increase the randomness in the solution space. This is the simple design cycle of the genetic algorithm. The cycle is repeated until the stopping criterion is reached and a global optimum solution is found. Since there is no differentiation in this process, genetic algorithm is not stuck in a local optimum and finds the global minimum of the problem directly. It should be noted that the genetic algorithm does not give the exact solution for the problem. It only converges to the global optimum very fast. In genetic algorithm optimization one must solve the optimization problem more than once, since it is an evolutionary algorithm.

Variable Stiffness Concept

Variable stiffness feature of a ply is achieved by curvilinear fiber paths which cannot be described by a single orientation angle. In the literature, a convenient continuous fiber path is described such that the fiber orientation of the reference fiber path varies linearly from one value at the center of the panel to another at a specified distance [Gürdal, Z. and Tatting, B. F,1998].

The advantage of the linearly varying fiber orientation angle is that, the fiber orientation of the composite lamina can be determined with just three variables. These variables are defined as $T_0, T_1, \emptyset$ [Gürdal, Z. and Tatting, B. F,1998; Langley, P. T., 1996]. T_0 is the fiber orientation angle of the reference fiber path measured at the center of the plate, T_1 is the fiber orientation angle measured at $x' = a/2$, where a is the reference dimension of the composite laminate and x' and y' are the rotated axes of the plate shown in Figure 2. Finally, as shown in Figure 2, ∅ is the rotation angle of the tow placement axis [Langley, P. T., 1996]. For a composite lamina, the representation of this concept is denoted by "$\emptyset\langle T_0|T_1\rangle$". A symmetric composite laminate with a zero degree of rotation angle ∅ is represented as $[\pm\langle T_0|T_1\rangle]_s$ [Langley, P. T., 1996]. A "$\pm$" sign in front of the rotation angle ∅ represents that reference fiber paths for two successive layers are rotated by equal and opposite amount such that the fiber orientation angle of the outer and inner lamina have positive and negative signs, and "s" represents that the composite laminate is mid-plane symmetric. Figure 3 shows the effect of the rotation angle ∅ on the composite laminate.

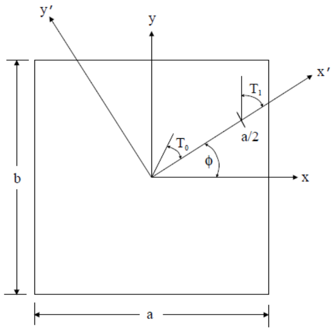
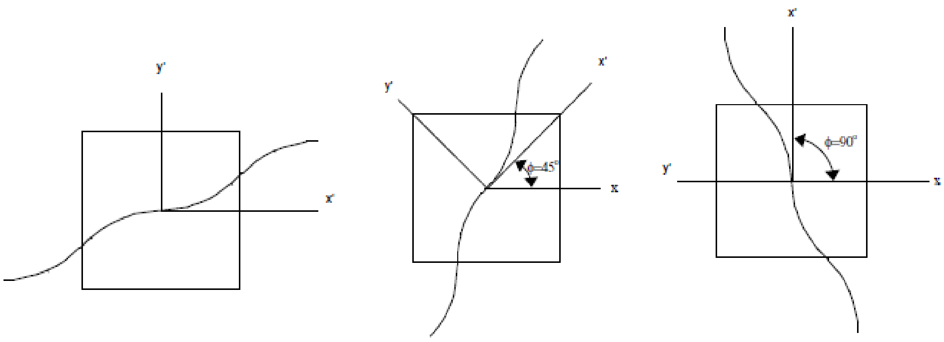


Figure 2: *The variables used in the variable stiffness definition of the composite laminate*



(1) (2) (3)

Figure 3: *The effect of the rotation angle ∅, in case (1) $\emptyset = 0°$, in case (2) $\emptyset = 45°$, in case (3) $\emptyset = 90°$ for the $\langle 0°|45°\rangle$ reference fiber path [Waldhart, C, 1996].*

These fiber orientation angles are calculated with respect to the equation below[Waldhart, C, 1996].

$$\theta(x') = \begin{cases} \emptyset + \frac{2}{a}(T_1 - T_0)x' + T_0 - 2(T_0 - T_1), & for \ -a \le x' \le -\frac{a}{2} \\ \emptyset + \frac{2}{a}(T_0 - T_1)x' + T_0, & for \ -\frac{a}{2} \le x' \le 0 \\ \emptyset + \frac{2}{a}(T_1 - T_0)x' + T_0, & for \ 0 \le x' \le \frac{a}{2} \\ \emptyset + \frac{2}{a}(T_0 - T_1)x' + T_0 - 2(T_0 - T_1), & for \ \frac{a}{2} \le x' \le a \end{cases} \quad Eqn. (1)$$

where,

$$x' = x\cos\emptyset + y\sin\emptyset \quad Eqn. (2)$$

To calculate the x' value for all elements, the Nastran input file is read. With the middle x and y values for the elements, the rotated x' axis coordinates are calculated. After calculating the x' values, they are implemented into the $\theta(x')$ equation, Eqn (1), within the code and written to the Nastran input file.

A general curvature constraint for the tow placement machines is taken from the references [Gürdal, Z. and Tatting, B. F,1998; Waldhart, C, 1996]. The maximum allowable curvature for the tow placement machine that is used is taken as 1/(12 inches). In SI units the maximum allowable curvature is 1/ (30.48 cm). According to the curvature value the minimum radius of curvature is 30.38 cm. For a function of single variable, the curvature equation is given by Eqn (3).

$$K = \frac{f''(x)}{(1+\left(f'(x)\right)^2)^{3/2}} \qquad Eqn.(3)$$

where, the function f(x) represents y(x') for the variable stiffness case [Waldhart, C, 1996].

Solution Procedure Used for the Continuous Fiber Path Optimization

A flowchart for the continuous fiber path optimization procedure is given in Figure 4. For the optimization of continuous fiber orientation, the steps for reading the geometry and element properties, calculating the mid-point of the elements and calculation of the fiber orientation angles of the elements are added to the analysis and optimization procedure [İnci, H. 2012].

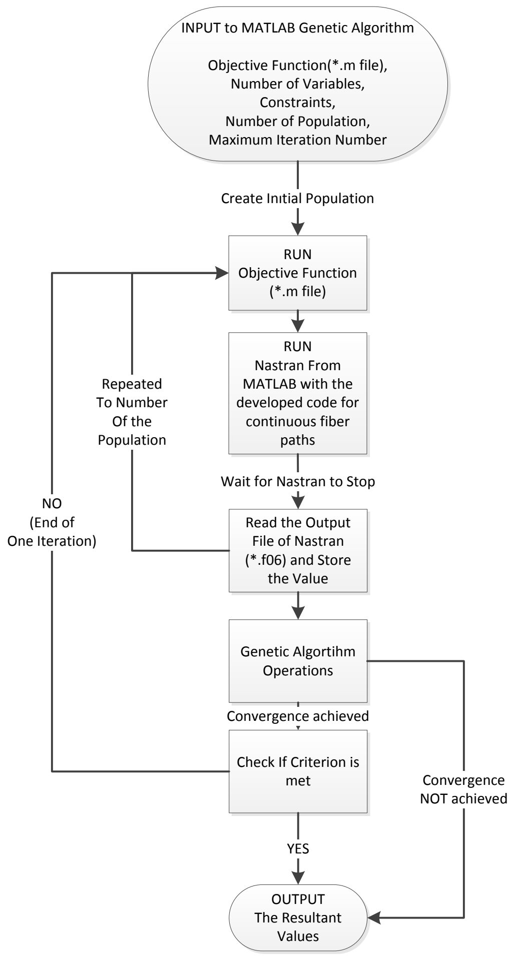


Figure 4: *Flowchart for the continuous fiber path optimization procedure [İnci,H.,2012]*

RESULTS

There are several case studies conducted utilizing the variable stiffness concept defined by the variables T_0, T_1. The studies can be grouped in two; total strain energy minimization problems, buckling load maximization problems. The study for optimization of total strain energy of rectangular plate under in-plane bending load is originated from the cases in the studies in the literature [Setoodeh, S., Gürdal, Z. and L.T., Watson, 2006]. The thickness for this case study is taken as 2mm.The study for optimization of buckling load of a rectangular multilayer composite plate under in-plane compressive loading is studied by Gürdal et. al.[Gürdal, Z., Tatting, B. F. and Wu, C. K. s.l, 2008]. Reference fiber path and stacking sequence definition is given by$[\pm\langle T_0|T_1\rangle]_{3s}$ for both of the case studies. The total thickness is kept as the same. For both case studies the material used is taken as carbon/epoxy composite.

Total Strain Energy Optimization

The loading and the constraints are shown in Figure 5. There are 736 elements in this case study, referenced from studies in the literature [Setoodeh, S., Gürdal, Z. and L.T., Watson, 2006]. The plate dimensions are 4mx1m. These dimensions are chosen according to the aspect ratio value given in the studies in the literature [Setoodeh, S., Gürdal, Z. and L.T., Watson, 2006]. There is a distributed load upper side of the composite plate which is 2000N/m.

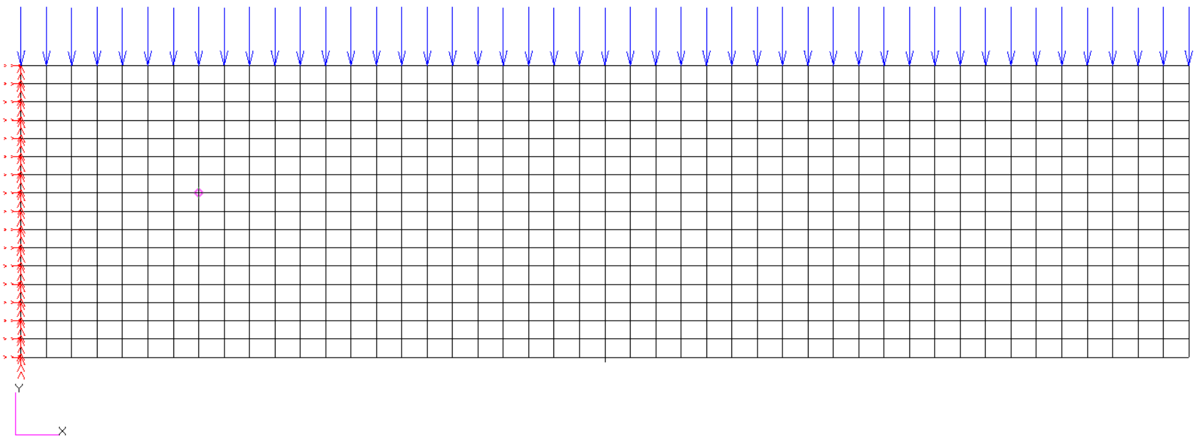


Figure 5: *Geometry, mesh and loading of the composite plate under in-plane load*

First to see the effect of the fiber paths, the composite plate with zero degree fibers is analyzed and the total strain energy is determined 1913.12 Joules.

For the total strain energy minimization, first two variables are considered, T_0, T_1. After these calculations, ∅ is inserted into the problem as the third variable.

<u>2 Variable Case Study:</u> For the two variable case study, seven optimizations are done with 30 populations. The optimization process is shown in Figure 6. The best solution within the seven optimizations is selected as the result.

The total strain energy value for this case is determined as 1265.2 Joules. The resultant orientation for the 2 variable case is $[\pm\langle 13.39|17.27\rangle]$.

<u>3 Variable Case Study:</u> For the three variable case study, again seven optimizations are performed with 50 populations. The optimization process is shown in Figure 7. The best solution within the seven optimizations is selected as the result.

The total strain energy value for this case is determined 1265 Joules. The resultant orientation for the 3 variable case is $\pm 80.49[\pm\langle 89.94|83.44\rangle]$.

Looking at Figures 6 and 7, it is clearly seen that the total strain energy value is dropping and approaching to an optimum solution. On the other hand, the optimized total strain energy value does not change significantly, by inserting the third variable, ∅. It is true to say that these solutions are close to each other by looking at their geometry.

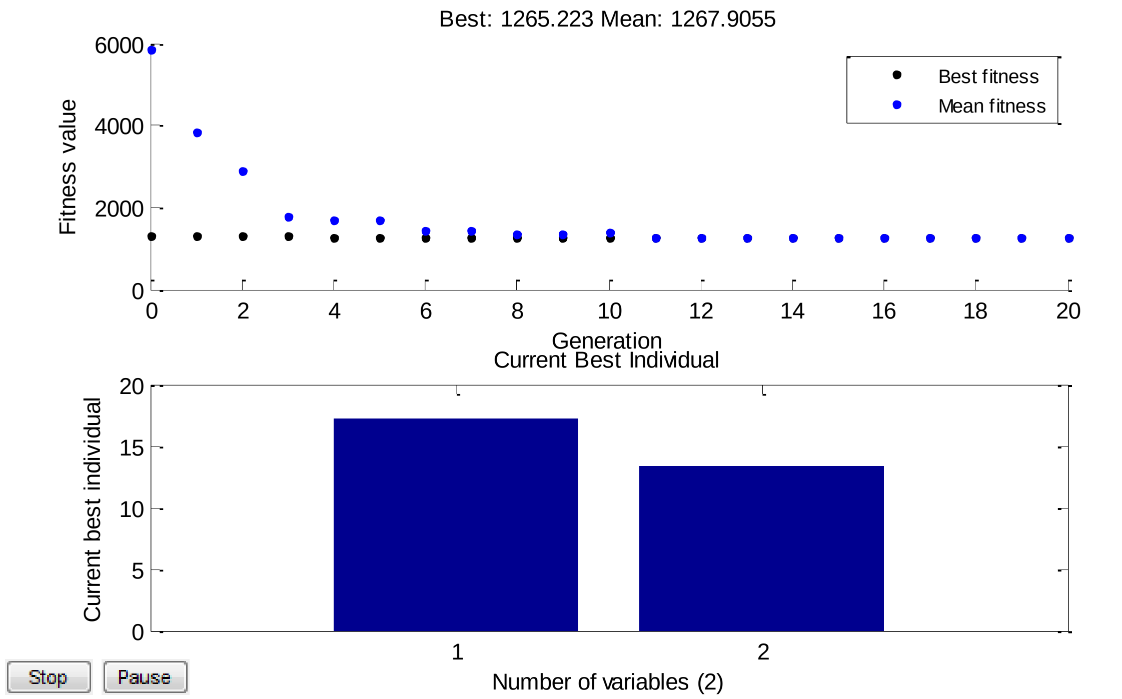


Figure 6: *Total strain energy minimization process for the composite plate with 2 variables*

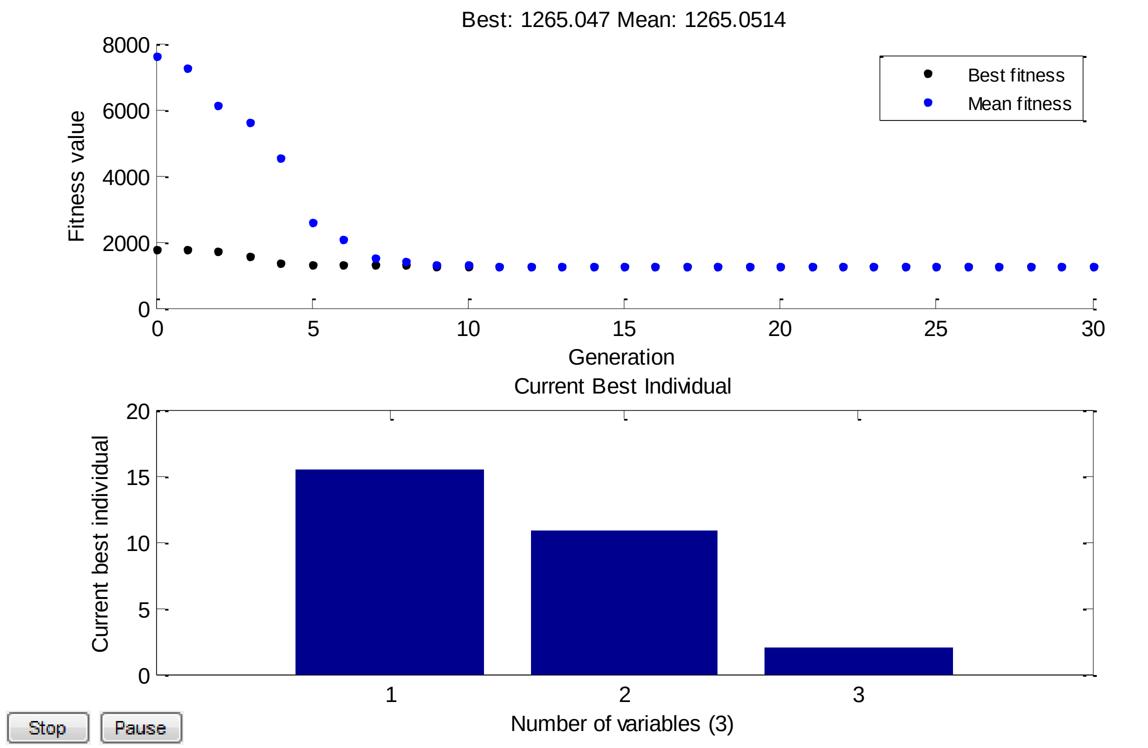


Figure 7: *Total strain energy minimization process for the composite plate with 3 variables*

Buckling Load Optimization

The loading and the constraints are shown in Figure 8. There are 400 elements in this case study, referenced from [Gürdal, Z., Tatting, B. F. and Wu, C. K. s.l, 2008]. The plate dimensions are 4mx4m. This dimensions are directly taken from the reference [Gürdal, Z., Tatting, B. F. and Wu, C. K. s.l, 2008]. The plate is under 1000N/m distributed load.

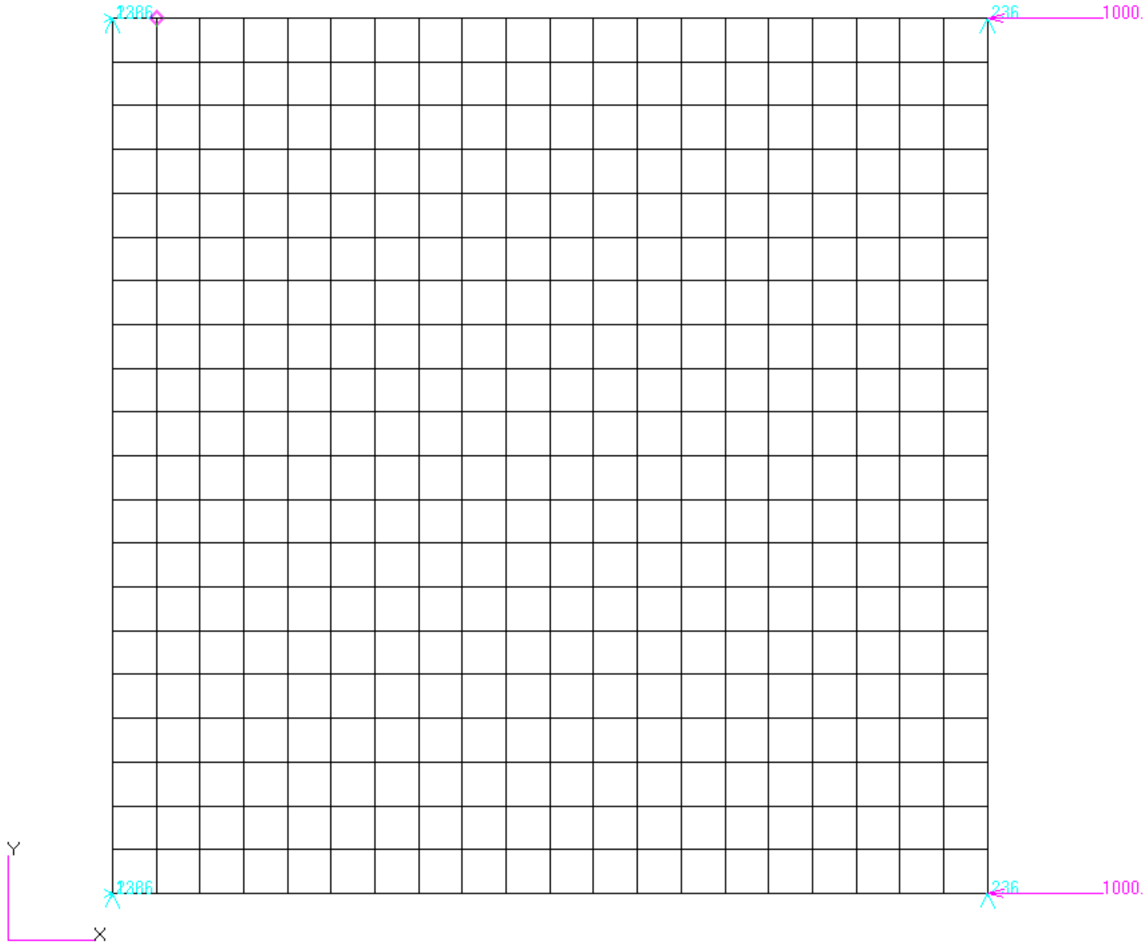


Figure 8: *Geometry, mesh and loading of the composite plate used for the buckling study*

This case is also optimized for two and three variables as in the previous one. For the buckling case study it is decided to first perform the optimization for the buckling case with 0^0 and 90^0 constant Ø angles in the 2 variable optimization case, to see the effect of the Ø angle more clearly. Since this is a continued study on the reference [İnci, H. 2012], to see the effect of the orientation angle, the curvature constraint is not taken account. Also this is a maximization problem, so the eigenvalues obtained is multiplied with -1 to reach the maximum value.

To compare the results an analysis is performed with 0^0 fiber angle square composite plate. The corresponding eigenvalue is 3.1464. [İnci,H., 2012]

<u>2 Variable Case Study with 0^0 Ø Angle:</u> Taking Ø as constant the optimization is performed. For this case, optimization is performed with 30 populations, and repeated 7 times. The optimization process is shown in Figure 9 below. The best solution within the seven optimizations is selected as the result.

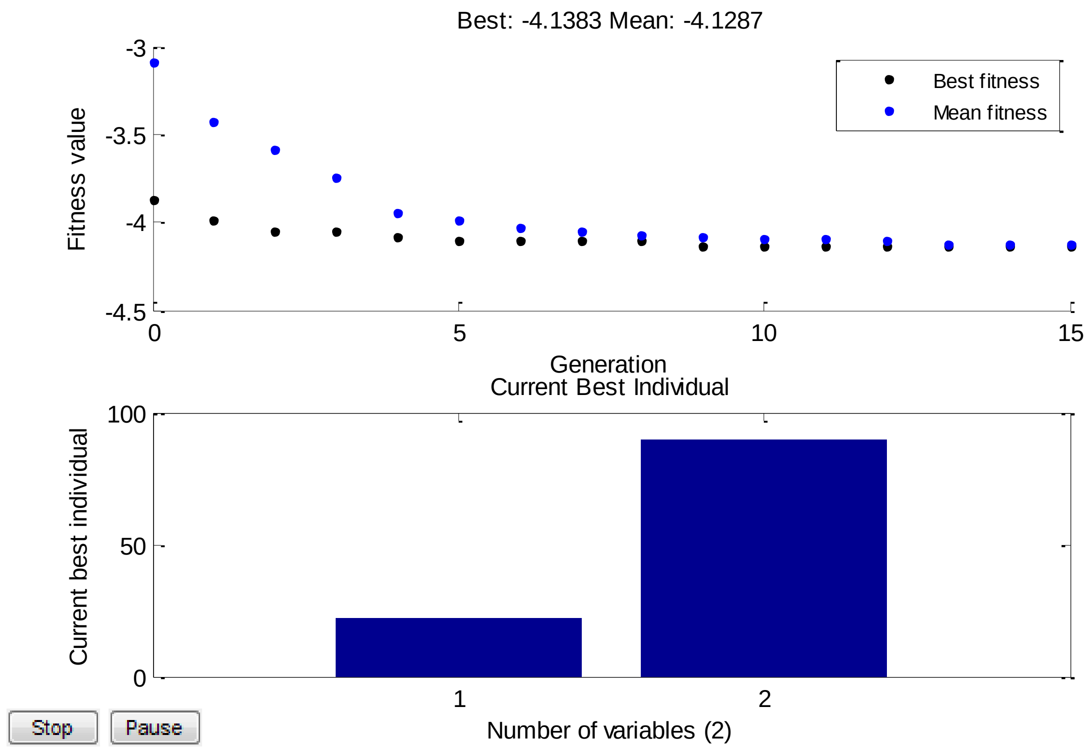


Figure 9: *Optimization process for the buckling load maximization of the composite square plate with 2 variables with ∅ taken as* 0^0

The maximum eigenvalue for this case is determined as 4.1383. The resultant orientation for this case is $[\pm\langle 89.90|21.93\rangle]$.

<u>2 Variable Case Study with 90^0 ∅ Angle:</u> Taking ∅ as constant the optimization is performed. These optimizations are performed with 30 populations, and repeated 7 times. The optimization process is shown in Figure 10 below. The best solution within the seven optimizations is selected as the result.

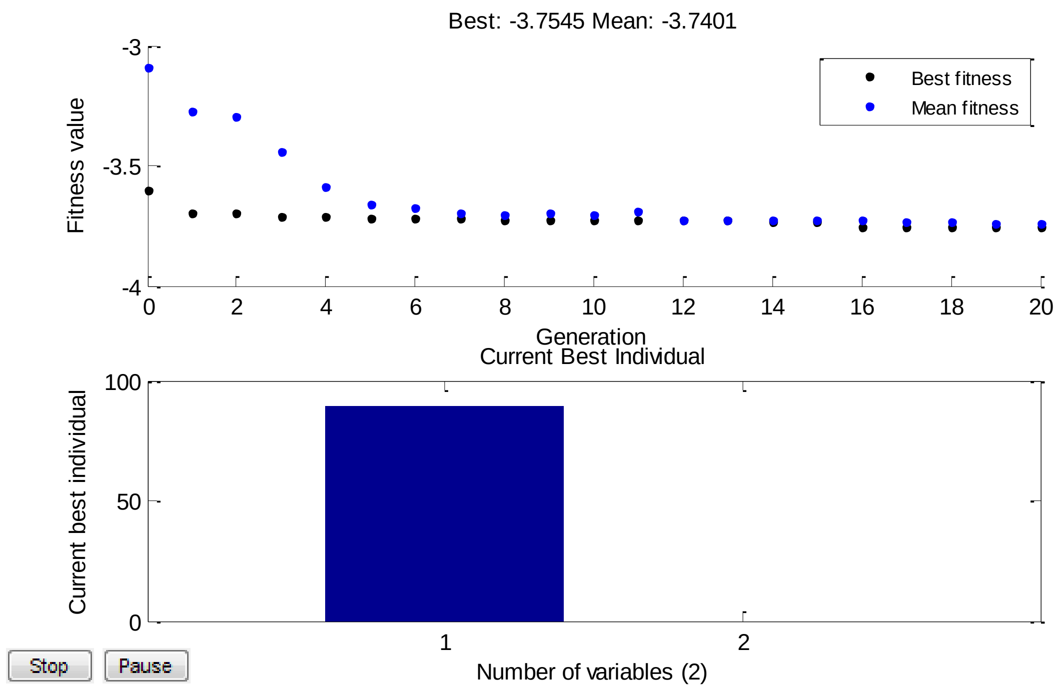


Figure 10: *Optimization process for the buckling loa maximization of the composite square plate with 2 variables with ∅ taken as* 90^0

The maximum eigenvalue for this case is determined as 3.7545. The resultant orientation for this case is $[\pm\langle 0|89{,}78\rangle]$.

3 Variable Case Study: Now, $\emptyset$ angle is also taken as the design variable of the optimization problem. Up to now the optimizations are performed with no constraints. In this part of the study, first the unconstrained optimization is performed and then the constrained optimization is performed.

First, the unconstrained optimization is performed 7 times with 50 populations. The best result's optimization process is shown below in Figure 11. The best solution within the seven optimizations is selected as the result.

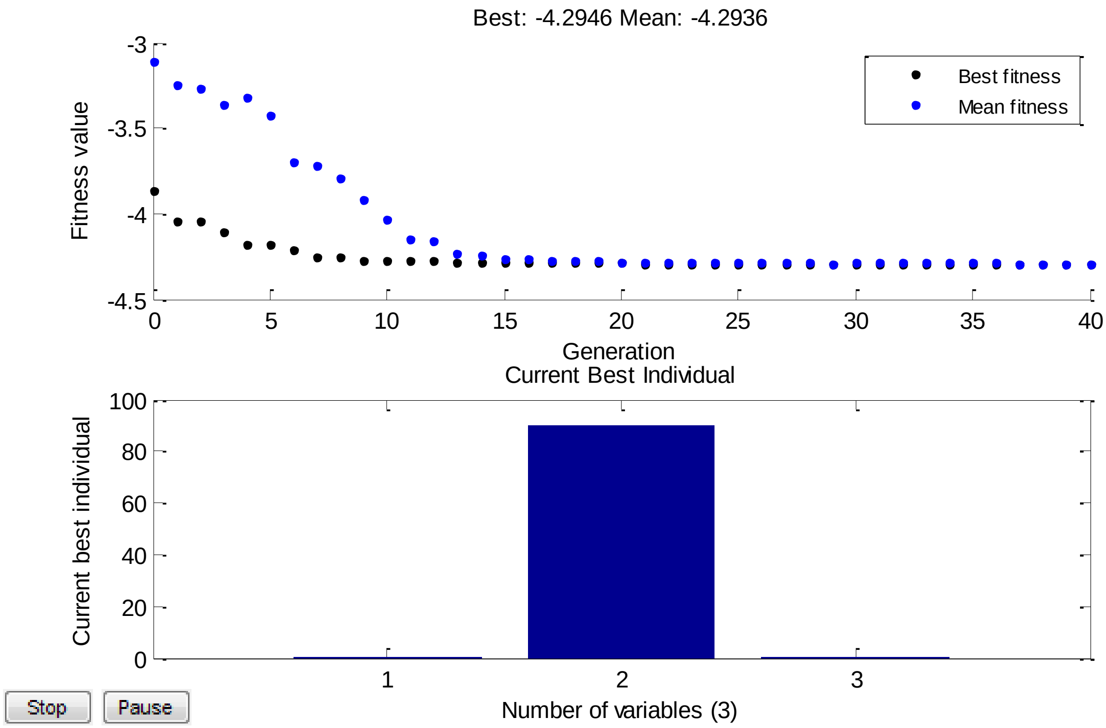


Figure 11: *Unconstrained optimization process for the composite square plate with 3 variables*

The maximum eigenvalue for this case is determined as 4.1383. The resultant orientation for this case is $\pm 0.02[\pm\langle 89.98|0.04\rangle]$.

Secondly, to design a composite plate which can be manufactured, one must consider the manufacturing constraint. The manufacutring constraint is explained in "Variable Stiffness Concept" part of this work and the equation used for the constrain is given in Eqn. (3). For this purpose, the constrained optimization is performed for 7 times with 50 populations. The best result out of 7 optimization processes is shown below in Figure 12.

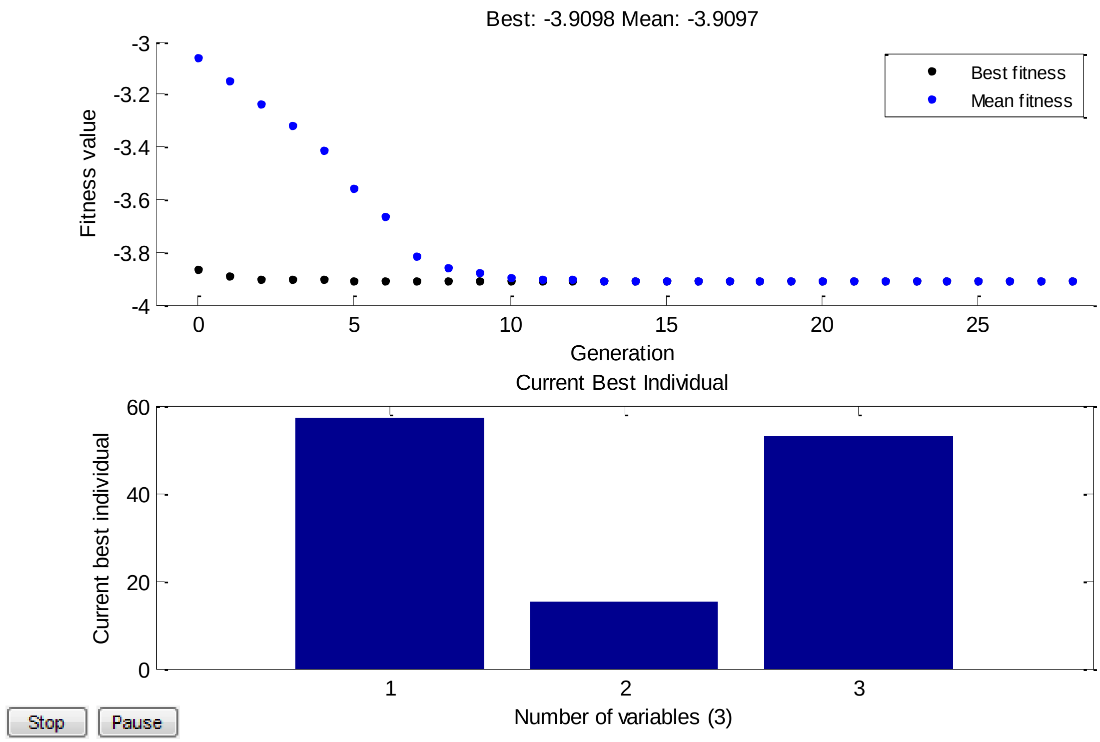
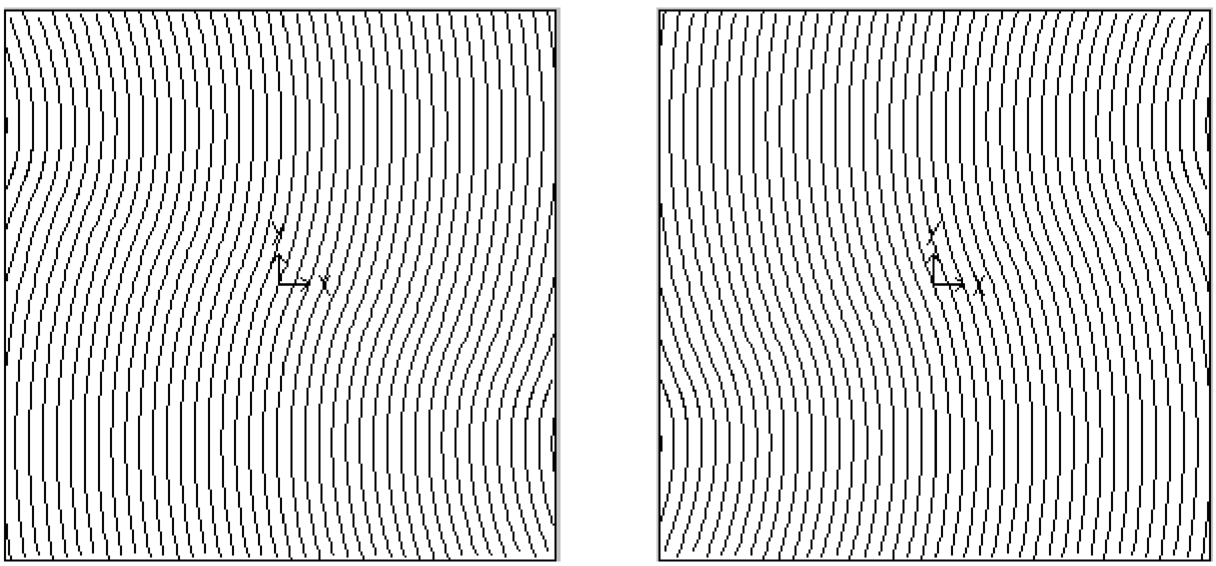


Figure 12: *Constrained optimization process for the composite square plate with 3 variables*

The maximum eigenvalue for this case is determined as 3.9098. The resultant orientation for this case is $\pm 53{,}09[\pm\langle 15.36|57.13\rangle]$. The corresppnding fiber paths are given in Figure 13. These fiber paths are drawn using a special tool [Adoptech, 2013].



(a) *(b)*

Figure 13: Fiber paths of the maximum buckling load for square plate (a) for the $\mathbf{53{,}09[\langle 15.36|57.13\rangle]}$ *(b)* $\mathbf{-53{,}09[-\langle 15.36|57.13\rangle]}$.

It is seen that, the rotation angle $\emptyset$**,** is more significant for the maximization of buckling load. Even for two variable cases, there is a significant difference between 0^0 and 90^0 $\emptyset$ angle. The unconstrained optimization is also making significant difference with the constrained optimization. There is a maximum of 31,5% of buckling load increase with respect to the zero degree fiber orientation case.

CONCLUSION

Fiber orientation angle is one the most significant variables affecting the mechanical behavior of composite laminates, since fibers are the main load carrying component of composites. The curvilinear fiber paths in composite structures allow the modification of load paths within the composite laminate. With the curvilinear fiber paths more favorable stress distributions and improved laminate performance can be obtained. This study presents a fundamental study on continuous fiber path optimization of composite structures.

There are several case studies which are conducted within this study but as indicated before the most significant one is the buckling load maximization of a multilayered composite plate. This case is important because one can compare the results with a previous literature study [Gürdal, Z., Tatting, B. F. and Wu, C. K. s.l, 2008].

Case study on the maximization of the buckling load by optimizing continuous fiber paths shows the great potential that the variable stiffness concept presents in terms of maximizing the buckling load. Buckling load is increased by 24,2% for the constrained optimization case study with three variables and 31,5% for the unconstrained optimization case study with three variables. This study presents a fundamental study to introduce the potentials of the variable stiffness composite structures which can be manufactured by the fiber placement machines. It is considered that in the future fiber placement technology will be the major production method of composite structures due to the automation that they offer. However more importantly, with the fiber placement machines it is possible to optimize a specific response or responses of a composite structure much more effectively compared to the straight fiber case using stacking sequence optimization.

In the present study, the effect of the curvilinear fiber path is investigated. Two different problems are investigated within this study with two and three design variables. By looking at the results, curvilinear fiber paths improve the plate characteristics in terms of strain energy stored or buckling load. On the other hand, it is seen that the third variable, $\emptyset$, which is also called the direction angle, is not very effective on the objective function value. But the direction angle affects the T_0, T_1 parameters which define the fiber path on the composite laminate. It is concluded that more case studies have to be solved with different load cases to see the effect of the direction angle more clearly.

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