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DEVELOPMENT OF TWO NONLINEAR CONTROLLERS FOR THE CONTROL OF AIRCRAFT FORMATIONS

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ABSTRACT

In this paper, a leader-follower approach is employed to make two unmanned aircrafts fly in a fixed geometrical formation. The first aircraft in the formation is designated as leader and the second is treated as the follower. The leader maintains a prescribed trajectory while the follower tracks and maintains a fixed relative distance from its leader. Since the associated kinematic equations are nonlinear, the relative guidance of the follower using two nonlinear control approaches, the Lyapunov based control algorithm and the SDRE based algorithms are proposed. These algorithms are tested through simulations for three cases. The results are given, discussed and the effectiveness of the proposed algorithms in realizing the desired formations is examined.

Keywords: Formation Flight, SDRE Control, Lyapunov Control

NOMENCLATURE

v_L, v_f	Leader and follower aircraft velocities
ψ_L, ψ_f	Leader and follower aircraft heading angles
$\dot{\psi}_f$	Follower heading rate
$\psi_e \equiv \psi_L - \psi_f$	
h_L, h_f	Leader follower aircraft altitudes
$\dot{\psi}_{fc}$	Reference heading rate for follower aircraft to track desired formation Geometry
h_{fc}	Reference altitude for follower aircraft to track desired formation Geometry
v_{fc}	Reference velocity for follower aircraft to track desired formation Geometry
ϕ_{fc}	Reference Roll angle for follower aircraft to track desired formation Geometry
r_{fc}	Reference yaw rate for a coordinated turn for ϕ_{fc}
(x, y, z)	Relative position of leader with respect to follower resolved in the follower's reference frame
x_f, y_f, z_f	Coordinate axis for the follower reference frame

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$(x_{ref}, y_{ref}, z_{ref})$ Coordinates of the desired relative position between the leader and follower in the follower's reference frame

INTRODUCTION

Formation flight has sparked several interests over the past decades. Formation flights of unmanned aircrafts find their use in surveillance and exploration missions, reconnaissance missions as well as rescue in hostile environments. In surveillance missions, unmanned aircrafts in formation are able to synthesize an antenna of dimension far larger than an individual agent, consequently leading to an improved sensitivity. Flying aircrafts in a formation, specifically the V-flight formation has the beneficial effect of reducing the induced drag of the individual members of the formation. NASA Dryden Research Center carried out a formation flight test on two F/A-18 aircrafts which have the same configuration [Ray and Cobleigh et al., 2002]. Performance benefits obtained from the flight test phase included a drag reduction of more than 20% and a reduction in fuel consumption by more than 18% at a flight condition of Mach 0.56 and altitude of 25000ft [Ray and Cobleigh et al., 2002]. Fowler, J.M. and Andrea, R.D. conducted an experiment on a large formation of 31 wings; the result showed an induced drag reduction of up to 41% is possible [Fowler, Andrea, 2002].

In this paper, control of the relative position of a pair of unmanned aircraft using Lyapunov control method is investigated. In this paper, comparisons between the lyapunov approach and the SDRE approach are made. The formation geometry kinematic equations are taken from reference [Dargan, 1991]. These equations represent the relative position of the follower with respect to the leader in the follower's reference frame. The formation-hold controller is based on the state dependent Riccati equation approach as well as the lyapunov approach, implemented on the follower to realize a fixed relative position between the leader and follower in spite of maneuvers carried out by the leader aircraft. In the simulations, the linear model of the SIG RASCAL 110 model aircraft is used [sinem, 2010].

In the following, first the problem formulation containing formation model, SDRE control method and Lyapunov control method, and individual aircraft controllers are presented. Then simulation results are given and analyzed. Finally conclusions are given.

FORMULATION OF THE PROBLEM

Formation Kinematics

The formation geometry kinematics is shown in Figure 1. Based on this figure, the kinematics equations may be written as [Dargan, 1991]:

$$\dot{x} = v_L \cos\psi_e + \dot{\psi}_f y - v_f \quad (1)$$

$$\dot{y} = v_L \sin\psi_e - \dot{\psi}_f x \quad (2)$$

In the derivation process, two reference frames were used. An inertial reference frame is affixed on the leader UAV and a rotating reference frame is affixed on the follower UAV. The rotating reference frame of the follower UAV has its x axis, x_f, aligned with the follower UAV's instantaneous velocity vector, v_f, the y axis, y_f, is perpendicular to this in the horizontal plane, the z axis, z_f, is in the down direction. Equations 1 and 2 give us the relative position of the follower UAV with respect to the leader UAV in x and y components. For the z component of the relative position, we simply take the difference in altitude of the two UAVs since the z-axis of both the follower and leader UAVs are perfectly aligned and both point towards the down direction. Thus,

$$\dot{z} = \dot{h}_f - \dot{h}_L \quad (3)$$

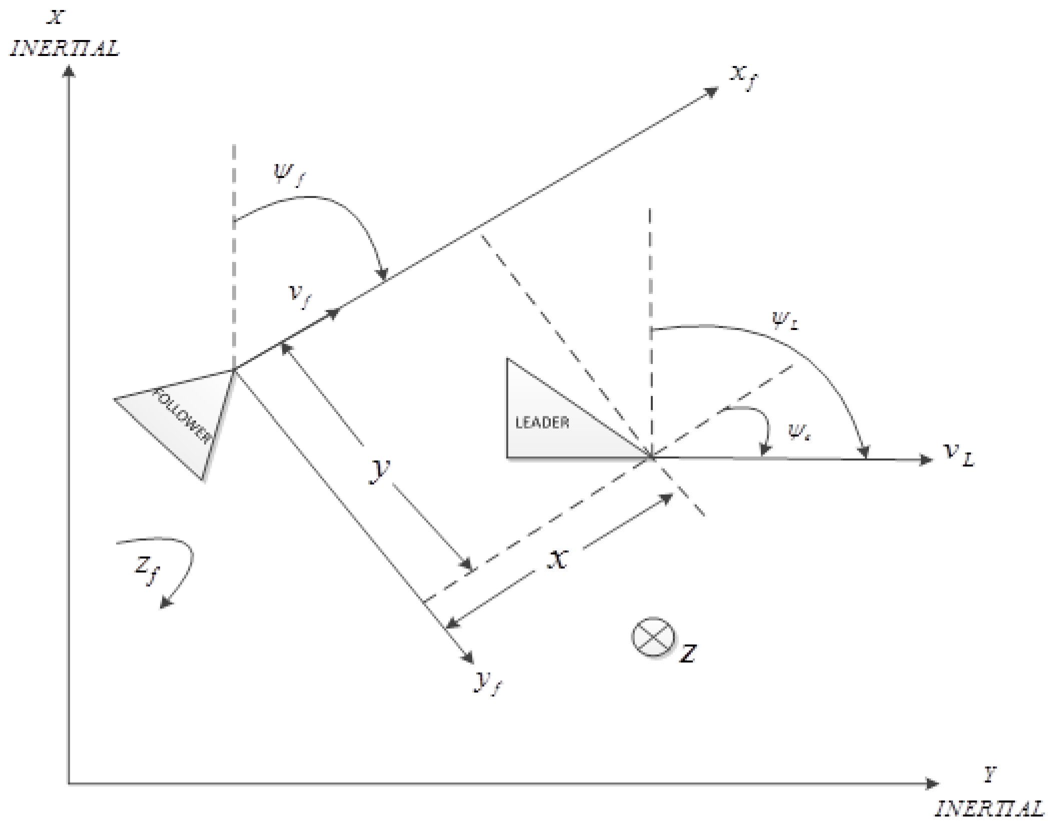

Figure 1: Formation geometry

SDRE Approach to Control the Formation

The State Dependent Riccati Equation (SDRE) control methodology uses extended linearization as the key concept in formulating the nonlinear optimal control problem [5]. At each instant, the method treats the state-dependent coefficient matrices as being constant, and computes a control action by solving a Linear Quadratic optimal control problem. The system has to be full-state observable, controllable, and affine in input. The weighing matrices of the quadratic cost function may be state dependent. Consider a system of the following form [Anderson and Moore, 2007]:

$$\dot{x}(t) = Ax + Bu + f \quad (4)$$

where $x \in \Re^n$ is the state vector, $u \in \Re^m$ is the input vector and $t \in [0, \infty]$, with functions $A: \Re^n \to \Re^n$, $B: \Re^n \to \Re^{nxm}$ and $f \in \Re^n$. The minimization of the following performance index is considered.

$$J = \frac{1}{2}\int_0^T [(Cx - r)^t Q(Cx - r) + u^t Ru]dt \quad (5)$$

where $r \in \Re^m$ is the reference input, and as defined by the control objectives, $C: \Re^n \to \Re^m$. Q must be positive definite, and R must be at least positive semi definite at all times. In other to obtain the control law, the Hamiltonian and its derivatives with respect to the control input u, states x and co-state variables λ are needed [6].

$$H = \frac{1}{2}(Cx - r)^t Q(Cx - r) + \frac{1}{2}u^t Ru + \lambda^t (Ax + Bu + f) \quad (6)$$

$$\dot{x} = \frac{\partial H}{\partial \lambda} = Ax + f - E\lambda \quad (8)$$

$$\dot{\lambda} = \frac{\partial H}{\partial x} = -(A^t\lambda + C^t Q(Cx - r)) \quad (9)$$

Combing equations (4), (5) and (6), we have:

$$H = (Cx - r)^t Q(Cx - r) + \lambda^t (Ax + f) - \frac{1}{2}\lambda^t E\lambda \tag{10}$$

where $E = BR^{-1}B^t$. Define the function λ including a bias term due to the tracking problem. Let the solution be:

$$\lambda = Px - g \tag{11}$$

If we take the derivative of λ and use equations (5) and (6), we have:

$$-A^t(Px - g) - C^t Q(Cx - r) = P(Ax + f - E(Px - g)) - \dot{g} + \dot{P}x \tag{12}$$

Collecting the terms with state variables and equating them to zero, we have:

$$\dot{P} + PA - PEP + A^t P + C^t QC = 0 \tag{13}$$

which is the differential state dependent Riccati equation. The remaining terms become:

$$\dot{g} - Pf + PEg + A^t g + C^t Qr = 0 \tag{14}$$

If we search for steady state solutions, assuming that f and r are constants, equation (13) becomes algebraic Riccati equation with well-known solution methods.

$$PA - PEP + A^t P + C^t QC = 0 \tag{15}$$

Then the solution of the auxiliary equation and the control law becomes:

$$g = -(PE - A^t)^{-1}(Pf - C^t Qr) \tag{16}$$

$$u = -R^{-1}B^t(Px - g) \tag{17}$$

Using equations (1) and (2), a formation-hold controller may be designed. To do this, equations (1) and (2) are written in the state-dependent coefficient form.

$$\begin{Bmatrix} \dot{x} \\ \dot{y} \end{Bmatrix} = \begin{bmatrix} y & -1 \\ -x & 0 \end{bmatrix} \begin{Bmatrix} \dot{\psi}_f \\ v_f \end{Bmatrix} + \begin{Bmatrix} v_L \cos\psi_e \\ v_L \sin\psi_e \end{Bmatrix} \tag{18}$$

Comparing equation 18 with equation 4, it may be observed that in this factorization, the system matrix, A, is zero. For a given reference formation, $r = \{x_{ref}, y_{ref}\}$, the SDRE controller may be designed to generate the necessary control commands $u = \{\dot{\psi}_f, v_f\}$ to realize the desired formation in a stable manner. Thus, the proposed formation control is a guidance methodology used to generate the necessary heading rate and velocity commands to the follower.

Lyapunov Control Approach

Let $x = 0$ be an equilibrium point of a nonlinear system $\dot{x} = f(x)$. Let $V: D \rightarrow R$ be a continuously differentiable function on a neighborhood D of $x = 0$, such that $V(0) = 0$ and $V(x) > 0$ in $D - \{0\}$. If $\dot{V}(x) \leq 0$, then $x = 0$ is stable. Moreover, if $\dot{V}(x) < 0$ in $D - \{0\}$, then $x = 0$ is asymptotically stable. This approach may also be used to design stabilizing controllers for nonlinear systems [Khalil, 2002].

Consider the following Lyapunov function,

$$V(\Delta x, \Delta y) = \frac{1}{2}(\Delta x)^2 + \frac{1}{2}(\Delta y)^2 \tag{19}$$

Where, $\Delta x = x - x_{ref}$ and $\Delta y = y - y_{ref}$. Taking the derivative of equation 19, we have

$$\dot{V}(\Delta x, \Delta y) = \Delta x \Delta\dot{x} + \Delta y \Delta\dot{y} = \begin{bmatrix} \Delta x & \Delta y \end{bmatrix} \begin{Bmatrix} \Delta\dot{x} \\ \Delta\dot{y} \end{Bmatrix} \tag{20}$$

To ensure that equation 20 is negative definite, we make

$$\begin{bmatrix} \Delta x & \Delta y \end{bmatrix} \begin{Bmatrix} \dot{x} \\ \dot{y} \end{Bmatrix} = -\begin{bmatrix} \Delta x & \Delta y \end{bmatrix} Q \begin{bmatrix} \Delta x \\ \Delta x \end{bmatrix} \tag{21}$$

where, Q is a positive definite matrix. Simplifying equation 21, we have

$$\begin{Bmatrix} \dot{x} \\ \dot{y} \end{Bmatrix} = -Q \begin{bmatrix} x - x_{ref} \\ y - y_{ref} \end{bmatrix} \tag{22}$$

Substituting equation 18 into 22 and solving for $\begin{Bmatrix} \dot{\psi}_f \\ v_f \end{Bmatrix}$, we have

$$\begin{Bmatrix} \dot{\psi}_f \\ v_f \end{Bmatrix} = -\begin{bmatrix} y & -1 \\ -x & 0 \end{bmatrix}^{-1} Q \begin{Bmatrix} x - x_{ref} \\ y - y_{ref} \end{Bmatrix} - \begin{bmatrix} y & -1 \\ -x & 0 \end{bmatrix}^{-1} \begin{Bmatrix} v_L \cos\psi_e \\ v_L \sin\psi_e \end{Bmatrix} \tag{23}$$

$\begin{Bmatrix} \dot{\psi}_f \\ v_f \end{Bmatrix}$ is the desired control law to bring the aircraft to the desired formation in a stable manner. Thus, the proposed formation control is a guidance methodology used to generate the necessary heading rate and velocity commands to the follower.

Flight Control Algorithms

As stated earlier, this paper uses a leader-follower approach for the formation control problem. In the Leader-follower approach to formation flight, the leader maintains a prescribed trajectory while the followers track a fixed relative distance from the neighboring aircraft. We only have two UAVs in our case with one of them designated as the leader while the other is designated as the follower. The follower UAV must always maintain the desired relative distance from the leader UAV in spite of maneuvers carried out by the leader UAV.

To enable the leader UAV carry out desired flight maneuvers (prescribing desired trajectory), three LQR controllers are implemented on the leader UAV. These controllers are an altitude-hold controller, a velocity-hold controller and a heading-hold controller. To maintain the desired formation geometry, i.e. the desired relative position of the follower UAV from the leader UAV, the formation-hold controller computes the desired heading rate and velocity the follower UAV must track in order to keep up with the leader UAV. Thus, controllers must be implemented on the follower UAV as well to enable track the commands from the formation-hold controller. Considering the turn geometry, where the lateral acceleration is zero, the following relation between the roll angle and heading rate may be written [McLean, 1990]:

$$\phi_{fc} = \arctan\left(\frac{v_f . \dot{\psi}_f}{g} \right) \tag{24}$$

The body fixed yaw rate to be realized by the follower then may be calculated from:

$$r_{fc} = \dot{\psi}_f . \cos(\theta_f) . \cos(\phi_{fc}) \tag{25}$$

where $\dot{\psi}_f , v_f$ are obtained from the formation-hold controller. ϕ_{fc}, r_{fc} are then sent to the follower controllers. The LQR controllers implemented on the follower UAV are, a velocity-hold controller, a roll-hold controller and a yaw rate controller. These controllers receive their input signals from the formation-hold controller. As stated before, the formation-hold controller provides reference signals to the follower UAV to enable it track (x_{ref}, y_{ref}), which is the x and y components of the desired relative position of the follower UAV with respect to the leader UAV. To enable the follower track z_{ref}, an altitude controller is implemented on the follower UAV as well. Input to this altitude controller is the leader UAV's altitude plus the desired offset, z_{ref} .

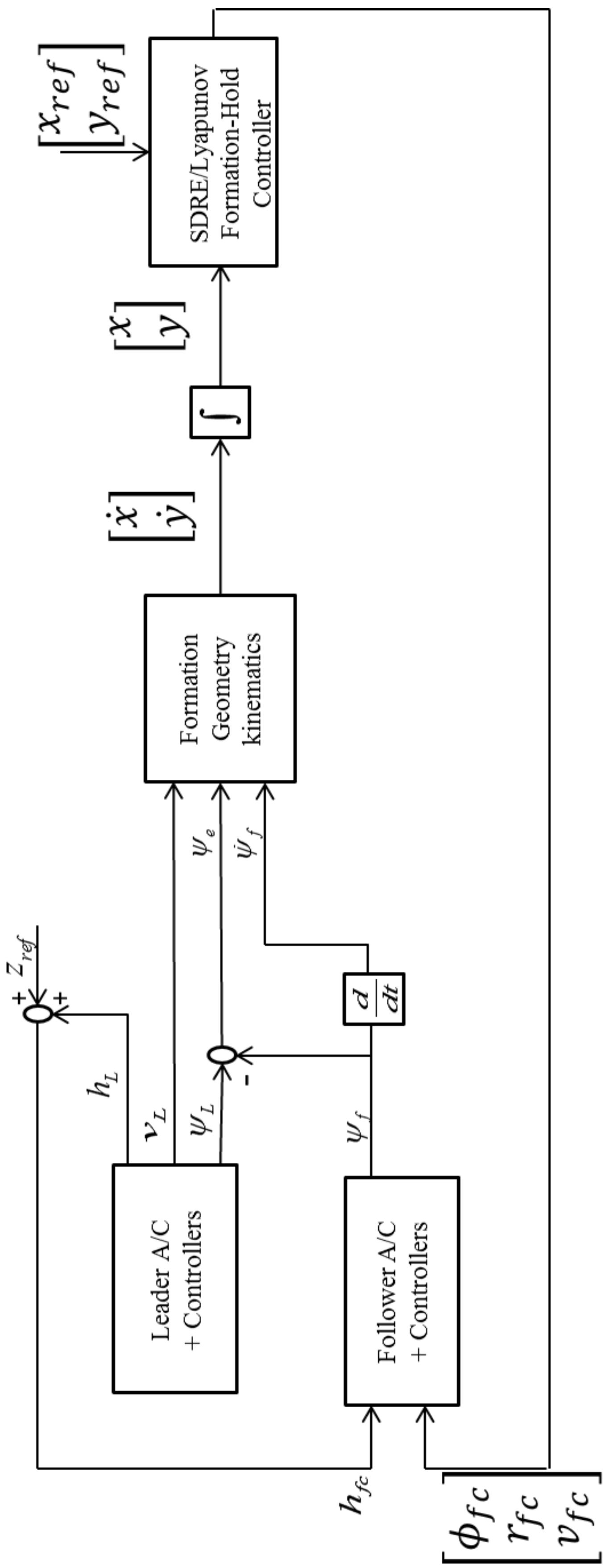

Figure 2: Block Diagram of the Formation Control System

The formation control system proposed in this manuscript has a two-loop structure. The outer loop generates the guidance commands to hold the formation. This formation-hold controller ensures that (x, y), which is the x and y component of the follower UAV's relative position to the leader UAV, track the desired (x_{ref}, y_{ref}). The formation-hold controller is designed using both the SDRE based control algorithm and the lyapunov based control algorithm. The formation-hold controller determines the necessary commands that the follower UAV must realize in order to maintain the x and y components of its relative position to the leader UAV. The commands from the formation-hold controller are sent into the inner loop of the overall formation control structure. The inner loop contains the LQR controllers discussed earlier. To enable the follower track z_{ref}, an altitude controller is implemented on the follower UAV in the inner loop.

RESULTS AND DISCUSSION

The formation-hold controller is tested using the linearized model of the SIG RASCAL 110 model aircraft for both the leader and follower UAVs. For this purpose, a simulation code is written in Matlab. To examine the effectiveness of the SDRE approach as well as the lyapunov approach in realizing the desired formation geometry in spite of maneuvers carried out by the leader aircraft, three cases are considered. In each case, SDRE update rate was set to 1 Hz. In addition, the control surface deflections for the follower UAV are limited to ± 20 degrees for both the elevator and aileron and ± 15 degrees for the rudder. The throttle maximum value is also constrained. The SIG RASCAL model was linearized at 1000m altitude and 20m/s velocity, and it is given in the Appendix.

Case 1: The follower relative position is desired to be brought to $(x_{ref}, y_{ref}, z_{ref}) = (8,8,5)$, while the Leader UAV is flown at constant velocity, altitude and heading.

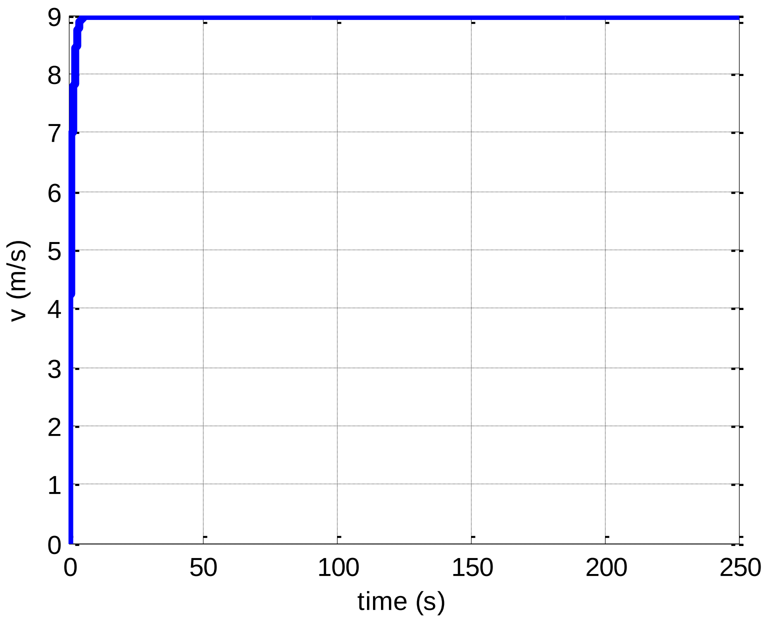

Figure 3: Time history of the Leader's velocity

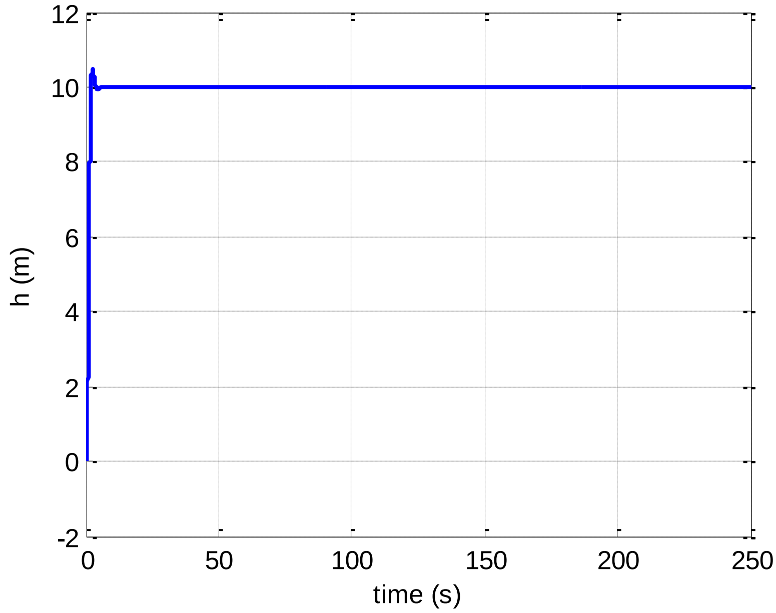

Figure 4: Time history of the Leader’s Altitude

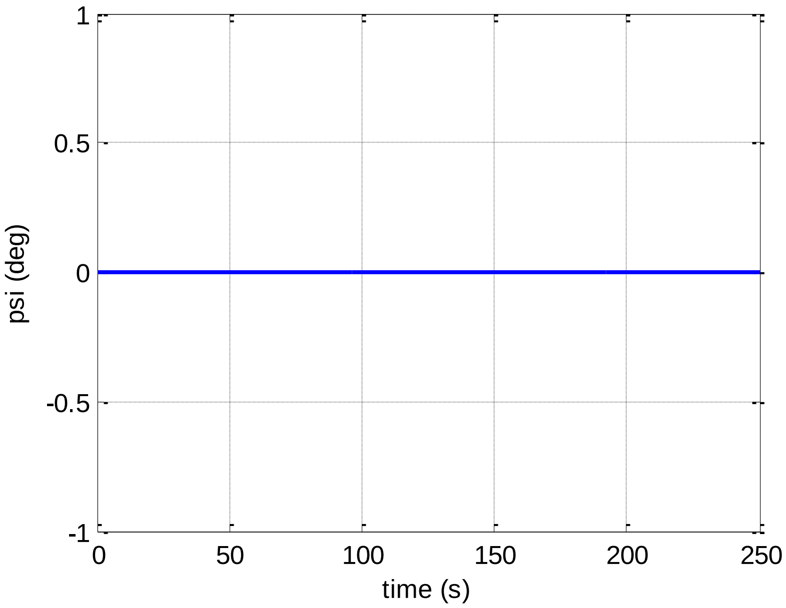

Figure 5: Time history of the Leader’s heading

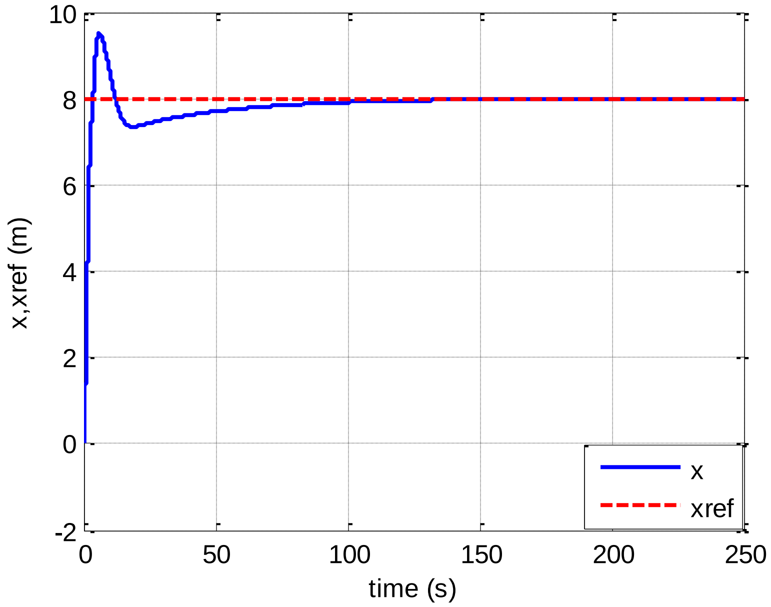

Figure 6: Time history of follower UAV's relative position, x-component, SDRE

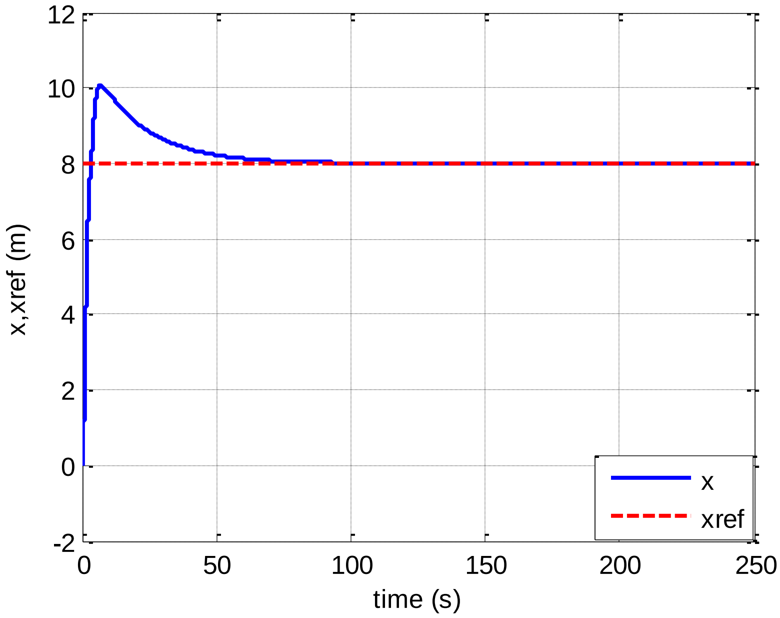

Figure 7: Time history of follower UAV's relative position, x-component, lyapunov

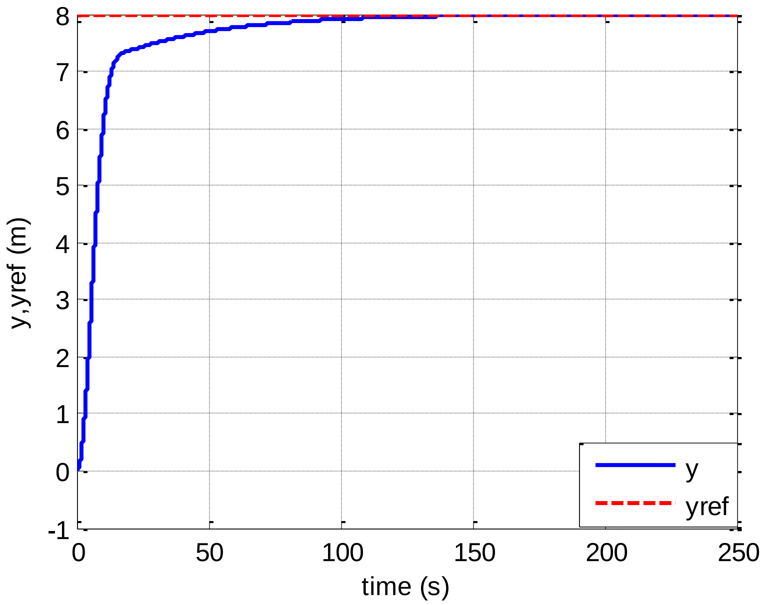

Figure 8: Time history of follower UAV’s relative position, y-component, SDRE

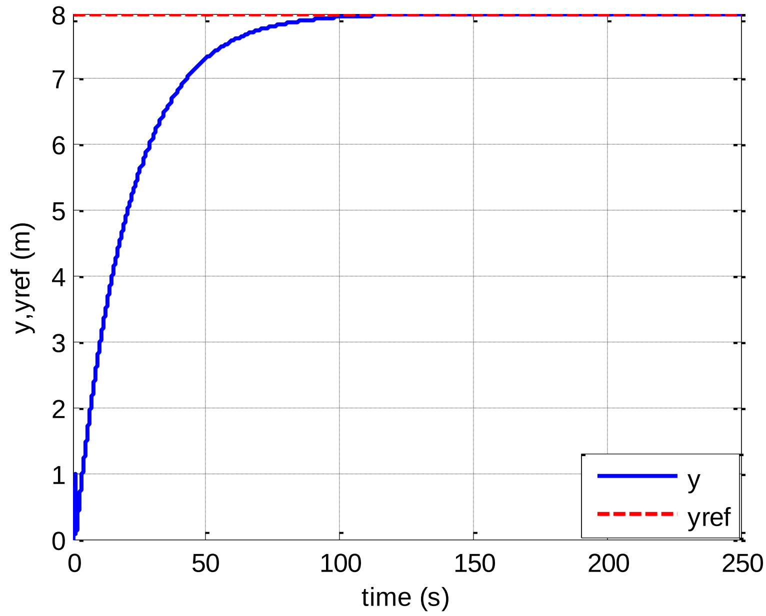

Figure 9: Time history of follower UAV’s relative position, y-component, Lyapunov

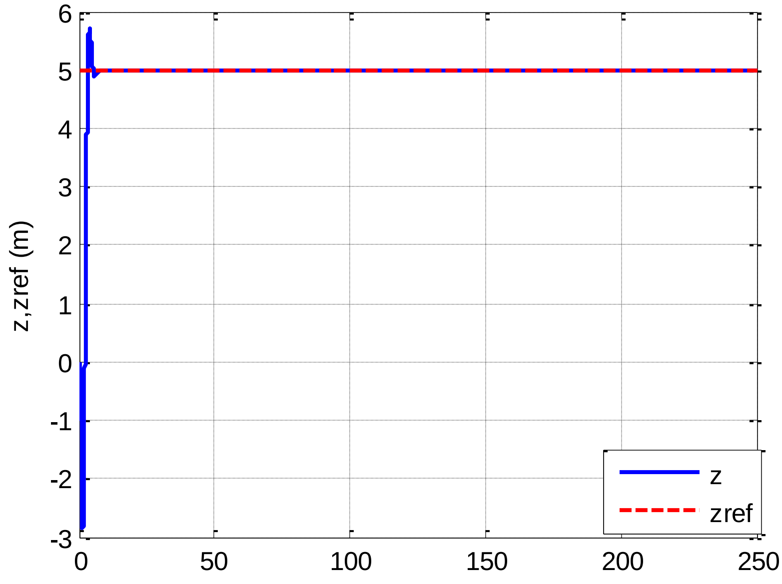

Figure 10: Time history of follower UAV's relative position, z-component, (SDRE,Lyapunov)

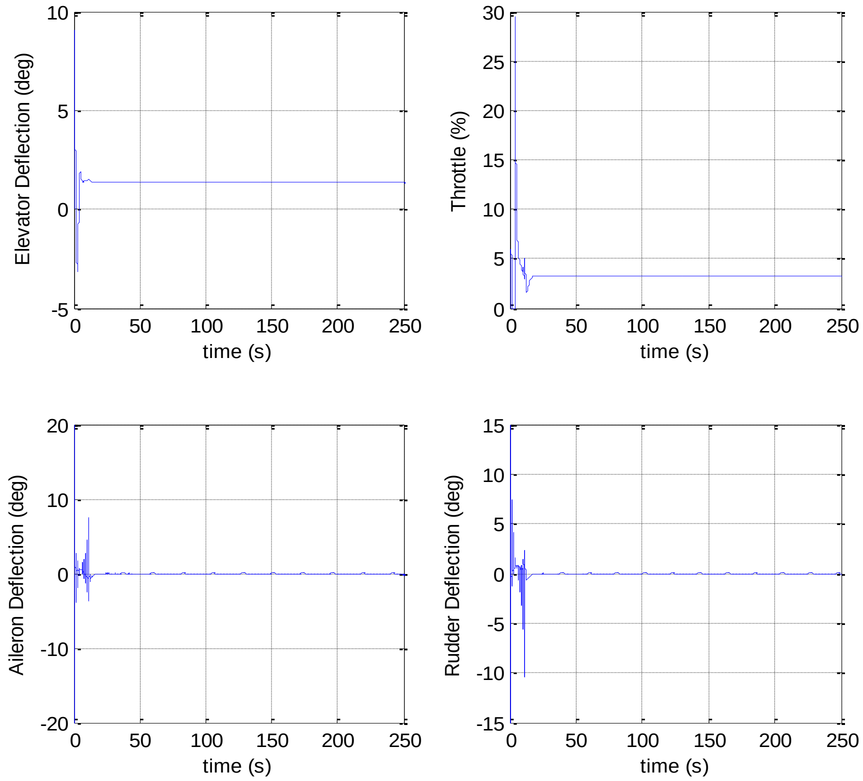

Figure 11: Time history of the control surfaces deflections and throttle of the Follower aircraft, SDRE

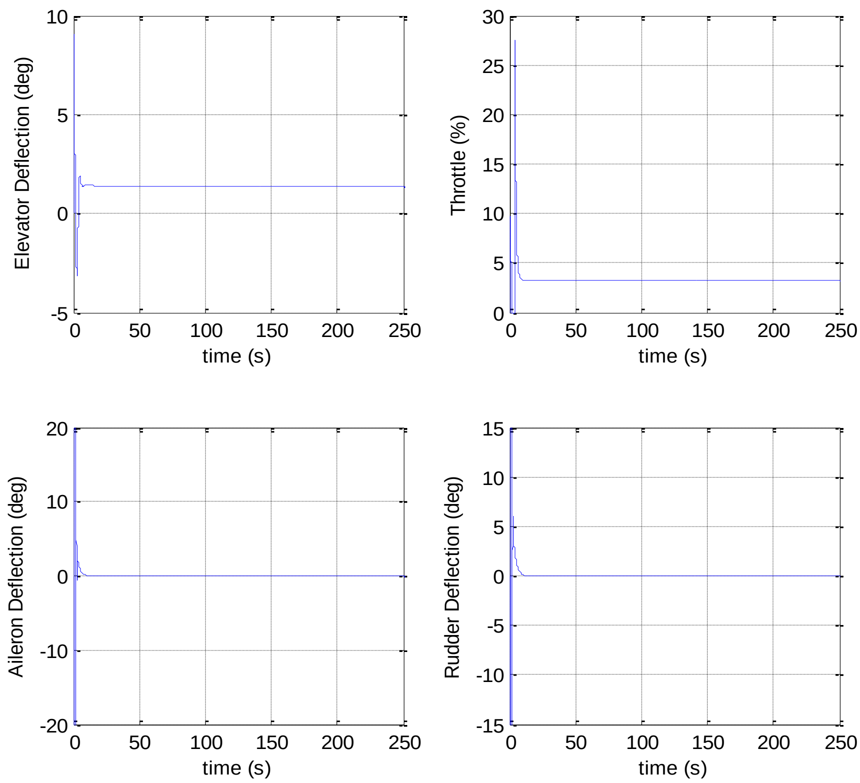

Figure 12: Time history of the control surfaces deflections and throttle of the Follower aircraft, Lyapunov

From figures 6 to 10, it may be observed that both the SDRE and Lyapunov formation-hold controller, together with the inner loop controllers enables the follower aircraft to settle to the reference formation geometry. Saturations in the control surface deflections can be seen for both the SDRE and lyapunov approach, however, these saturations do not adversely affect the performances of both formation control systems.

Case 2: In this case, the relative position of the follower was desired to be brought to $(x_{ref}, y_{ref}, z_{ref}) = (10,10,0)$ while the leader aircraft changes its velocity, altitude and heading.

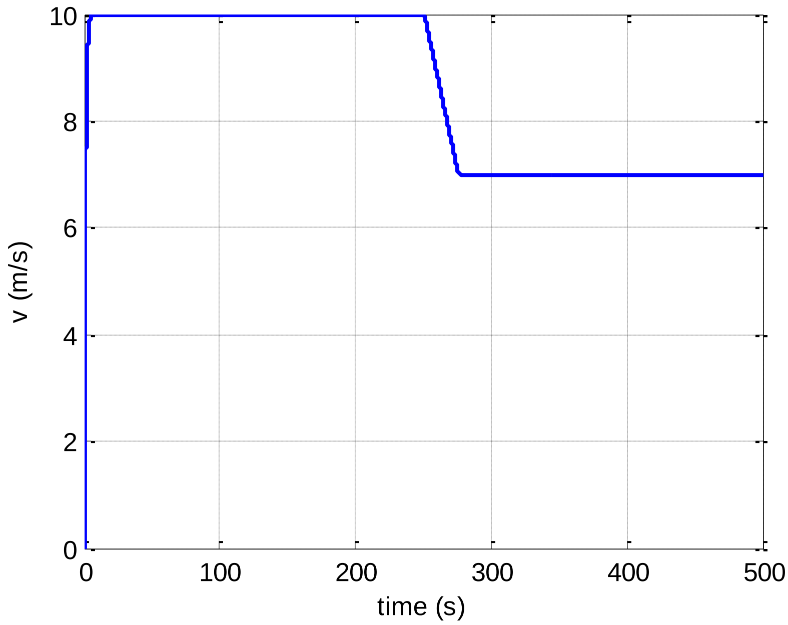

Figure 13: Time history of leader UAV’s velocity

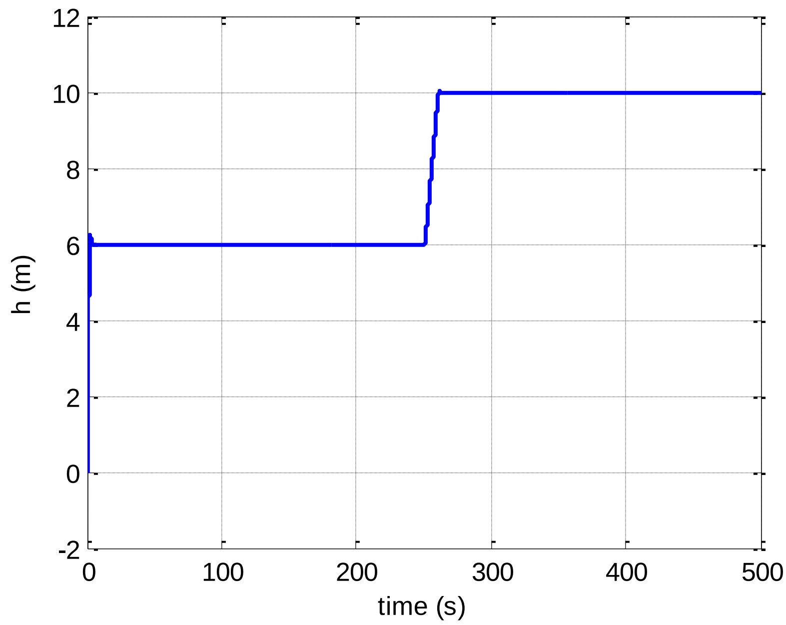

Figure 14: Time history of leader UAV’s altitude

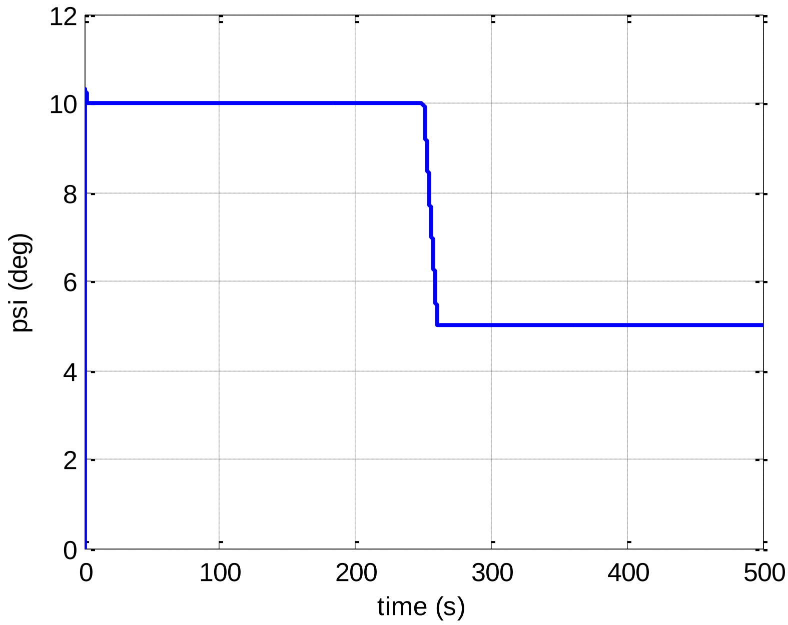

Figure 15: Time history of leader UAV’s heading

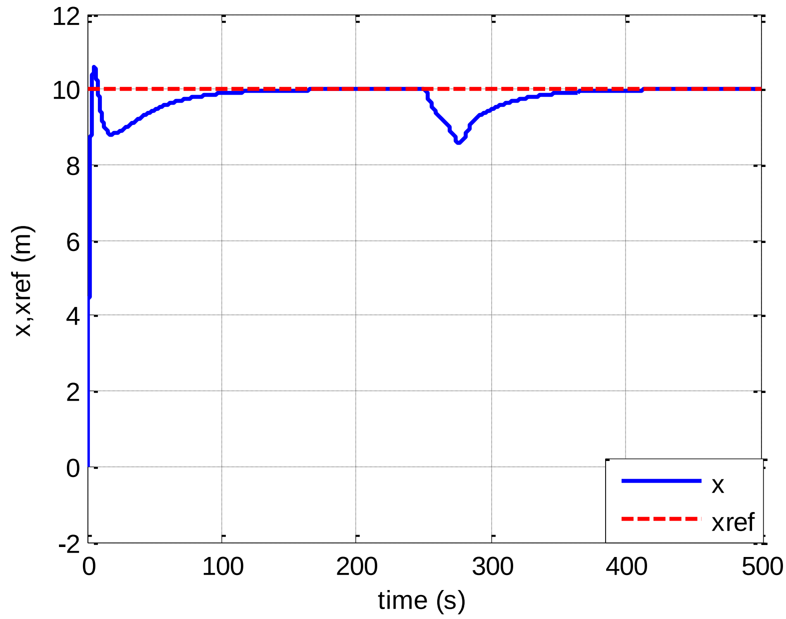

Figure 16: Time history of follower UAV’s relative position, x-component, SDRE

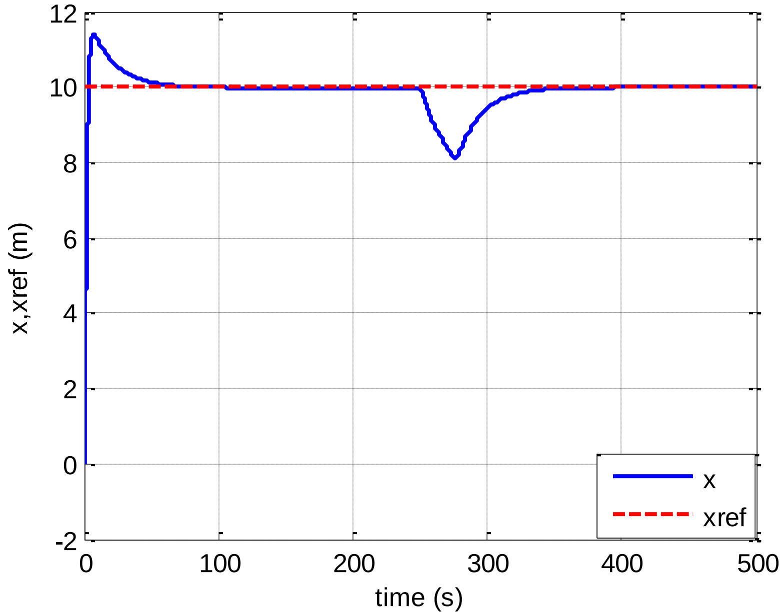

Figure 17: Time history of follower UAV's relative position, x-component, lyapunov

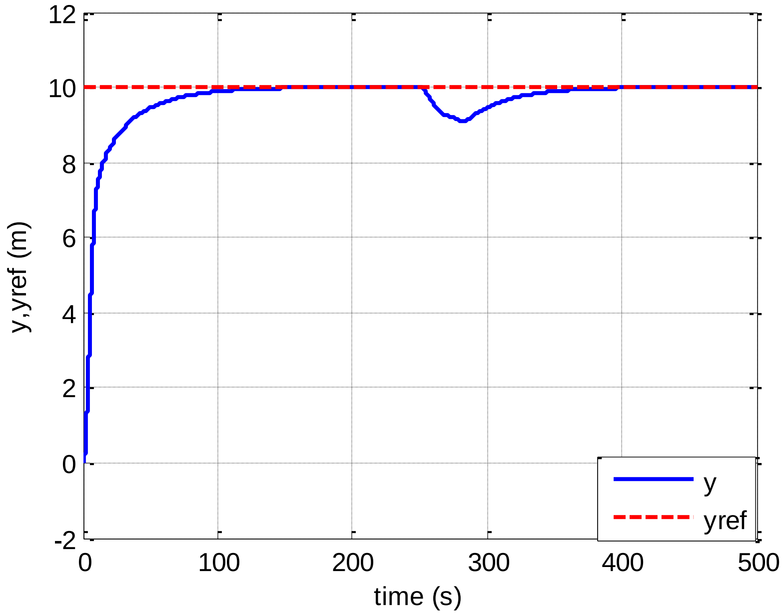

Figure 18: Time history of follower UAV's relative position, y-component, SDRE

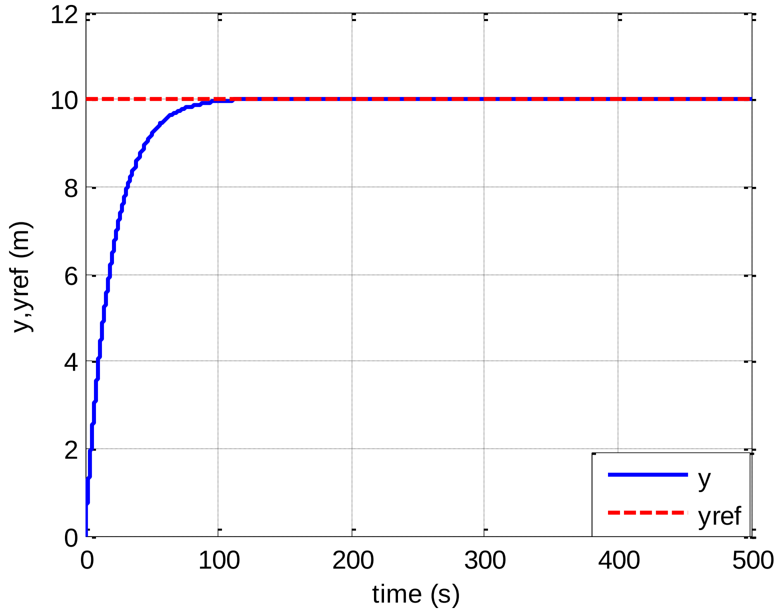

Figure 19: Time history of follower UAV's relative position, y-component, Lyapunov

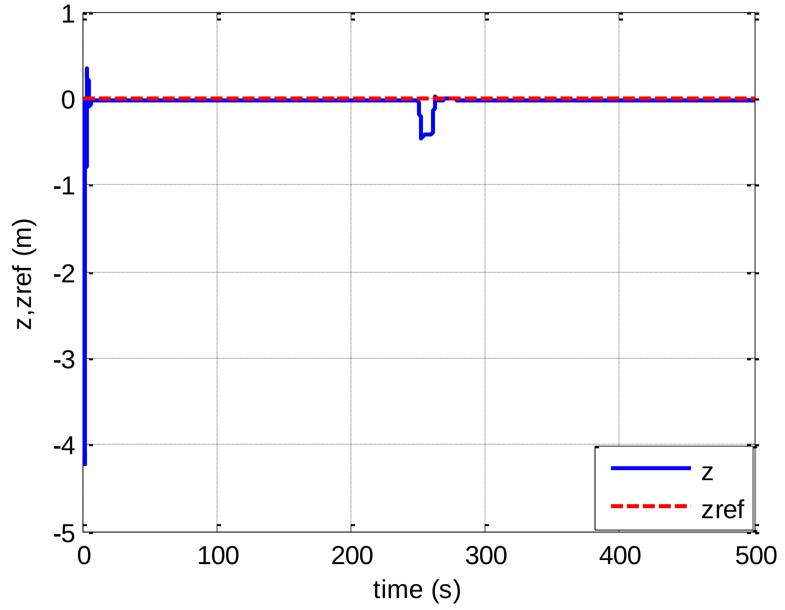

Figure 20: Time history of follower UAV's relative position, z-component, (SDRE,Lyapunov)

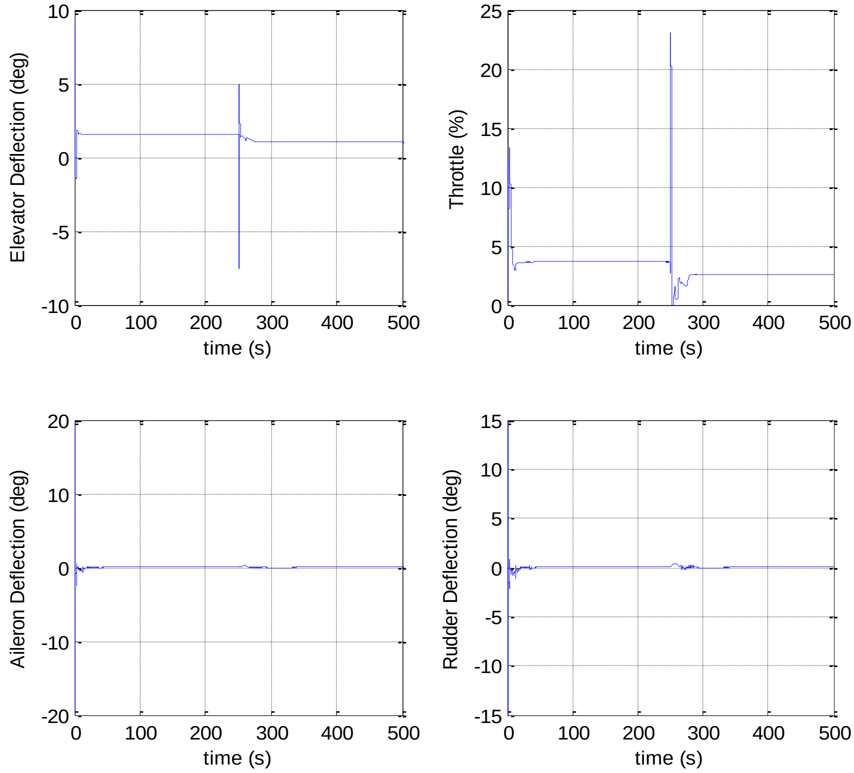

Figure 21: Time history of the control surfaces deflections and throttle of the Follower aircraft, SDRE

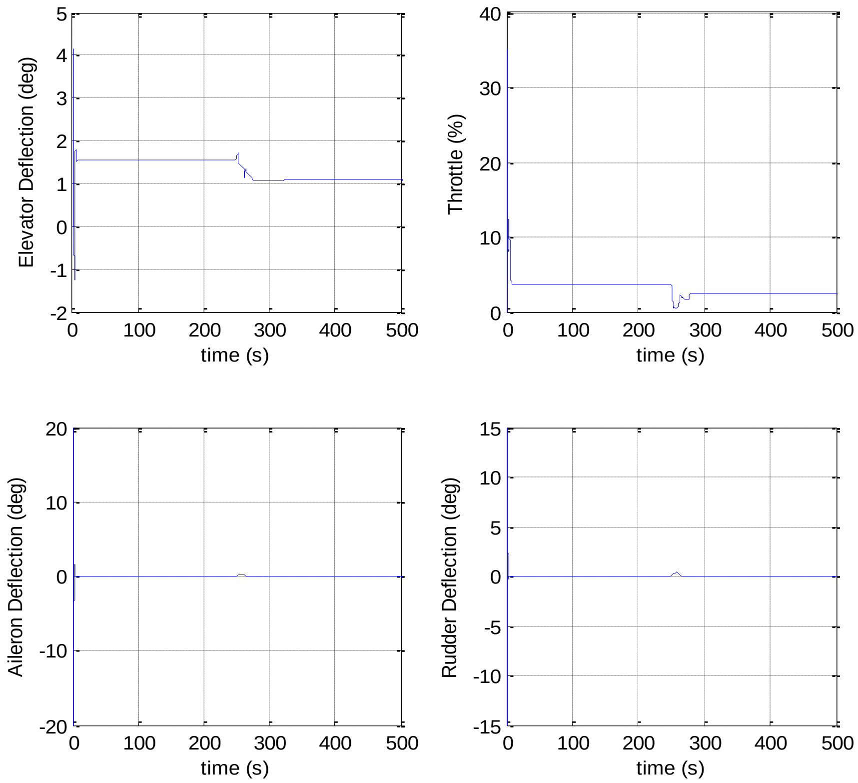

Figure 22: Time history of the control surfaces deflections and throttle of the Follower aircraft, Lyapunov

It may be observed from Figures 16 to 20 that both the SDRE and Lyapunov formation-hold controller enable the follower aircraft to maintain the desired relative position with the leader aircraft in spite of the maneuvers of the leader aircraft. Both formation hold controllers lagged behind a bit in maintaining the desired relative position when the flight parameters of the leader aircraft changed, however, they both soon catch up in maintaining the desired separation distance. The lyapunov controller performs slightly better in this case in the sense that, there was not any lag in maintaining the desired y-component of the desired relative position. Again, saturations are seen for both algorithms but these saturations have no substantial impact of the overall performance of the formation control system.

Case 3: In the first 250s, relative position of the follower is desired to be kept at $(x_{ref}, y_{ref}, z_{ref}) = (8,5,5)$ and then steadily switch to $(x_{ref}, y_{ref}, z_{ref}) = (5,0,0)$. The Leader UAV is flown at constant velocity, constant altitude and varying heading as shown in Figures 23, 24 and 25.

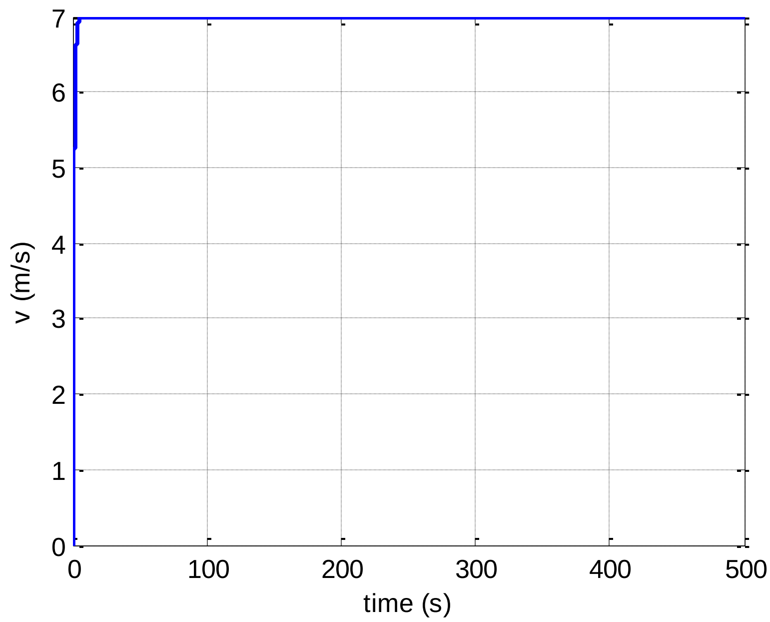

Figure 23: Time history of leader UAV's velocity

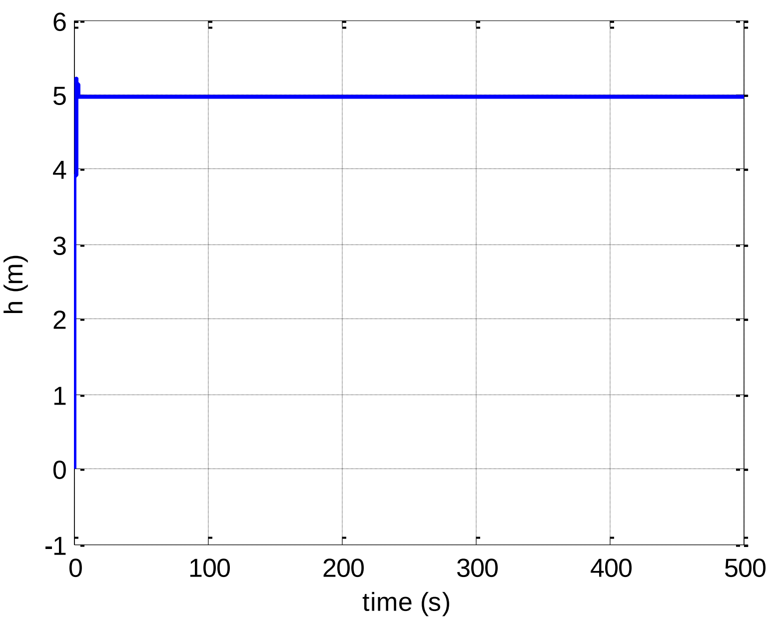

Figure 24: Time history of leader UAV's altitude

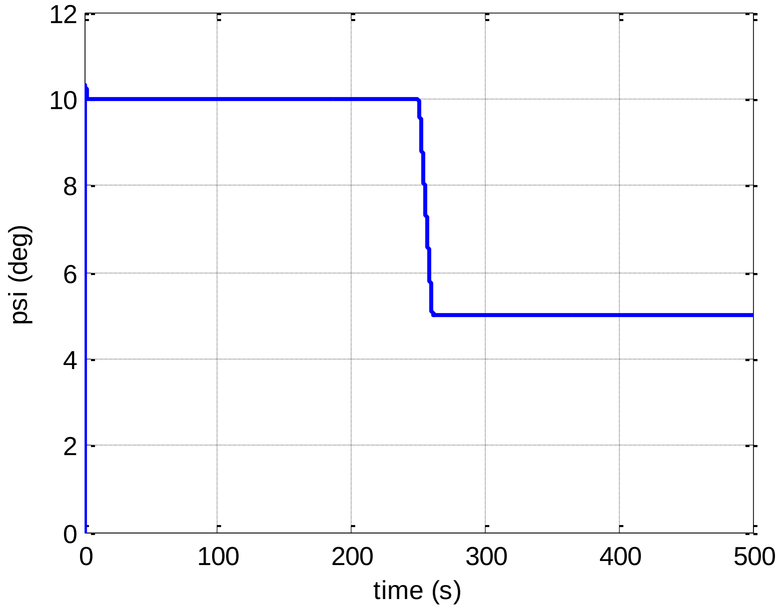

Figure 25: Time history of leader UAV's heading

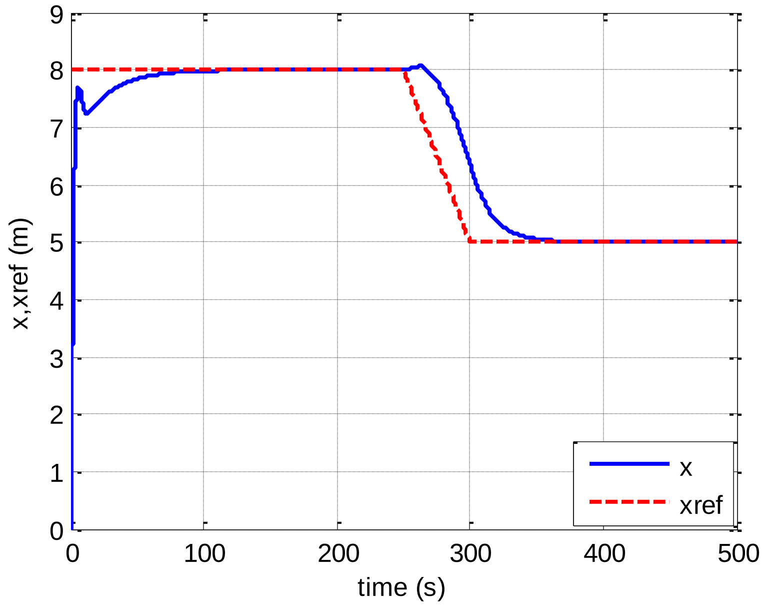

Figure 26: Time history of follower UAV's relative position, x-component, SDRE

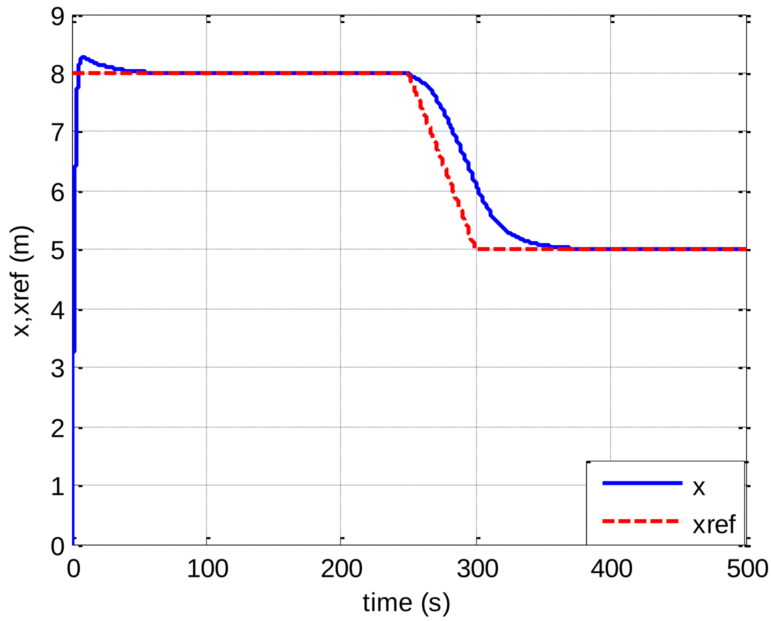

Figure 27: Time history of follower UAV's relative position, x-component, lyapunov

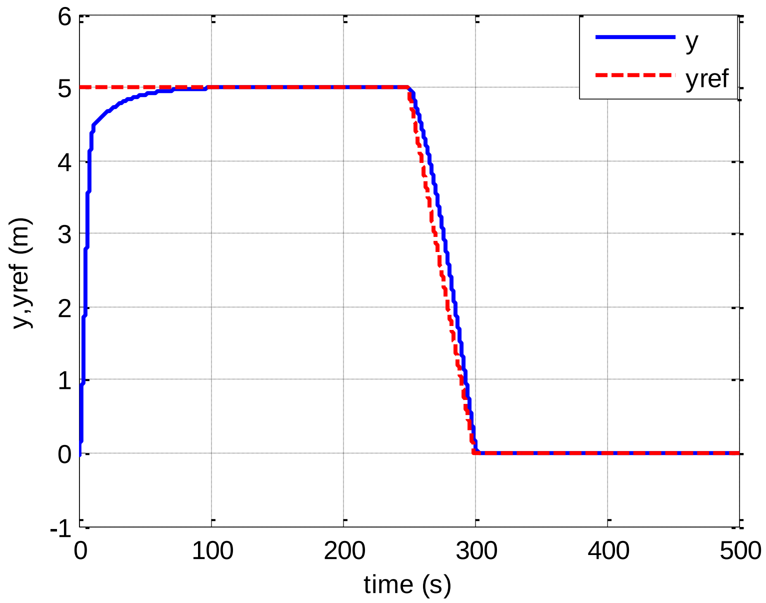

Figure 28: Time history of follower UAV's relative position, y-component, SDRE

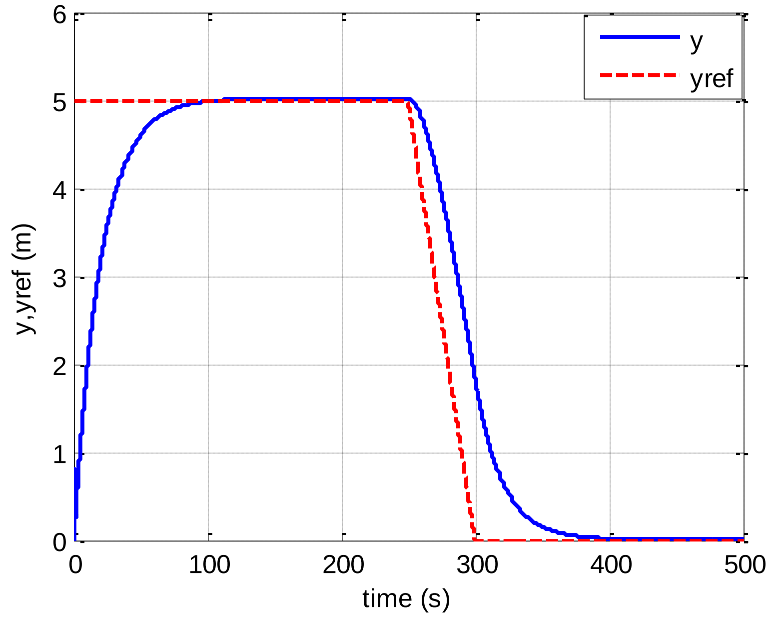

Figure 29: Time history of follower UAV's relative position, y-component, Lyapunov

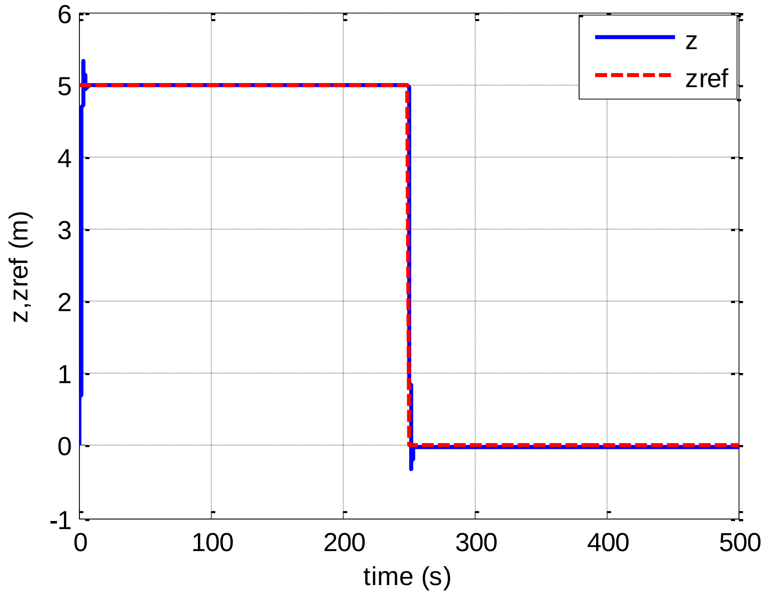

Figure 30: Time history of follower UAV's relative position, z-component, (SDRE, Lyapunov)

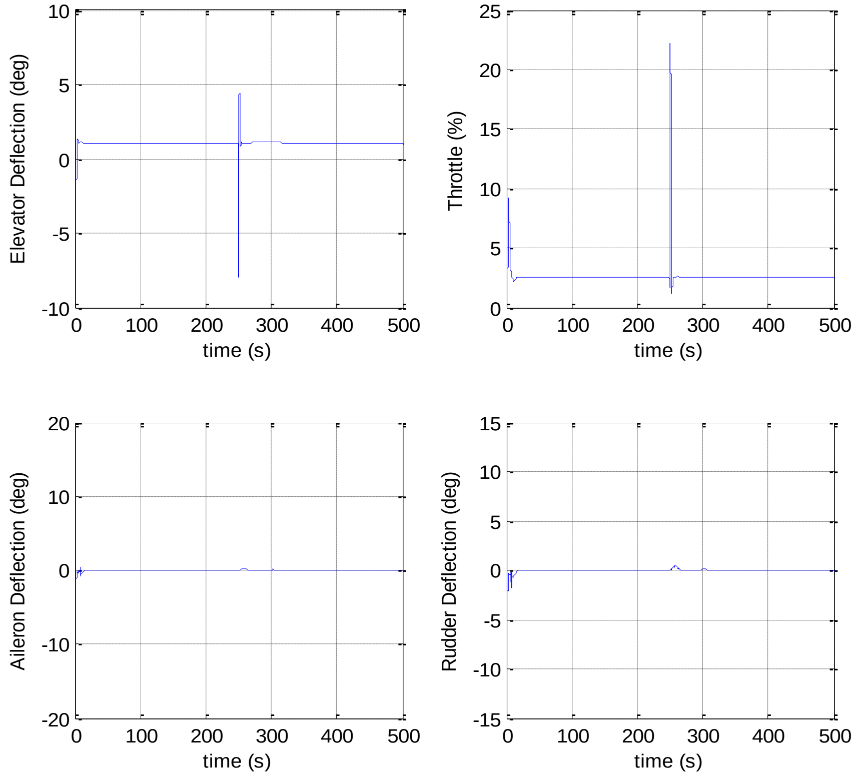

Figure 31: Time history of the control surfaces deflections and throttle of the Follower aircraft, SDRE

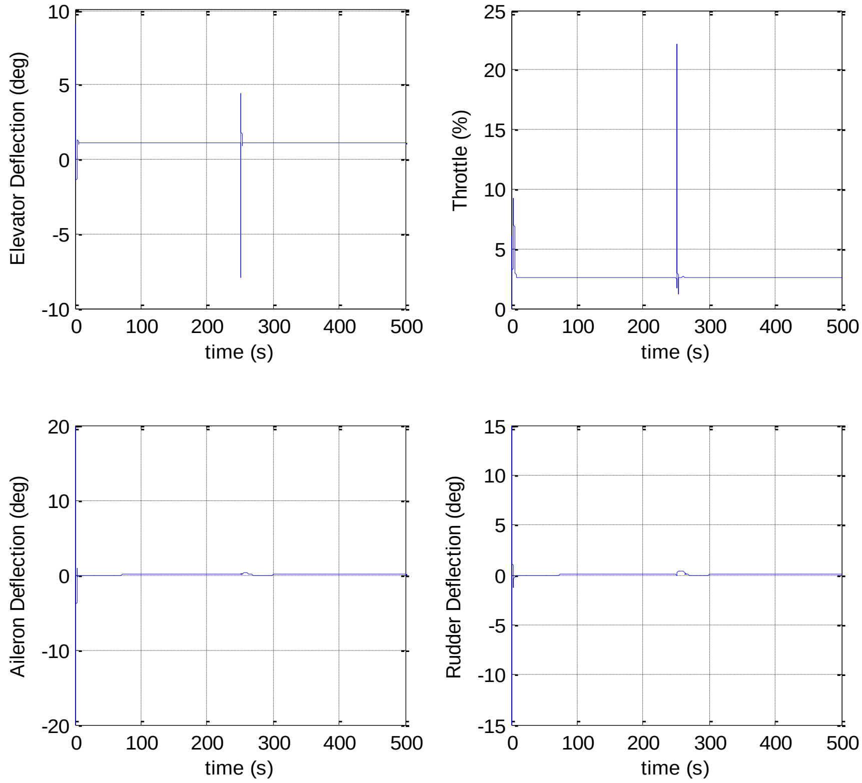

Figure 32: Time history of the control surfaces deflections and throttle of the Follower aircraft, Lyapunov

From figures 26 to 30, it may be observed that both the SDRE and Lyapunov formation-hold controller enables follower aircraft to successfully track varying formation geometry in spite of the constant maneuvers of the leader aircraft. However, the SDRE formation-hold controller switches from the initial formation geometry to the final formation geometry much faster than the lyapunov formation-hold controller.

CONCLUSIONS

In this paper, two formation control algorithms based on SDRE and Lyapunov control approaches are developed and applied to guide the follower to the desired relative formation. The performance of both algorithms is presented for three different flight scenarios. The simulations results demonstrates the success of both approaches to successfully realize the formation geometry. Comparisons between the two control approaches are then made and discussed.

APPENDIX

Longitudinal Dynamics of the Leader and Follower UAVs (SIG RASCAL 110)

$$Along = \begin{bmatrix} -0.0893 & 0.1064 & 0.3701 & -9.8039 & -0.0001 \\ -1.1273 & -7.4207 & 17.7733 & 0.2041 & -0.0010 \\ 0.0406 & -0.3144 & -8.0281 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0.0208 & 0.9988 & 0 & -20.0043 & 0 \end{bmatrix}$$

$$Blong = \begin{bmatrix} -0.5302 & 0.1135 \\ -9.8269 & -0.0001 \\ -33.0768 & -0.0003 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad Xlong = \begin{bmatrix} u \\ w \\ q \\ \theta \\ h \end{bmatrix} \quad ulong = \begin{bmatrix} \delta elev \\ \delta th \end{bmatrix}$$

Lateral Dynamics of the Leader and Follower UAVs (SIG RASCAL 110)

$$Alate = \begin{bmatrix} -0.4807 & -0.6362 & -20 & -9.8039 & 0 \\ -0.9700 & -8.2639 & 0.1482 & 0 & 0 \\ 0.2808 & -0.3477 & -0.5848 & 0 & 0 \\ 0 & 1 & -0.0208 & 0 & 0 \\ 0 & 0 & 1.0002 & 0 & 0 \end{bmatrix}$$

$$Blate = \begin{bmatrix} -0.5302 & 0.1135 \\ -9.8269 & 0.0001 \\ -33.0768 & -0.0003 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad Xlate = \begin{bmatrix} v \\ p \\ r \\ \phi \\ \psi \end{bmatrix} \quad ulate = \begin{bmatrix} \delta ail \\ \delta rud \end{bmatrix}$$

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