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A MULTIPLE-LOOP AIRCRAFT FLIGHT CONTROLLER USING SDRE TECHNIQUE

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ABSTRACT

In this paper an application of a multiple-loop State-Dependent Riccati Equation control algorithm is proposed for the longitudinal control for a fixed-wing aircraft. The purpose of designing a multiple-loop control systems is to decrease the sizes of the system's state vectors, that leads to reduction of the computational cost of the solution of an Algebraic Riccati equation. The problem of tracking the desired trajectory using a nonlinear control is also covered while developing an altitude controller for an aircraft

INTRODUCTION

State-Dependent Riccati Equation (SDRE) control technique provides an approximate solution for the nonlinear optimal control problem with a linear quadratic cost function. The first work introducing SDRE method belongs to [Pearson J.D. , 1962], followed by Wernli and Cook [Wernli A., Cook G. , 1975]. The basic idea of an SDRE control as well as representation of the system dynamics in the state-dependent coefficient (SDC) form may be found in [Cloutier, 1997]. The SDC matrices can be treated as constant in discrete time steps, and thus an approximate solution to a nonlinear state-dependent Riccati equation can be obtained. Considering application of the control theory to aerospace systems, classical sequential loop closing techniques based on trim flight equations of motion can be found to be the most popular [MacLean , 1990; Roskam, 1995]. The changing flight conditions are usually handled by the gain scheduling. Nonlinear controllers, on the other hand, might eliminate the need for gain scheduling, which typically requires extensive flight testing and tuning. Lyapunov function based nonlinear control is generally used in spacecraft attitude control applications. The most well-known technique is the quaternion error feedback algorithm, [Tekinalp O. , 2010]. Applications of a SDRE control method to flight control systems are highlighted in literature by a number of examples. [Bogdanov A. and Wan E., 2004] demonstrates application of an SDRE control approach to an autonomous helicopter. An SDRE controller along with a nonlinear feed-forward compensator applied to a small helicopter is present in [Bogdanov A. and Wan E., 2003]. [Chang D. Y. et al , 2009] presents an application of the SDRE method to a rotorcraft guidance and control. However, there are two difficulties in an SDRE approach. The first problem is related to the controllability issue, which arises when a physically controllable system may lose controllability due to the choice of factorization. Then the solution of the ARE is not possible. The second difficulty is the dimension of the matrices. For the systems of large orders, when dimensions of the state vectors are high, computing a solution of the algebraic state-dependent Riccati equation is computationally costly. To simplify the solution of the state-dependent Riccati equation some iterative techniques were also proposed by [Benner P. et al , 2008]. Introducing a multiple-loop structure of the feedback control system allows to reduce dimensions of the state vectors by separating them into the outer and inner loop state vectors, in such a way that the computational efforts on the solution of the state-dependent Riccati equation may be sufficiently reduced. Also, separation of the states that are related to faster or slower dynamics using a multi-loop structure, allows setting the feedback gain update frequency independently, reducing the workload on the

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flight computer. This approach is recently applied to the spacecraft attitude control problem [Kayastha,S. , 2010]. A definition of the tracking problem and its solution for a general case of a linear observable system is given in [Anderson B. D. O. and Moore J.B. , 1989; Naidu D.S. , 2002]. In this paper we extend the results to a state-dependent system model and also introduce a nonlinear compensator, which is used to cancel the differences that may occur between the original nonlinear model and an SDC form used by SDRE technique. The paper is organized as follows. Firstly, a background to the extended linearization and SDRE control is introduced. Definition of the tracking controller and the nonlinear compensator design are presented in the following section. Then control system structure along with the state-dependent models are given. Finally, concluding remarks are given.

SDRE NONLINEAR REGULATOR PROBLEM

SDC Parametrization

Extended linearization, also known as apparent linearization or SDC parameterization is the process of factorizing a nonlinear system into a linear-like structure, which contains SDC matrices [Friedland B. , 1996]. Consider the system, which is full-state observable, autonomous, nonlinear in the state, and affine in the input, represented in the following form [Cimen T., 2008] :

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}) + \mathbf{B}(\mathbf{x})\mathbf{u}(t), \mathbf{x}(\mathbf{0}) = \mathbf{x_0} \tag{1}$$

where $\mathbf{x} \in \mathbf{R^n}$ is the state vector, $\mathbf{u} \in \mathbf{R^m}$ is the input vector, function $\mathbf{f} : \mathbf{R^n} \to \mathbf{R^n}$, $\mathbf{B} : \mathbf{R^n} \to \mathbf{R^{n\times m}}$ and $\mathbf{B}(\mathbf{x}) \neq \mathbf{0}, \forall \mathbf{x}$.

Under the assumptions $\mathbf{f}(\mathbf{0}) = \mathbf{0}$, and $\mathbf{f}(\cdot) \in \mathbf{C^1}(\mathbf{R^n})$ a continuous nonlinear matrix-valued function $\mathbf{A}(\mathbf{x})$ always exists such that

$$\mathbf{f}(\mathbf{x}) = \mathbf{A}(\mathbf{x})\mathbf{x} \tag{2}$$

where $\mathbf{A}$, the $n \times n$ matrix, is found by mathematical factorization and is non unique when $n > 1$. Hence, extended linearization of the input-affine nonlinear system (1) becomes

$$\dot{\mathbf{x}}(t) = \mathbf{A}(\mathbf{x})\mathbf{x}(t) + \mathbf{B}(\mathbf{x})\mathbf{u}(t), \mathbf{x}(\mathbf{0}) = \mathbf{x_0} \tag{3}$$

which has a linear structure with SDC matrices $\mathbf{A}(\mathbf{x})$, $\mathbf{B}(\mathbf{x})$. The application of any linear control synthesis method to the linear-like SDC structure, where A(x) and B(x) are treated as constant matrices, forms an extended linearization control method [Cimen T., 2008].

Control Problem Formulation

SDRE control is referred to a nonlinear optimal control problem, and is based on the same approach as a linear quadratic optimal control theory. The performance cost function to be minimized is defined as follows:

$$\mathbf{J}(\mathbf{x_0}, \mathbf{u}) = \frac{\mathbf{1}}{\mathbf{2}} \int_{\mathbf{0}}^{\infty} \Big\{ \mathbf{x^T}(t)\mathbf{Q}(\mathbf{x})\mathbf{x}(t) + \mathbf{u^T}(t)\mathbf{R}(\mathbf{x})\mathbf{u}(t) \Big\} \mathbf{dt} \tag{4}$$

where $\mathbf{Q}(\mathbf{x}) \in \mathbf{R^{n\times m}}$ is symmetric positive semidefinite, $\mathbf{R}(\mathbf{x}) \in \mathbf{R^{m\times m}}$ is symmetric positive definite matrix, which in general may be state dependent. For the calculation of instantaneous feedback gains, the weighting matrices, $\mathbf{Q}$ and $\mathbf{R}$ as well as system matrices, $\mathbf{A}$ and $\mathbf{B}$ are assumed to be constant. Then, for the given sate, the feedback gain is calculated as it is done for an infinite horizon LQR controller:

$$\mathbf{u}(\mathbf{x}) = -\mathbf{K}(\mathbf{x})\mathbf{x} = -\mathbf{R^{-1}}(\mathbf{x})\mathbf{B^T}(\mathbf{x})\mathbf{P}(\mathbf{x})\mathbf{x} \tag{5}$$

where $\mathbf{P}(\mathbf{x})$ is the solution of the following Algebraic State Dependent Riccati Equation:

$$\mathbf{P}(\mathbf{x})\mathbf{A}(\mathbf{x})\mathbf{A^T}(\mathbf{x})\mathbf{P}(\mathbf{x}) - \mathbf{P}(\mathbf{x})\mathbf{B}(\mathbf{x})\mathbf{R^{-1}}(\mathbf{x})\mathbf{B^T}(\mathbf{x})\mathbf{P}(\mathbf{x}) + \mathbf{Q}(\mathbf{x}) = \mathbf{0} \tag{6}$$

At each time step, the matrices $\mathbf{A}$ and $\mathbf{B}$ are treated as being constant, thus a control problem reduces to a linear quadratic optimal control problem [Cimen T., 2008]. However, the controller generated this way is a nonlinear controller with the convenience that it does not require the linearization of the system equations. One important issue is to make sure that the system matrices and form a fully controllable pair.

TRACKING CONTROLLER

A regulator theory can be extended to solve the wider class of control problems that involve achieving a desired trajectory. If the desired trajectory is a particular prescribed function of time, the problem is known as a tracking problem. In [Naidu D.S. , 2002] and [Anderson B. D. O. and Moore J.B. , 1989] provide derivation of the optimal control law for a linear quadratic tracking system. Consider a linear time invariant observable system (7)

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{A}(t)\mathbf{x}(t) + \mathbf{B}(t)\mathbf{u}(t)\\ \mathbf{y}(t) &= \mathbf{C}(t)\mathbf{x}(t)\end{aligned} \tag{7}$$

where $\mathbf{x}(t)$ is an n^{th} order state vector, $\mathbf{u}(t)$ is the r^{th} order control vector, and $\mathbf{y}(t)$ is the m^{th} order output vector.

The optimal tracking problem implies finding the optimal control $\mathbf{u}(t)$ for the system(7) such that the output $\mathbf{y}(t)$ tracks the desired signal $\mathbf{r}(t)$ minimizing the performance index given as:

$$\mathbf{J} = \frac{\mathbf{1}}{\mathbf{2}}\mathbf{e^T}(\mathbf{t_f})\mathbf{F}(\mathbf{t_f})\mathbf{e}(\mathbf{t_f}) + \frac{\mathbf{1}}{\mathbf{2}}\int_{\mathbf{t_0}}^{\mathbf{t_\infty}} \Big\{\mathbf{e^T}(t)\mathbf{Q}(t)\mathbf{e}(t) + \mathbf{u^T}(t)\mathbf{R}(t)\mathbf{u}(t)\Big\}\mathbf{dt} \tag{8}$$

where $\mathbf{e}(t) = \mathbf{z}(t) - \mathbf{y}(t)$ is the error vector.

It is assumed that $\mathbf{F}(\mathbf{t_f})$ and $\mathbf{Q}(t)$ are $m \times m$ symmetric positive semidefinite matrices, and $\mathbf{R}(t)$ is $r \times r$ symmetric positive definite matrix. A detailed solution for the optimal control law for the finite time and infinite time cases are presented in [Naidu D.S. , 2002] and [Anderson B. D. O. and Moore J.B. , 1989] .

In this paper a modified problem is considered and the system is assumed to be given in the following form:

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{A}(t)\mathbf{x}(t) + \mathbf{B}(t)\mathbf{u}(t) + \mathbf{f}\\ \mathbf{y}(t) &= \mathbf{C}(t)\mathbf{x}(t)\end{aligned} \tag{9}$$

where $\mathbf{f}$ represents the mismatch that appears as a result of the factorization of the nonlinear system equations in the form of (7) provided that $\mathbf{f}$ is a slowly varying signal that may be assumed constant at certain time intervals, and $\mathbf{f}$ is bounded. For the performance index given by Eq.(8), the Hamiltonian of the minimum cost function then may be defined as follows:

$$\begin{aligned}\mathbf{H}(\mathbf{x}(t), \mathbf{u}(t), \lambda(t)) = {}& \tfrac{\mathbf{1}}{\mathbf{2}}[\mathbf{z}(t) - \mathbf{C}(t)\mathbf{x}(t)]^{\mathbf{T}}\mathbf{Q}(t)[\mathbf{z}(t) - \mathbf{C}(t)\mathbf{x}(t)] + \tfrac{\mathbf{1}}{\mathbf{2}}\mathbf{u^T}(t)\mathbf{R}(t)\mathbf{u}(t)\\ &+\lambda^{\mathbf{T}}(t)[\mathbf{A}(t)\mathbf{x}(t) + \mathbf{B}(t)\mathbf{u}(t) + \mathbf{f}]\end{aligned} \tag{10}$$

The optimal control equation is obtained from $\frac{\partial \mathbf{H}}{\partial \mathbf{u}} = \mathbf{0}$, which results into the optimal control given in the following form:

$$\mathbf{u}^*(t) = -\mathbf{R^{-1}}(t)\mathbf{B^T}(t)\lambda^*(t) \tag{11}$$

Then the Hamiltonian canonical form for the optimal state and costate equations may be written:

$$\begin{bmatrix} \dot{\mathbf{x}}^*(t) \\ \dot{\lambda}^*(t) \end{bmatrix} = \begin{bmatrix} \mathbf{A}(t) & -\mathbf{E}(t) \\ -\mathbf{V}(t) & -\mathbf{A^T}(t) \end{bmatrix} \begin{bmatrix} \mathbf{x}^*(t) \\ \lambda^*(t) \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{W}(t) \end{bmatrix}\mathbf{z}(t) + \begin{bmatrix} \mathbf{f} \\ \mathbf{0} \end{bmatrix} \tag{12}$$

where

$$\begin{aligned}\mathbf{E}(t) &= \mathbf{B}(t)\mathbf{R^{-1}}(t)\mathbf{B^T}(t)\\ \mathbf{V}(t) &= \mathbf{C^T}(t)\mathbf{Q}(t)\mathbf{C}(t)\\ \mathbf{W}(t) &= \mathbf{C^T}(t)\mathbf{Q}(t)\end{aligned} \tag{13}$$

The boundary conditions for the state and costate equations are defined by the initial condition on the state: $\mathbf{x}(\mathbf{t} = \mathbf{t_0}) = \mathbf{x}(\mathbf{t_0})$ and the final condition on the costate:

$$\lambda(\mathbf{t}_f) = \frac{\partial}{\partial(\mathbf{x}(\mathbf{t}_f))}\left[\frac{\mathbf{1}}{\mathbf{2}}\mathbf{e^T}(\mathbf{t}_f)\mathbf{F}(\mathbf{t}_f)\mathbf{e}(\mathbf{t}_f)\right] = \mathbf{C^T}(\mathbf{t}_f)\mathbf{F}(\mathbf{t}_f)\mathbf{C}(\mathbf{t}_f)\mathbf{x}(\mathbf{t}_f) - \mathbf{C^T}(\mathbf{t}_f)\mathbf{F}(\mathbf{t_f})\mathbf{z}(\mathbf{t}_f) \tag{14}$$

The boundary condition Eq. (14) and the solution of the system (12) indicate that the optimal state and costate are linearly related as

$$\lambda^*(t) = \mathbf{P}(t)\mathbf{x}^*(t) - \mathbf{g}(t) \tag{15}$$

where $\mathbf{P}(t)$ is a square matrix of size n and $\mathbf{g}(t)$ is a vector of length n, are to be determined such that the canonical system (12) is satisfied. As a result it may be shown that if $\mathbf{P}(t)$ can be found as solution to a matrix differential Riccati equation (16), and $\mathbf{g}(t)$ is a solution to a vector differential equation [Naidu D.S. , 2002] (17) :

$$\dot{\mathbf{P}}(t) = -\mathbf{P}(t)\mathbf{A}(t) - \mathbf{A^T}(t)\mathbf{P}(t) + \mathbf{P}(t)\mathbf{E}(t)\mathbf{P}(t) - \mathbf{V}(t) \tag{16}$$

$$\dot{\mathbf{g}}(t) = \left[\mathbf{P}(t)\mathbf{E}(t) - \mathbf{A^T}(t)\right]\mathbf{g}(t) - \mathbf{W}(t)\mathbf{z}(t) + \mathbf{P}(t)\mathbf{f} \tag{17}$$

The optimal control is obtained in the form given by Eq. (18).

$$\mathbf{u}^*(t) = -\mathbf{R^{-1}}(t)\mathbf{B^T}(t)\left[\mathbf{P}(t)\mathbf{x}^*(t) - \mathbf{g}(t)\right] = -\mathbf{K}(t)\mathbf{x}^*(t) + \mathbf{R^{-1}}(t)\mathbf{B^T}(t)\mathbf{g}(t) \tag{18}$$

For the infinite-horizon problem formulation, consider the system Eq. (9) but with the system matrices being time invariant, and the performance index chosen as

$$\lim_{t_f\to\infty}\mathbf{J} = \lim_{t_f\to\infty}\frac{\mathbf{1}}{\mathbf{2}}\int_{\mathbf{t_0}}^{\mathbf{t}_f}\left\{\mathbf{e^T}(t)\mathbf{Q}(t)\mathbf{e}(t) + \mathbf{u^T}(t)\mathbf{R}(t)\mathbf{u}(t)\right\}\mathbf{dt} \tag{19}$$

Using the results for a finite-time case above and let $\mathbf{t}_f \to \infty$ will lead to the infinite-time case solution. Thus, the matrix function $\mathbf{P}(t)$ in Eq. (16) will result to the steady-state value $\mathbf{P}$ as the solution of the following algebraic Riccati equation:

$$-\mathbf{PA} - \mathbf{A^TP} + \mathbf{PBR^{-1}B^TP} - \mathbf{C^TQC} = 0 \tag{20}$$

For slowly varying input signals $\mathbf{z}(t)$ solution of matrix differential equation (17) can be obtained by setting the derivative to zero and solving Eq. (17) for $\mathbf{q}(t)$:

$$\mathbf{g}(t) = \left[\mathbf{PE} - \mathbf{A^T}(t)\right]^{\mathbf{-1}}(\mathbf{Wz}(t) + \mathbf{Pf}) \tag{21}$$

where

$$\begin{array}{rcl}\mathbf{E} & = & \mathbf{BR^{-1}B^T} \\ \mathbf{W} & = & \mathbf{C^TQ}\end{array} \tag{22}$$

The final expression for the optimal control is obtained in the following form:

$$\mathbf{u}(t) = \mathbf{Kx}(t) + \mathbf{K_z z}(t) + \mathbf{K_f f} \tag{23}$$

and the corresponding controller gains are defined as:

$$\begin{array}{rcl}\mathbf{K} & = & -\mathbf{R^{-1}BP} \\ \mathbf{K_z} & = & \mathbf{R^{-1}B}\left[\mathbf{PE} - \mathbf{A^T}\right]^{\mathbf{-1}}\mathbf{W} \\ \mathbf{K_f} & = & -\mathbf{R^{-1}B}\left[\mathbf{PE} - \mathbf{A^T}\right]^{\mathbf{-1}}\mathbf{P}\end{array} \tag{24}$$

In this paper an application of the introduced tracking algorithm is illustrated for the case when the system matrices $\mathbf{A}$ and $\mathbf{B}$ as well as the additional term $\mathbf{f}$ are state dependent. At every time instant the system may be considered as linear time invariant. However, the overall control strategy results in a nonlinear controller with a nonlinear compensator. It should be noted that one of the important issues to consider is the boundedness of the performance index and the input signals, as well as the steady state tracking error. Extension of the finite-time results to the infinite-time case, and applying restrictions on the reference signal will result in the approximately optimal tracking control system

DUAL-LOOP SDRE CONTROLLER DESIGN

A dual-loop structure of the controller involves separation of the system states in such a way that the states of the inner loop are used as the controls for the outer loop states. The control for the inner loop states is achieved using the available actuators. In general, this separation of states may be based on the dynamics of the states as well as actuators used for the control. A nonlinear tracking system with a compensator is used for a design of the longitudinal controller for a fixed-wing aircraft.

A full set of nonlinear 6 degrees-of-freedom equations of motion of an aircraft written in the body-fixed coordinate system are given in [MacLean , 1990; Roskam, 1995]. Only longitudinal dynamics of the aircraft is considered to obtain the SDC model. Implementing a dual-loop structure of the control system, the linear and angular velocities of the aircraft are referred to the inner loop states for which control is achieved using elevator and thrust. While the pitch angle and altitude are referred to the outer loop states. Thus, the state and control vectors for the inner and outer loops dynamics are defined as:

$$\mathbf{x_{in}} = [u, w, q]^T, \mathbf{u_{in}} = [\delta_e, \delta_T]^T \tag{25}$$

$$\mathbf{x_{out}} = [\theta, h]^T, \mathbf{u_{in}} = [u, w, q]^T \tag{26}$$

Then a possible set of SDC matrices of the inner and outer loop may be written in the form:

$$\mathbf{A}_{in} = \begin{bmatrix} \frac{\frac{1}{2}\rho VS(-C_{D_0}-C_{D_\alpha}\alpha)}{m} & \frac{\bar{q}S(C_{L_\alpha}\alpha-C_{L_0})}{mu} & \frac{\bar{q}S\bar{c}(C_{L_q}+C_{L_{\dot{\alpha}}})\alpha}{m}\frac{\bar{c}}{2V_0} - w \\ \frac{\frac{1}{2}\rho VS(-C_{D_0}\alpha-C_{L_0})}{m} & \frac{\bar{q}S(-C_{D_\alpha}\alpha-C_{L_\alpha})}{mu} & \frac{\bar{q}S\bar{c}(-C_{L_q}-C_{L_{\dot{\alpha}}})\alpha}{m}\frac{\bar{c}}{2V_0} + u \\ \frac{\frac{1}{2}\rho VS\bar{c}C_{m_0}}{I_{yy}} & \frac{\bar{q}S\bar{c}C_{m_\alpha}}{I_{yy}u} & \frac{\bar{q}S\bar{c}(C_{m_q}+C_{m_{\dot{\alpha}}})}{I_{yy}}\frac{\bar{c}}{2V_0} \end{bmatrix} \tag{27}$$

$$\mathbf{B}_{in}(\mathbf{x}_{in}) = \begin{bmatrix} \frac{\bar{q}S(C_{L_{\delta_e}}\alpha-C_{D_{\delta_e}})}{m} & \frac{C_T}{m} \\ \frac{\bar{q}S(-C_{L_{\delta_e}}-C_{D_{\delta_e}}\alpha)}{m} & 0 \\ \frac{\bar{q}S\bar{c}C_{m_{\delta_e}}}{I_{yy}} & 0 \end{bmatrix} \tag{28}$$

$$\mathbf{A}_{out}(\mathbf{x}_{out}) = [\mathbf{0}] \tag{29}$$

$$\mathbf{B}_{out}(\mathbf{x}_{out}) = \begin{bmatrix} 0 & 0 & 1 \\ \sin\theta & -\cos\theta & 0 \end{bmatrix} \tag{30}$$

The mismatch between the actual dynamics and the SDC parameterization includes terms that appear due to the gravitational acceleration is given by Eq.(31), and is handled by the nonlinear compensator.

$$\mathbf{f}_{in} = \begin{bmatrix} -g\sin\theta \\ g\cos\theta \\ 0 \end{bmatrix} \tag{31}$$

SIMULATION RESULTS

A nonlinear simulation model of a fixed-wing UAV, with a mass of $105kg$ and a wing span of $4.3m$ is developed and used to validate performance of the SDRE dual-loop controller. The elevator and throttle are modeled as first order systems, including the corresponding saturation limits. In the simulation the reference commands are given to the outer loop. Thus, a desired altitude and pitch angle trajectories are generated. The update rate of the state-dependent matrices is set to $1Hz$. The following state and control weighted matrices are chosen for the inner and the outer loops, respectively:

$$\mathbf{Q}_{in} = \begin{bmatrix} 1\times10^6 & 0 & 0 \\ 0 & 1\times10^5 & 0 \\ 0 & 0 & 1\times10^8 \end{bmatrix}, \mathbf{R_{in}} = \begin{bmatrix} 100 & 0 \\ 0 & 1 \end{bmatrix} \tag{32}$$

$$\mathbf{Q}_{out} = \begin{bmatrix} 10^4 & 0 \\ 0 & 1 \end{bmatrix}, \mathbf{R}_{out} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 10^2 & 0 \\ 0 & 0 & 10^3 \end{bmatrix} \tag{33}$$

<u>Simulation Scenario 1:</u> In the first scenario, reference pitch angle and altitude inputs are given to the control system. The simulations results for the outer-loop states are given in Fig. 1, from which it may be observed that the system executes nearly perfect tracking of the desired altitude profile, as well as the pitch angle reference input. Fig. 2 illustrates responses of the inner-loop states to the generated command inputs. Finally, elevator deflection and thrust responses are given in Fig. 3, from which it may be observed that no saturation occurs and the actuators demands are within the allowable limits.

<u>Simulation Scenario 2:</u> The second simulation scenario aims to illustrate the performance of the control system when the aircraft operates in a nonlinear flight regime. In this case, an aircraft is requested to fly with a pitch angle, while holding altitude constant. For this maneuver the values of the angle of attack go beyond the stall limit, and the aerodynamic lift coefficient is no longer a linear function. The aerodynamic lift coefficient curve is given in Fig. 4. From the response of the angle of attack, illustrated in Fig. 5, it may be observed that it significantly exceeds the stall value. Responses of the pitch angle and altitude are shown in Fig. 6. Though, an altitude tracking error appears in the response, however, simulations with longer simulation time show that this error vanishes. Fig. 7 provides the responses of the inner-loop states. It may be observed that some steady state errors are present in the linear velocities responses. It should be noted that the optimal quadratic regulator technique does not guarantee absence of steady state errors. The steady state errors may be minimized further by increasing the penalty on the state by increasing the corresponding

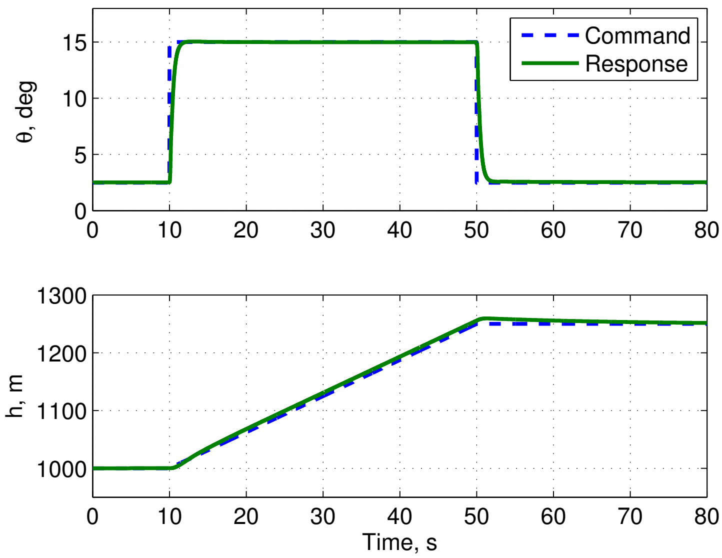

Figure 1: Outer Loop States Responses (Scenario 1)

value of the error weighted matrix, or eliminated by adding an integrator in the feedback controller. Control system's gains for the inner and outer loops are shown in Fig. 9- 12.

CONCLUSIONS

In this work, a dual-loop SDRE controller for for the longitudinal motion of a fixed-wing aircraft is proposed. In the proposed algorithm, translational velocity and pitch rate are controlled within the inner loop, whereas the outer loop controls the pitch attitude and altitude of an aircraft. The SDC models are developed for each control loop. The gravity term is treated as a slowly varying disturbance, and is handled by the nonlinear compensator. A tracking SDRE algorithm is proposed and implemented for a small UAV. The simulation results demonstrate the successful application of the algorithm, and particular its effectiveness in a nonlinear flight phases.

Acknowledgments

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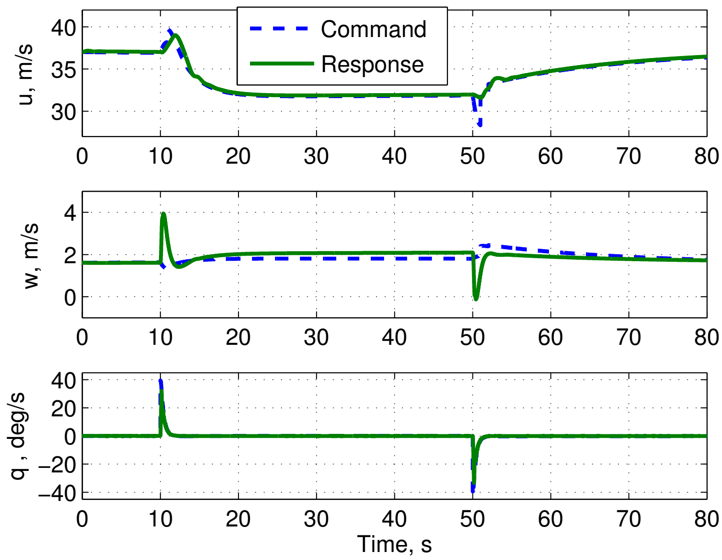

Figure 2: Inner Loop States Responses (Scenario 1)

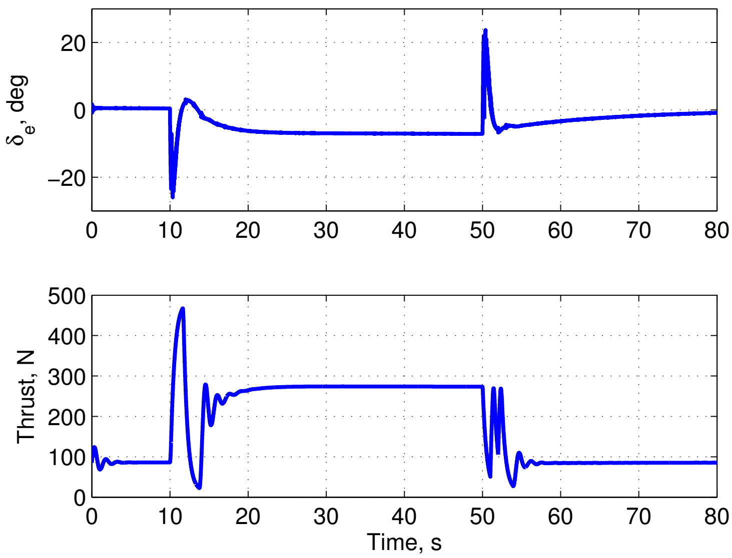

Figure 3: Actuators Responses (Scenario 1)

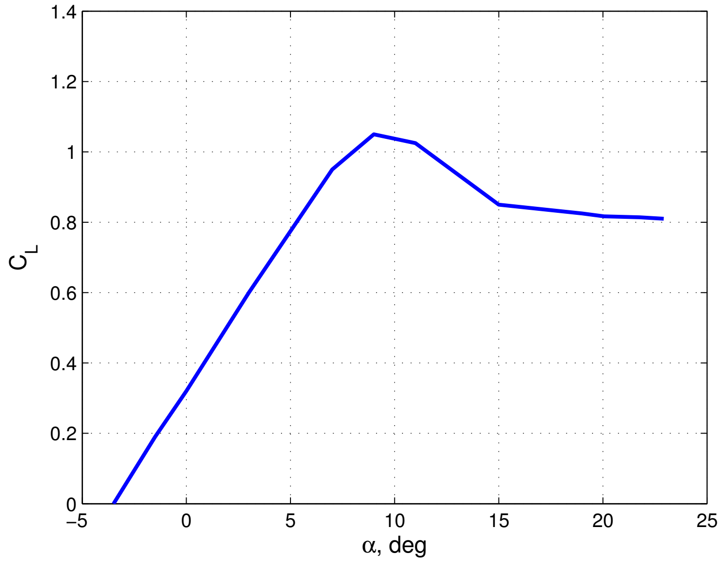

Figure 4: Aerodynamic Lift Coefficient vs AOA

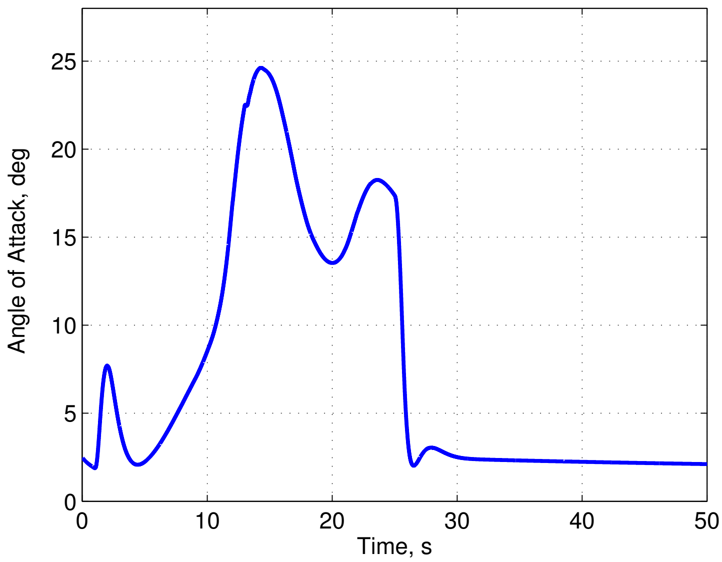

Figure 5: Angle of Attack Response (Scenario 2)

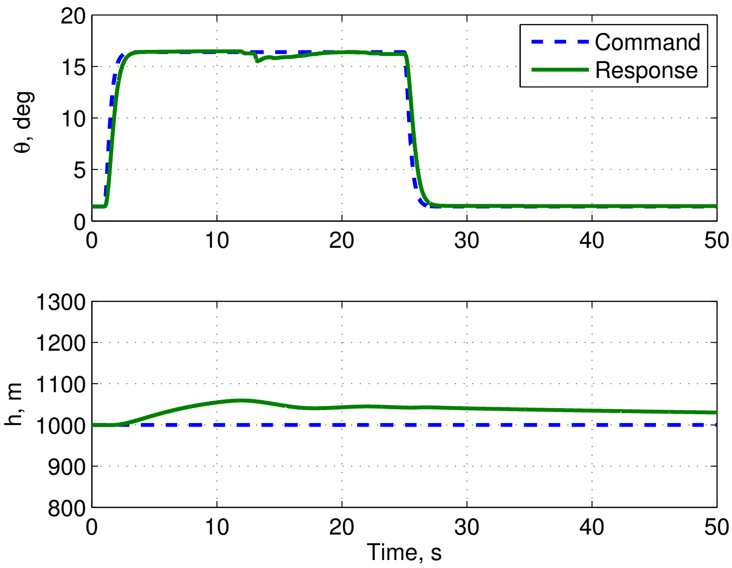

Figure 6: Outer Loop States Responses (Scenario 2)

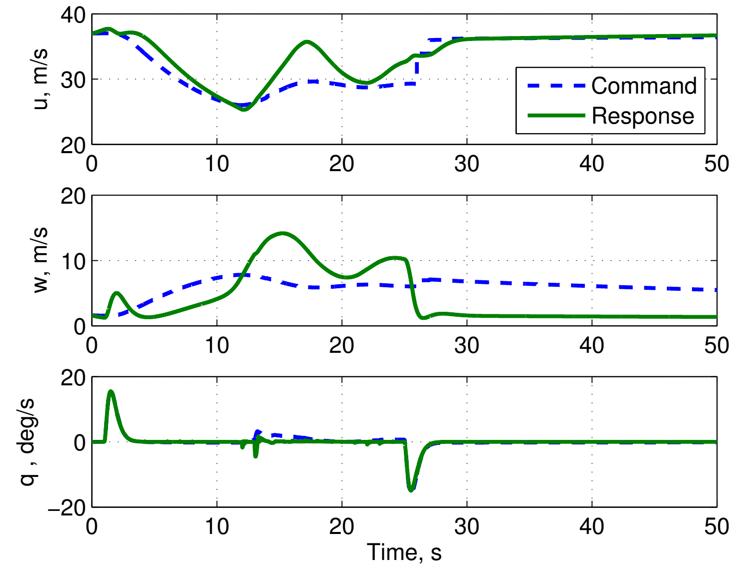

Figure 7: Inner Loop States Responses (Scenario 2)

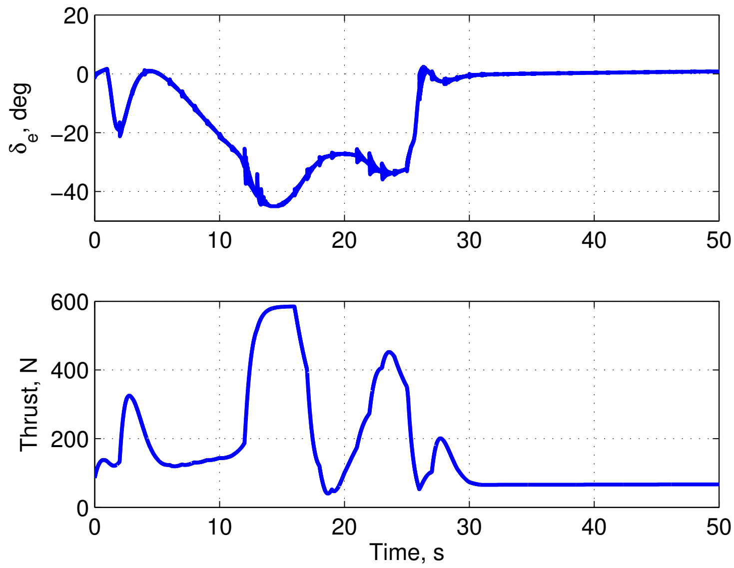

Figure 8: Actuators Responses (Scenario 2)

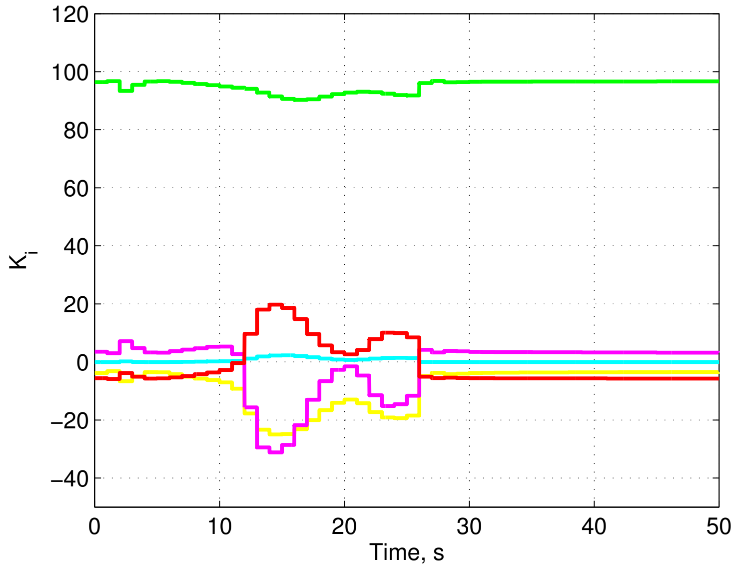

Figure 9: K Inner Loop (Scenario 2)

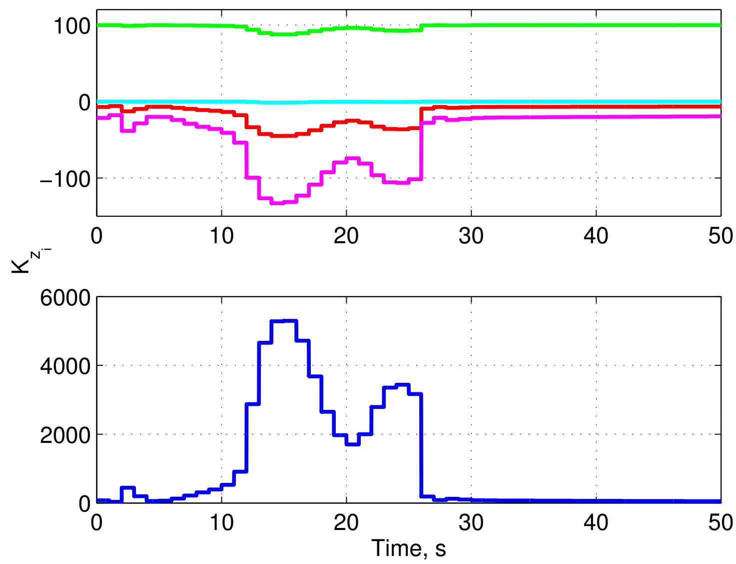

Figure 10: K_z Inner Loop (Scenario 2)

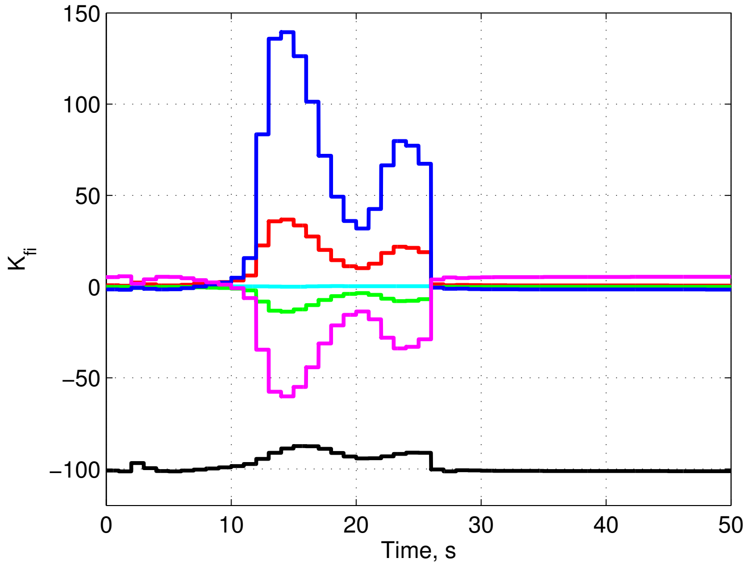

Figure 11: K_f Inner Loop (Scenario 2)

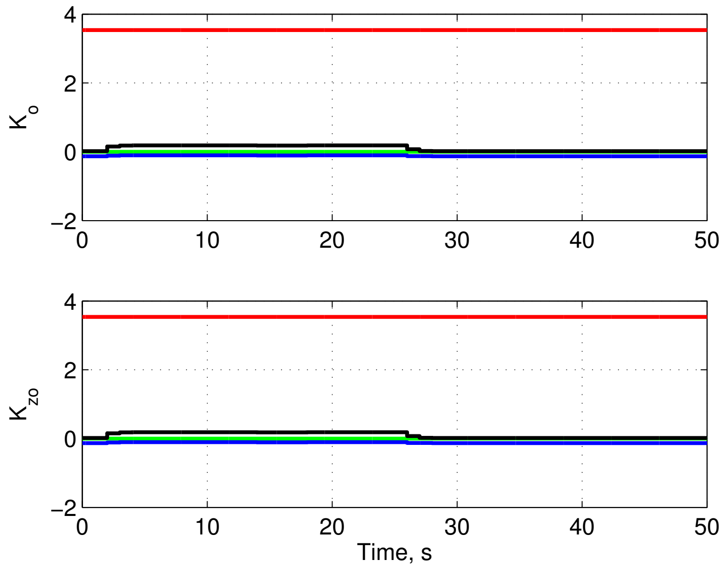

Figure 12: K, K_z Outer Loop (Scenario 2)

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