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TIME DOMAIN SYSTEM IDENTIFICATION FOR AN UAV USING FLIGHT TEST DATA

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ABSTRACT

System identification is the determination of a mathematical model from measured input-output data. In this paper, an overview of time domain system identification for an unmanned aerial vehicle (UAV) called Pelikan which is a fixed wing UAV developed by Turkish Aerospace Industries (TAI), is presented. The aim of this study is to estimate stability and control derivatives of Pelikan from flight test data. FVSysID software, provided by Dr. Ravindra Jategaonkar, is used for this purpose. Firstly, a brief summary about Pelikan and its instrumentation are given. Then, experiment design which is the initial step of system identification procedure is discussed. After that data compatibility analysis carried out to check suitability and consistency of recorded flight data is presented. Two different parameter estimation methods are used for parameter estimation. The first one is equation-error which is based on least square regression and the second one is output-error method which is based on maximum likelihood principle. Results of parameter estimation using equation-error method and output-error method for longitudinal-directional motion and lateral-directional motion are presented and discussed. Then, model validation studies to check accuracy and reliability of system identification results are achieved. Finally, it is concluded that output-error method is more suitable for system identification of Pelikan since recorded flight data is noisy and output-error method can handle measurement noise which equation-error method cannot handle.

Keywords: UAV, time domain system identification, data compatibility check, equation-error method, output-error method

INTRODUCTION

System identification is a general procedure to match the observed input-output response of a dynamic system by a proper choice of an input-output model and its physical parameters [Goes et al.,2006].It is possible to obtain accurate and comprehensive mathematical models of aircraft by means of system identification from flight data [Klein and Morelli, 2006].

Aircraft aerodynamic database is usually derived from analytical predictions, wind tunnel measurements or extrapolation of existing data from similar configurations However, such databases have some limitations like model scaling, obtaining dynamic derivative and cross coupling effects and assumptions in analytical equations. Using system identification, entire aerodynamic database of an aircraft can be derived from real flight data. [Jategaonkar and Thielecke, 2000].

Mathematical model of an aircraft can be modeled in time domain or frequency domain. In frequency domain, after transforming time domain recorded flight data to frequency domain data by means of Fourier transform, parameter estimation is performed. Although, it has some advantages like applicability to unstable system, faster computation and providing more robust results, some information may be lost during the transformation into the frequency domain [Klein and Morelli, 2006].This paper focuses on time domain identification. The purpose of this study is to obtain stability and control derivatives of Pelikan from flight test data by using time domain system identification approach.

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There are three commonly used parameter estimation methods. First one is equation-error method which is based on linear regression using ordinary least squares technique. It is deterministic and non-iterative so that computational procedure is low. However, it cannot handle both process and measurement noise. Second method is output-error method which is most frequently used method and mostly uses maximum likelihood principle. It is a deterministic method parameter estimates are asymptotically unbiased, consistent and efficient. Output-error method can handle only measurement error. The last method is filter-error method which is a stochastic method and accounts for both process and measurement noise [Peyada et al., 2008]. In this study, equation-error and output-error methods are used to estimate parameters.

Pelikan UAV and Instrumentation

Pelikan is a fixed wing UAV developed by Turkish Aerospace Industries (TAI) and is used as a test platform. Pelikan has high-wing and twin boom tail configurations. It is propelled by one pusher and one tractor piston-prop engines. Maximum power of each engine is 9.8hp. Pelikan's maximum speed is 95knots. A picture and specifications of Pelikan is given in Figure 1.

Weight (kg)	63
Length (m)	3.3
Wing chord (m)	0.39
Wing span (m)	4
Aspect ratio	10.7

Figure 1: *Pelikan*

Pelikan has an air data test boom which is placed on the wing. It measures static and total pressure, angle of attack and angle of sideslip. A GPS/INS (EGI) system is placed in the body, near cg location. EGI provides translational accelerations, angular rates and attitude angles data. Pilot commands on aileron, elevator, rudder and throttle are also recorded. Moreover, there are encoders on the control surfaces to measure control surface deflection. Frequency of the recorded flight data is 50Hz.

Software Tool

The name of the software used in current study is FVSysID (Flight Vehicle System Identification). It is for time domain system identification provided by Dr. Ravindra Jategaonkar. FVSysID is developed in Matlab and provides source codes so that input and output equations of both equation-error and output-error models can be modified.

It can be applied to both linear and nonlinear systems. It includes off-line algorithms, which are output-error algorithm and filter error algorithm, and recursive algorithms, which are recursive least square, Fourier transform regression and extended/unscented Kalman filters [Jategaonkar, 2006].

In this study, system is considered as linear. Least square algorithm is used in parameter estimation by means of equation-error method. In addition, output-error algorithm is used for both parameter estimation and data compatibility check.

METHOD

The first step of the aircraft system identification procedure is experiment design which consists of maneuvers design to excite desired aircraft mode. Then, data compatibility analysis is achieved to check suitability and consistency of recorded flight data and to determine instrumentation errors if necessary. After recorded flight data is corrected according to results of data compatibility check, parameter estimation is carried out to estimate desired aerodynamic parameters. Equation-error method and output-error method are two methods used to estimate parameters in this study. The final step is model validation which is required to confirm that parameter estimates can be applicable.

Experiment Design

Since stability and control derivatives are estimated form flight test data, maneuvers, which aircraft will perform in flight tests, should be chosen properly. Control inputs are designed such that dynamic motion of the aircraft is excited sufficiently to identify desired aerodynamic derivatives [Jategaonkar, 2006]. In flight tests, maneuvers should be initiated from horizontal level flight and repeated for different trim speeds. It is also suggested that during the maneuvers engine throttle level should be kept constant [Jategaonkar, 2006].

In order to identify longitudinal-directional derivatives, short period maneuvers are performed by applying doublet and multiplet inputs to elevator. Jategaonkar states that during the short period maneuver, angle of attack variation should be ± 3 – 4deg. and the load factor variation should be ± 0.4 – 0.5g [Jategaonkar, 2006]. An elevator doublet input applied to Pelikan and variations of angle of attack and load factor during the maneuver are given in Figure 2.

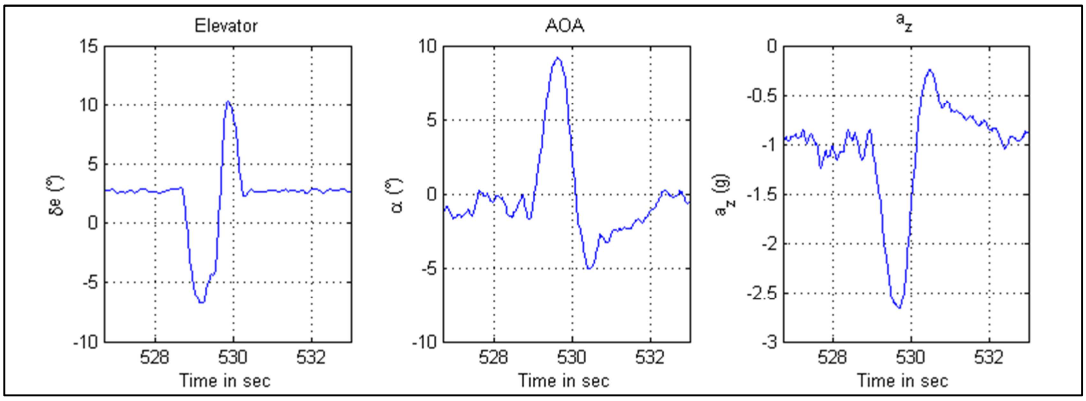

Figure 2: *Elevator doblet input*

For the identification of lateral-directional derivatives bank-to-bank maneuvers and Dutch roll maneuvers are performed. During the bank-to-bank maneuver, aircraft should be banked at least 30 degree [Jategaonkar, 2006]. An aileron input applied to Pelikan and variations of bank angle are given in Figure 3.

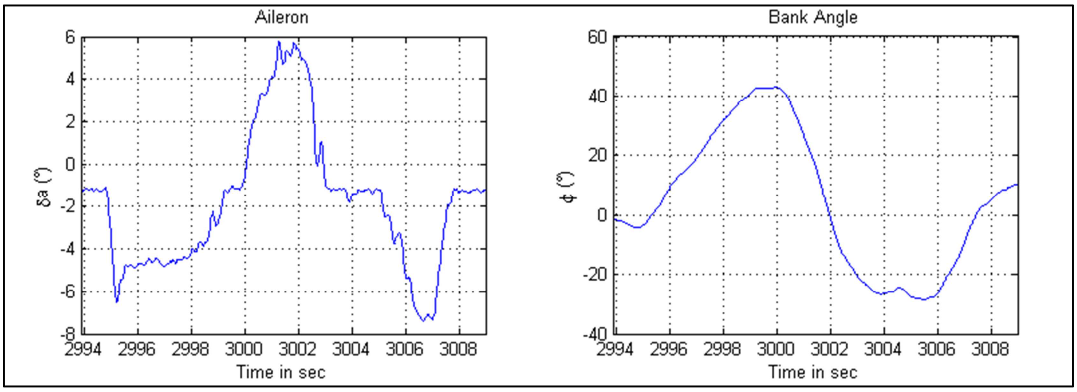

Figure 3: *Aileron input*

Dutch roll maneuvers are done through rudder doublet inputs. During the maneuver, sideslip angle variation should be ± 4deg and lateral acceleration variation should be ± 0.1g [Jategaonkar, 2006]. A rudder doublet input applied to Pelikan and variations of angle of sideslip and lateral acceleration during the maneuver are given in Figure 4.

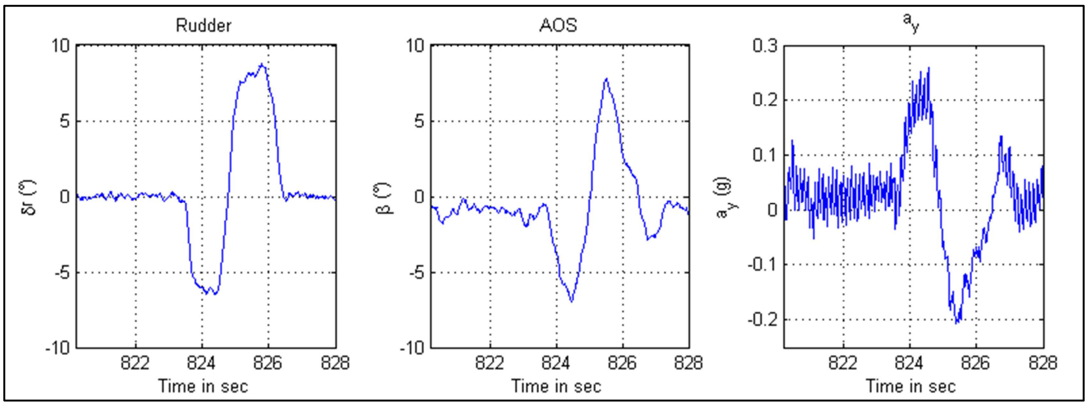

Figure 4: *Rudder doublet input*

Data Compatibility Check

The purpose of data compatibility check is to determine instrument errors like scale factors, zero shifts and time delays [Jategaonkar, 2006]. FVSysID data compatibility check model uses kinematic equations which are composed of translational equations, rotational equations and navigation equations.

Data compatibility check model uses output-error parameter estimation model. Using model inputs and measurements, which are also called as observation variables, state equations, which are kinematic equations, are solved. From the solution of kinematic equations, observation variables are recalculated. Differences between measured and reconstructed values are due to instrumentation errors and are estimated using optimization techniques [Klein and Morelli, 2006]. FVSysID uses Gauss-Newton optimization algorithm. State equations, observation equations are given below.

State equations;

$$\dot{x_1} = \dot{u} = (a_{xm} - \Delta a_x) - g\sin\theta + (r_m - \Delta r)v - (q_m - \Delta q)w \quad (1)$$

$$\dot{x_2} = \dot{v} = (a_{ym} - \Delta a_y) + g\sin\phi\cos\theta + (p_m - \Delta p)w - (r_m - \Delta r) \quad (2)$$

$$\dot{x_3} = \dot{w} = (a_{zm} - \Delta a_z) + g\cos\phi\cos\theta + (q_m - \Delta q)u - (p_m - \Delta p)v \quad (3)$$

$$\dot{x_4} = \dot{\phi} = (p_m - \Delta p) + \big((q_m - \Delta q)\sin\phi + (r_m - \Delta r)\cos\phi\big)\tan\theta \quad (4)$$

$$\dot{x_5} = \dot{\theta} = (q_m - \Delta q)\cos\phi - (r_m - \Delta r)\sin\phi \quad (5)$$

$$\dot{x_6} = \dot{\psi} = ((q_m - \Delta q)\sin\phi + (r_m - \Delta r)\cos\phi)/\cos\theta \quad (6)$$

$$\dot{x_7} = \dot{h} = u\sin\theta - v\sin\phi\cos\theta - w\cos\phi\cos\theta \quad (7)$$

Observation equations;

$$y_1 = V_m = \sqrt{u^2 + v^2 + w^2} \quad (8)$$

$$y_2 = \alpha_m = K_\alpha \tan^{-1}\left(\frac{w}{u}\right) + \Delta\alpha \quad (9)$$

$$y_3 = \beta_m = K_\beta \sin^{-1}\left(\frac{v}{\sqrt{u^2 + v^2 + w^2}}\right) + \Delta\beta \quad (10)$$

$$y_4 = \phi_m \quad (11)$$

$$y_5 = \theta_m \quad (12)$$

$$y_6 = \psi_m \quad (13)$$

$$y_7 = h_m \quad (14)$$

For longitudinal motion flight data, bias in a_z measurement (Δa_z), bias in q measurement (Δq), bias in α measurement $(\Delta\alpha)$ and scale factor in α measurement (K_α) are estimated. Estimation results are given in Table 1. Time histories of input and output variables are given in Figure 5 and Figure 6, respectively.

Table 1: *Data compatibility check results - longitudinal-directional motion*

	Parameter Estimation	Relative Std. Deviation (%)
K_α	1.29	0.63
$\Delta\alpha$ (deg)	0.91	4.5
Δa_z (m/s^2)	0.049	2.55
Δq (deg/s)	0.013	0.79

For lateral motion flight data, bias in a_y measurement (Δa_y), bias in p measurement (Δp), bias in r measurement (Δr), bias in β measurement $(\Delta\beta)$ and scale factor in β measurement (K_β) are estimated. Estimation results are given in Table 2. Time histories of input and output variables are given in Figure 7 and Figure 8, respectively.

Table 2: *Data compatibility check results - lateral-directional motion*

	Parameter Estimation	Relative Std. Deviation (%)
K_β	0.97	0.87
$\Delta\beta$ (deg)	0.115	3.2
Δa_y (m/s^2)	0.41	3.2
Δp (deg/s)	-0.032	11.5
Δr (deg/s)	0.376	5.3

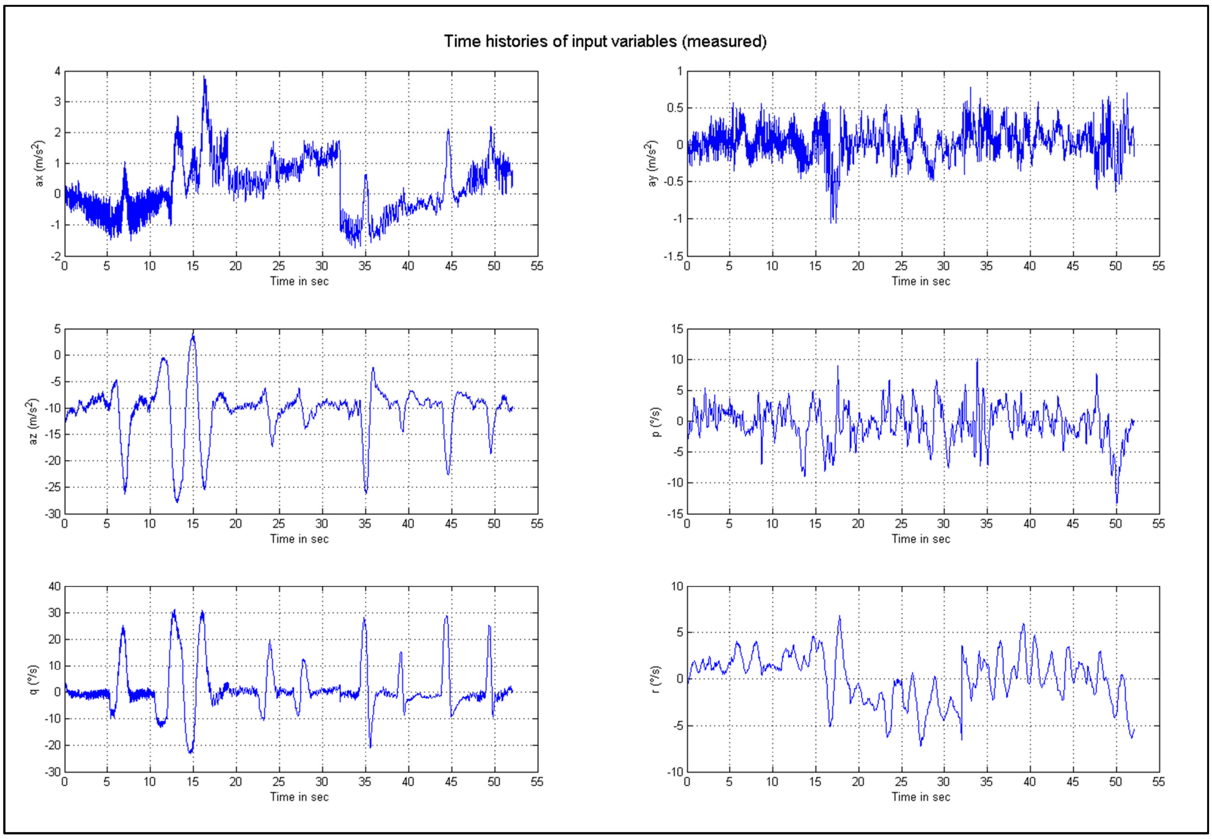

Figure 5: *Time histories of input variables – Data compatibility check (logitudinal-directional motion)*

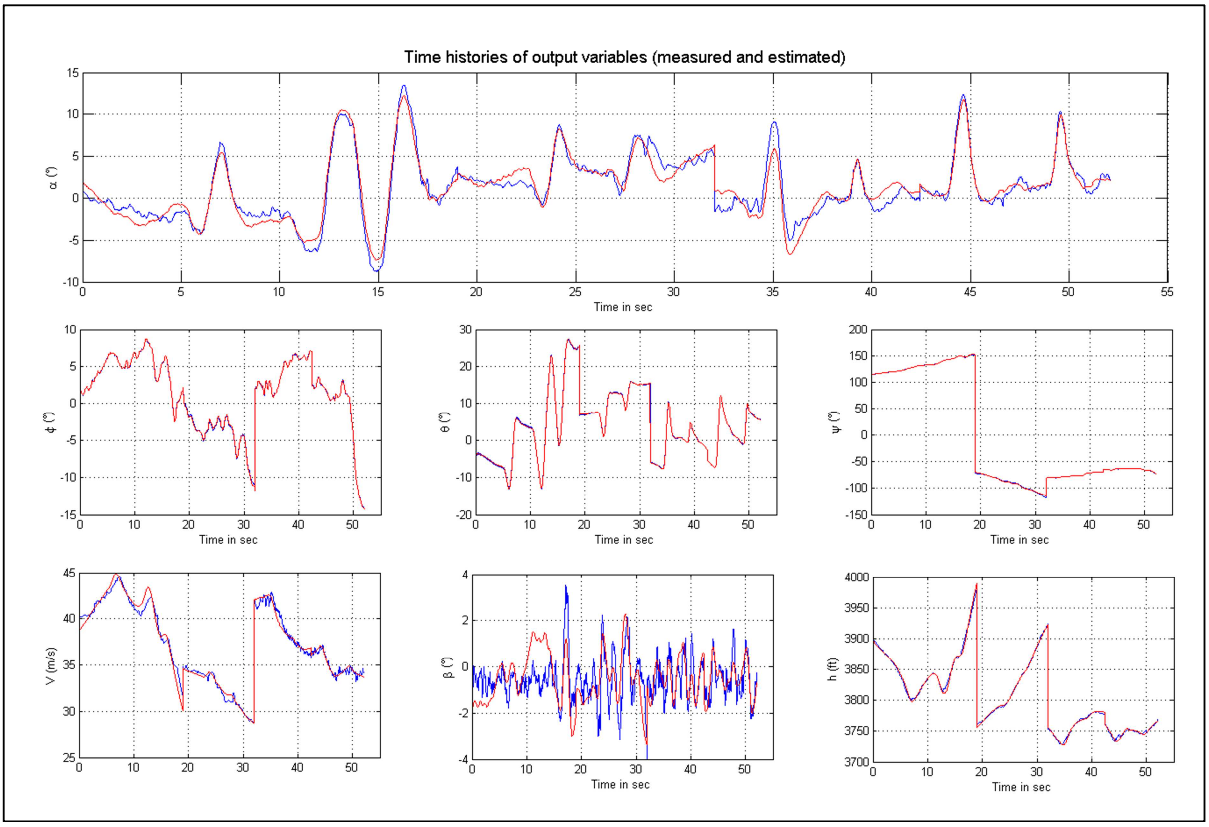

Figure 6: *Time histories of output variables (measured and estimated) – Data compatibility check (logitudinal-directional motion)*

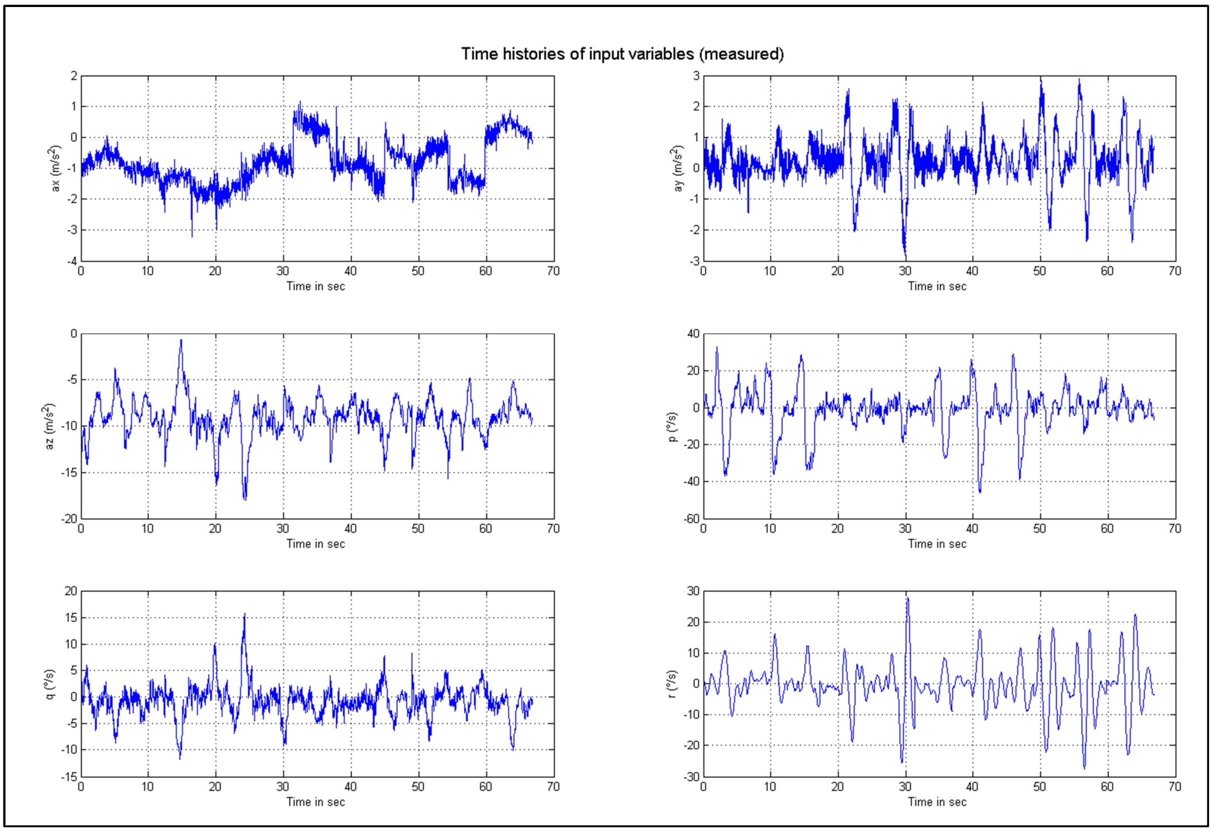

Figure 7: *Time histories of input variables – Data compatibility check (lateral-directional motion)*

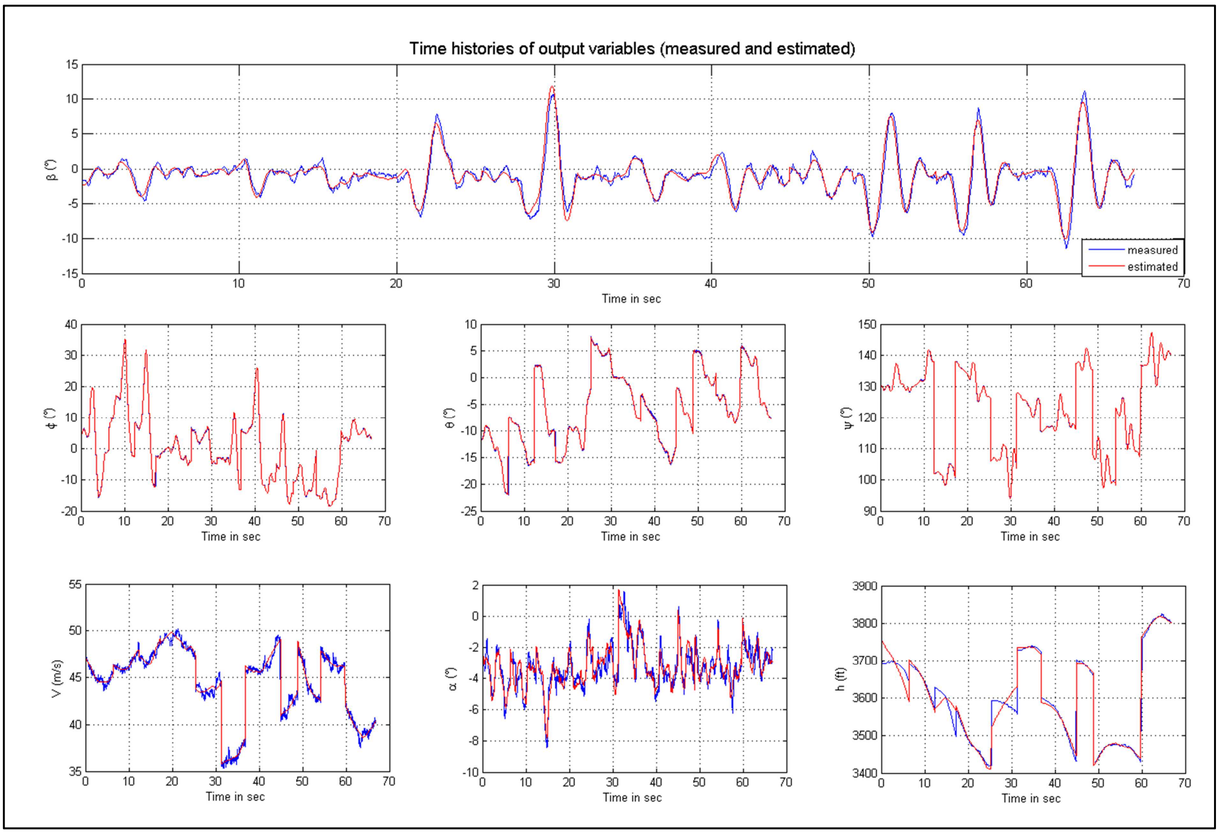

Figure 8: *Time histories of output variables (measured and estimated) – Data compatibility check (lateral-directional motion)*

Parameter Estimation

The system identification techniques are significant because they are the main source of the dynamic derivatives [Drobik and Brian, 2004]. As mentioned before, in this study, two methods, which are equation-error method and output-error method, are used in parameter estimations. Lateral dynamics and longitudinal dynamics are considered as decoupled and studied separately.

Equation-error Method

Equation-error method uses least squares techniques to estimate parameters. Using defined input-output equations, this method find parameter set which minimizes cost function. Unlike the output-error method, minimization of the cost function is not based on probability and computational procedure is shorter [Jategaonkar, 2006]. One limitation about equation-error method is that it does not take measurement and process noise into account. Therefore, equation-error results are very sensitive to noise and strongly affected by recorded data quality [Jategaonkar, 2006].

At time t_k dependent variable $y(t)$ can be written in terms of independent variables $x(t)$;

$$y(k) = \theta_1 x_1(k) + \theta_2 x_2(k) + \cdots + \theta_n x_n(k) + \varepsilon(k); k = 1,2, \ldots . N \quad (15)$$

Where ε is stochastic equation error and $\theta = [\theta_1 \ \theta_2 \ldots \theta_n]^T$ is vector of unknown parameters.

In matrix notation;

$$y(k) = x^T(k)\theta + \varepsilon(k) \quad (16)$$

For N discrete time points, equation error is

$$\varepsilon = Y - X\theta \quad (17)$$

The cost function is defined as

$$J(\theta) = \frac{1}{2}\varepsilon^T \varepsilon \quad (18)$$

Taking the derivative of cost function with respect to parameters and equating to zero, parameter estimates which minimize error are found.

$$\hat{\theta} = (X^T X)^{-1} X^T Y \quad (19)$$

Equation-error model for longitudinal parameter estimation is given below.

Input equations;

$$C_L = (\frac{ma_x}{\bar{q}\,S}\sin\alpha - \frac{ma_z}{\bar{q}\,S}cos\alpha) \quad (20)$$

$$C_m = (I_y\dot{q} - I_{xz}(p^2 - r^2) - (I_z - I_x)pr)/(\bar{q}S_{ref}c) \quad (21)$$

Output equations;

$$C_L = C_{L_0} + C_{L_\alpha}\alpha \quad (22)$$

$$C_m = C_{m_0} + C_{m_\alpha}\alpha + C_{m_q}q_n + C_{m_{\delta e}}\delta e \quad (23)$$

Time histories of output variables are given in Figure 9.

Equation-error model for lateral parameter estimation is given below.

Input equations;

$$C_Y = \frac{ma_y}{\bar{q}\,S} \quad (24)$$

$$C_l = (I_x\dot{p} - I_{xz}\dot{r} - I_{xz}pq - (I_y - I_z)qr - l_T - l_{LG})/(\bar{q}S_{ref}b) \quad (25)$$

$$C_n = (I_z\dot{r} - I_{xz}\dot{p} + I_{xz}qr - (I_x - I_y)pq - n_T - n_{LG})/(\bar{q}S_{ref}b) \quad (26)$$

Output equations;

$$C_Y = C_{Y_r}r_n + C_{Y_{\delta r}}\delta r + C_{Y_\beta}\beta + C_{Y_0} \quad (27)$$

$$C_l = C_{l_p}p_n + C_{l_r}r_n + C_{l_{\delta a}}\delta a + C_{l_{\delta r}}\delta r + C_{l_\beta}\beta + C_{l_0} \quad (28)$$

$$C_n = C_{n_p}p_n + C_{n_r}r_n + C_{n_{\delta r}}\delta r + C_{n_\beta}\beta + C_{n_0} \quad (29)$$

Time histories of output variables are given in Figure 10.

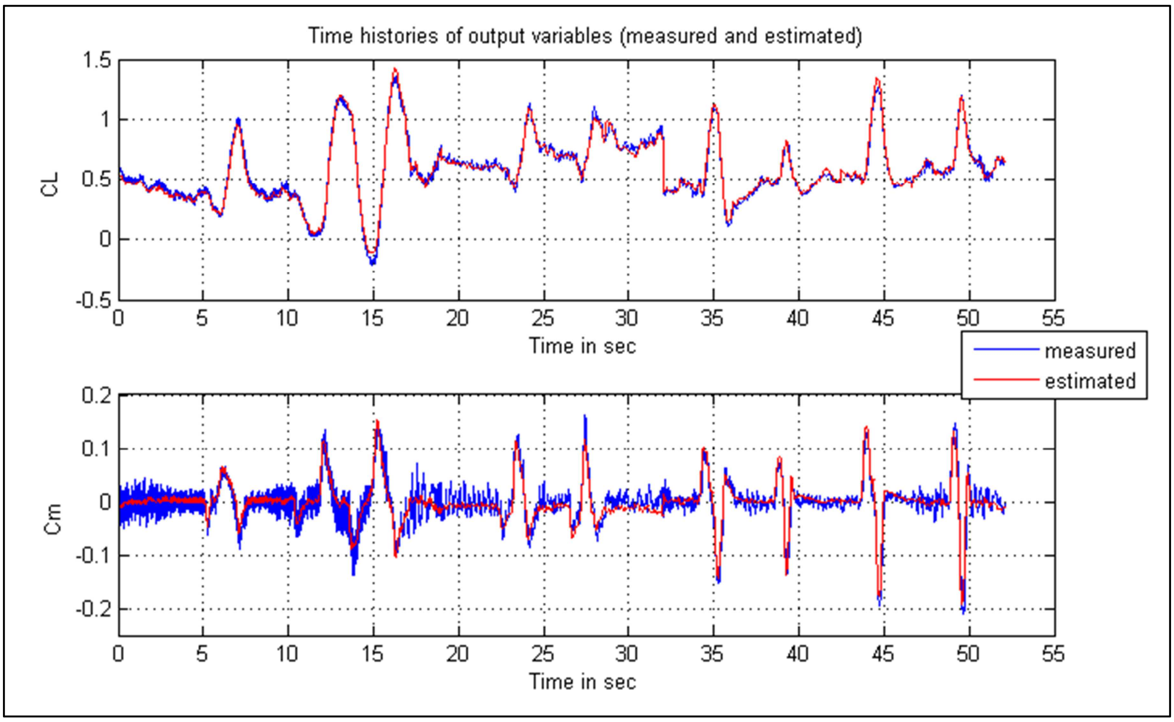

Figure 9*: Time histories of output variables (measured and estimated) – Parameter estimation with equation-error method (longitudinal-directional motion)*

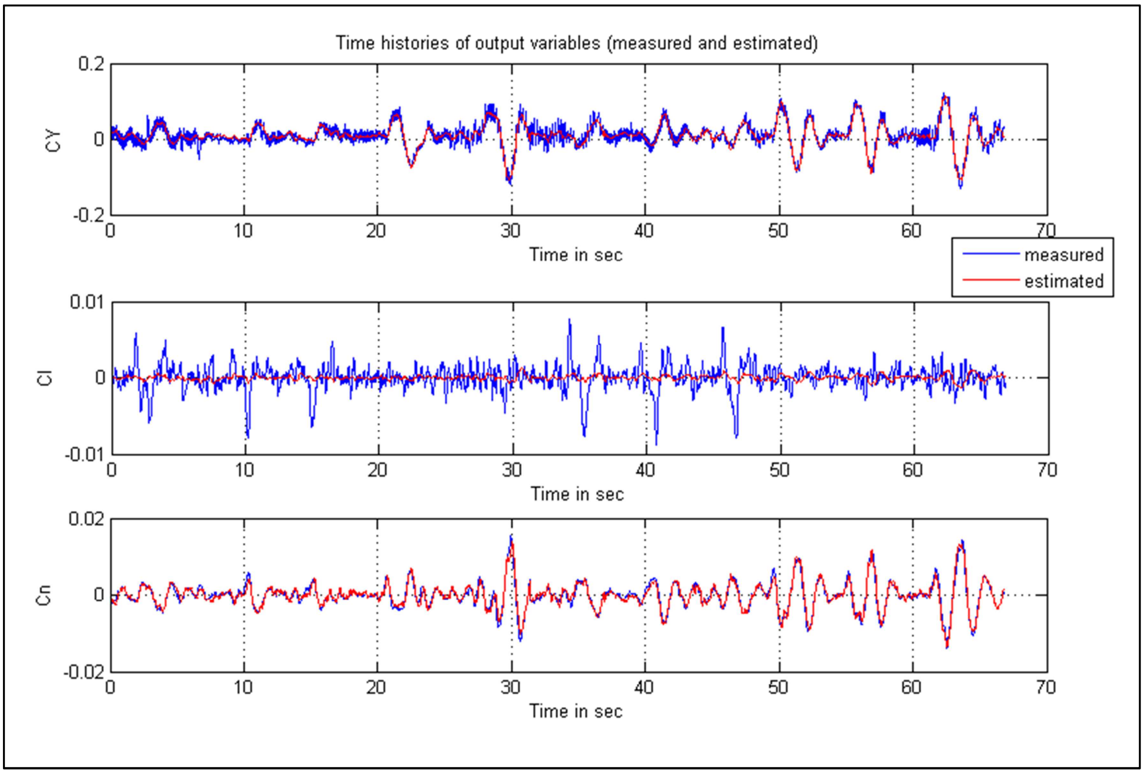

Figure 10: *Time histories of output variables (measured and estimated) – Parameter estimation with equation-error method (lateral-directional motion)*

Output-Error Method

Output-error method uses maximum likelihood principle and probability theory in parameter estimation process. In output-error method, model parameters are obtained iteratively to minimize the error between the measured variables and calculated results [Jategaonkar, 2006]. FVSysID uses Gauss-Newton optimization method to find parameters which minimizes error. Although this method provides more accurate parameter estimation, computational cost of the output-error method is higher than the cost of the equation-error method because of optimization process. In addition, output-error model in FVSysID considers measurement noise which is ignored in equation-error model [Jategaonkar, 2006].

Equations of motion of an aircraft in state space form are given below.

$$\dot{x}(t) = Ax(t) + Bu(t) + b_x, \;\; x(t_o) = x_0 \tag{30}$$

$$y(t) = Cx(t) + Du(t) + b_y \tag{31}$$

$$z(k) = y(k) + v(k) \tag{32}$$

Where x is state vector, y is observation vector, u is control input, z is the measured variable and v is measurement noise. A, B, C and D matrices contain unknown parameters = $[\theta_1 \, \theta_2 \dots \theta_n]^T$

Cost function is

$$J(\Theta, R) = \frac{1}{2}\sum_{k=1}^{N}[z(t_k) - y(t_k)][z(t_k) - y(t_k)]^T \tag{33}$$

Where R ismeasurement noise covariance matrix

$$R = \frac{1}{N}\sum_{k=1}^{N}[z(k) - y(k)][z(k) - y(k)]^T \tag{34}$$

Parameters are updated by using Gauss Newton formulation given below.

$$\Theta_{i+1} = \Theta_i + \Delta\Theta \text{ and } \mathrm{F}\Delta\Theta = -\mathrm{G} \tag{35}$$

Where

$$F = \sum_{k=1}^{N}\left[\frac{\partial y(t_k)}{\partial \Theta}\right]^T R^{-1}\left[\frac{\partial y(t_k)}{\partial \Theta}\right] \tag{36}$$

$$G = -\sum_{k=1}^{N}\left[\frac{\partial y(t_k)}{\partial \Theta}\right]^T R^{-1}\left[z(t_k) - y(t_k)\right] \tag{37}$$

Output-error model for longitudinal parameter estimation is given below.

State equations;

$$\dot{x_1} = \dot{\alpha} = -C_L\frac{\bar{q}S_{ref}}{mV\cos\beta} + q - \tan\beta\,(p\cos\alpha + r\sin\alpha) + \frac{g}{V\cos\beta}(\cos\phi\cos\theta\cos\alpha + \sin\theta\sin\alpha) \tag{38}$$

$$\dot{x_2} = \dot{q} = C_m\frac{\bar{q}S_{ref}}{cI_y} - \frac{(p^2 - r^2)I_{xz}}{I_y} + \frac{pr(I_z - I_x)}{I_y} \tag{39}$$

$$\dot{x_3} = \dot{\theta} = q\cos\phi - r\sin\phi \tag{40}$$

Observation equations;

$$y_1 = \alpha \tag{41}$$

$$y_2 = q \tag{42}$$

$$y_3 = a_z = -C_L\frac{\bar{q}S_{ref}}{m} \tag{43}$$

$$y_4 = \dot{q} = C_m\frac{\bar{q}S_{ref}}{cI_y} - (p^2 - r^2)I_{xz}/I_y + pr(I_z - I_x)/I_y \tag{44}$$

$$y_5 = \theta \tag{45}$$

where

$$C_L = C_{L_0} + C_{L_\alpha}\alpha \tag{46}$$

$$C_m = C_{m_0} + C_{m_\alpha}\alpha + C_{m_q}q_n + C_{m_{\delta e}}\delta e \tag{47}$$

$$q_n = \frac{qc}{2V} \tag{48}$$

Estimated parameters are $C_{L_0},\ C_{L_\alpha},\ C_{m_0},\ C_{m_\alpha},\ C_{m_q},\ C_{m_{\delta e}}$

Output-error model for longitudinal parameter estimation is given below.
State equations;

$$\dot{x_1} = \dot{p} = \bar{q}S_{ref}b\left(\frac{I_z}{I_xI_z - I_{xz}^2}C_l + \frac{I_{xz}}{I_xI_z - I_{xz}^2}C_n\right) + \left(\frac{(I_y - I_z)I_z - I_{xz}^2}{I_xI_z - I_{xz}^2}r + \frac{(I_x - I_y + I_z)I_{xz}}{I_xI_z - I_{xz}^2}p\right)q \tag{49}$$

$$\dot{x_2} = \dot{r} = \bar{q}S_{ref}b\left(\frac{I_x}{I_xI_z - I_{xz}^2}C_n + \frac{I_{xz}}{I_xI_z - I_{xz}^2}C_l\right) + \left(\frac{(I_x - I_y)I_x - I_{xz}^2}{I_xI_z - I_{xz}^2}p - \frac{(I_x - I_y + I_z)I_{xz}}{I_xI_z - I_{xz}^2}r\right)q \tag{50}$$

$$\dot{x_3} = \dot{\phi} = p + \tan\theta\,(q\sin\phi + r\cos\phi) \tag{51}$$

$$\dot{x_4} = \dot{\psi} = (q\sin\phi + r\cos\phi)\sec\theta \tag{52}$$

$$\dot{x_5} = \dot{\beta} = p\sin\alpha - r\cos\alpha + \frac{g}{V}\cos\beta\cos\theta\sin\phi + \frac{\sin\beta}{V}(g\cos\alpha\sin\theta - g\sin\alpha\cos\phi\cos\theta) + C_Y \tag{53}$$

Observation equations;

$$y_1 = p \tag{54}$$
$$y_2 = r \tag{55}$$
$$y_3 = \phi \tag{56}$$
$$y_4 = \psi \tag{57}$$
$$y_5 = \dot{p} = \bar{q}S_{ref}b\left(\frac{I_z}{I_xI_z - I_{xz}^2}C_l + \frac{I_{xz}}{I_xI_z - I_{xz}^2}C_n\right) + \left(\frac{(I_y - I_z)I_z - I_{xz}^2}{I_xI_z - I_{xz}^2}r + \frac{(I_x - I_y + I_z)I_{xz}}{I_xI_z - I_{xz}^2}p\right)q \tag{58}$$

$$y_6 = \dot{r} = \bar{q}S_{ref}b\left(\frac{I_x}{I_xI_z - I_{xz}^2}C_n + \frac{I_{xz}}{I_xI_z - I_{xz}^2}C_l\right) + \left(\frac{(I_x - I_y)I_x - I_{xz}^2}{I_xI_z - I_{xz}^2}p - \frac{(I_x - I_y + I_z)I_{xz}}{I_xI_z - I_{xz}^2}r\right)q \tag{59}$$

$$y_7 = a_y = C_Y\frac{\bar{q}S_{ref}}{m} \tag{60}$$

$$y_8 = \beta \tag{61}$$

where

$$C_Y = C_{Y_r}r_n + C_{Y_{\delta r}}\delta r + C_{Y_\beta}\beta + C_{Y_0} \tag{62}$$
$$C_l = C_{l_p}p_n + C_{l_r}r_n + C_{l_{\delta a}}\delta a + C_{l_{\delta r}}\delta r + C_{l_\beta}\beta + C_{l_0} \tag{63}$$
$$C_n = C_{n_p}p_n + C_{n_r}r_n + C_{n_{\delta r}}\delta r + C_{n_\beta}\beta + C_{n_0} \tag{64}$$
$$p_n = \frac{pb}{2V} \quad and \quad r_n = \frac{rb}{2V} \tag{65}$$

Estimated parameters are $C_{Y_r}, C_{Y_{\delta r}}, C_{Y_\beta}, C_{Y_0},\ C_{l_p},\ C_{l_r},\ C_{l_{\delta a}},\ C_{l_{\delta r}},\ C_{l_\beta},\ C_{l_0},\ C_{n_p},\ C_{n_r},\ C_{n_{\delta r}},\ C_{n_\beta},\ C_{n_0}$

In order to obtain longitudinal stability and control derivatives 7 elevator doublet and 1 elevator multiplet maneuvers are realized. Time histories of input and output variables are given in Figure 11 and Figure 12.

In order to obtain lateraal stability and control derivatives, flight dat of 6 aileron and 5 rudder doublet maneuvers are used. Time histories of input and output variables are given in Figure 13 and Figure 14.

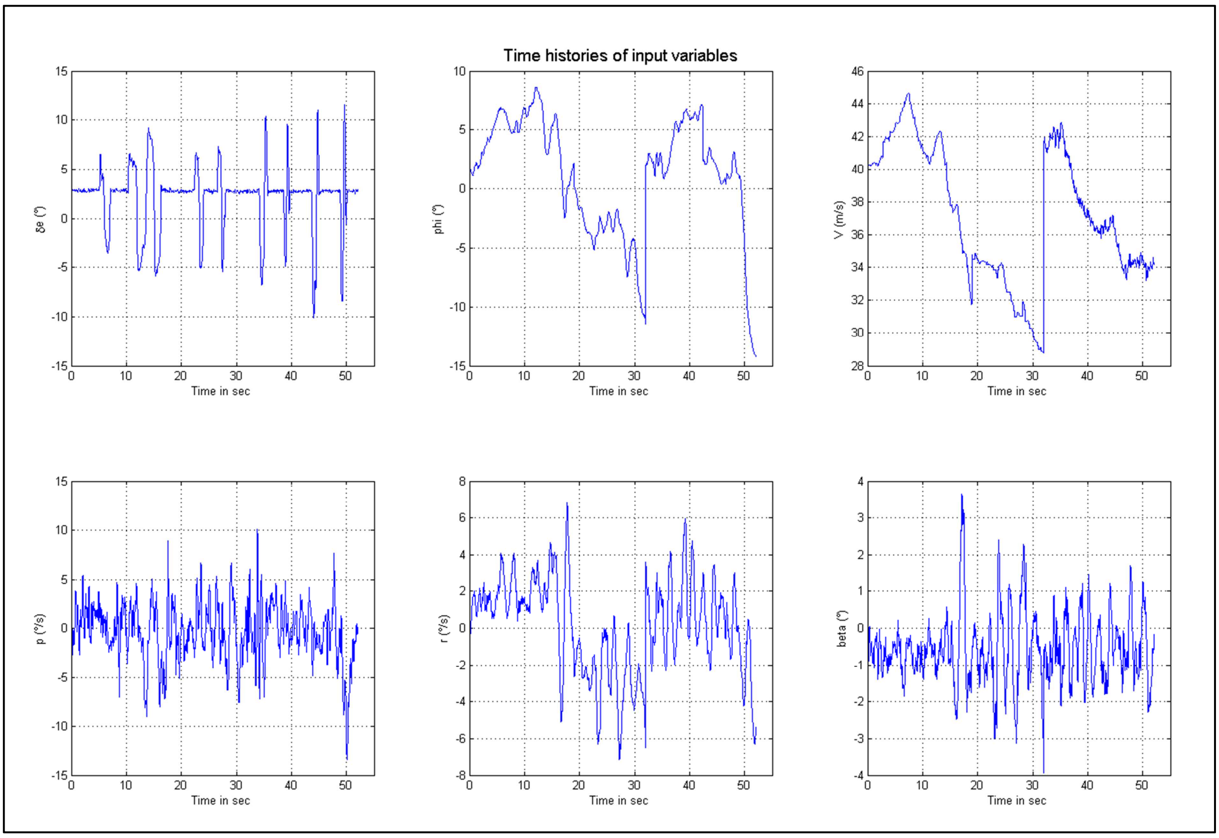

Figure 11: *Time histories of input variables – Parameter estimation with output-error method (longitudinal-directional motion)*

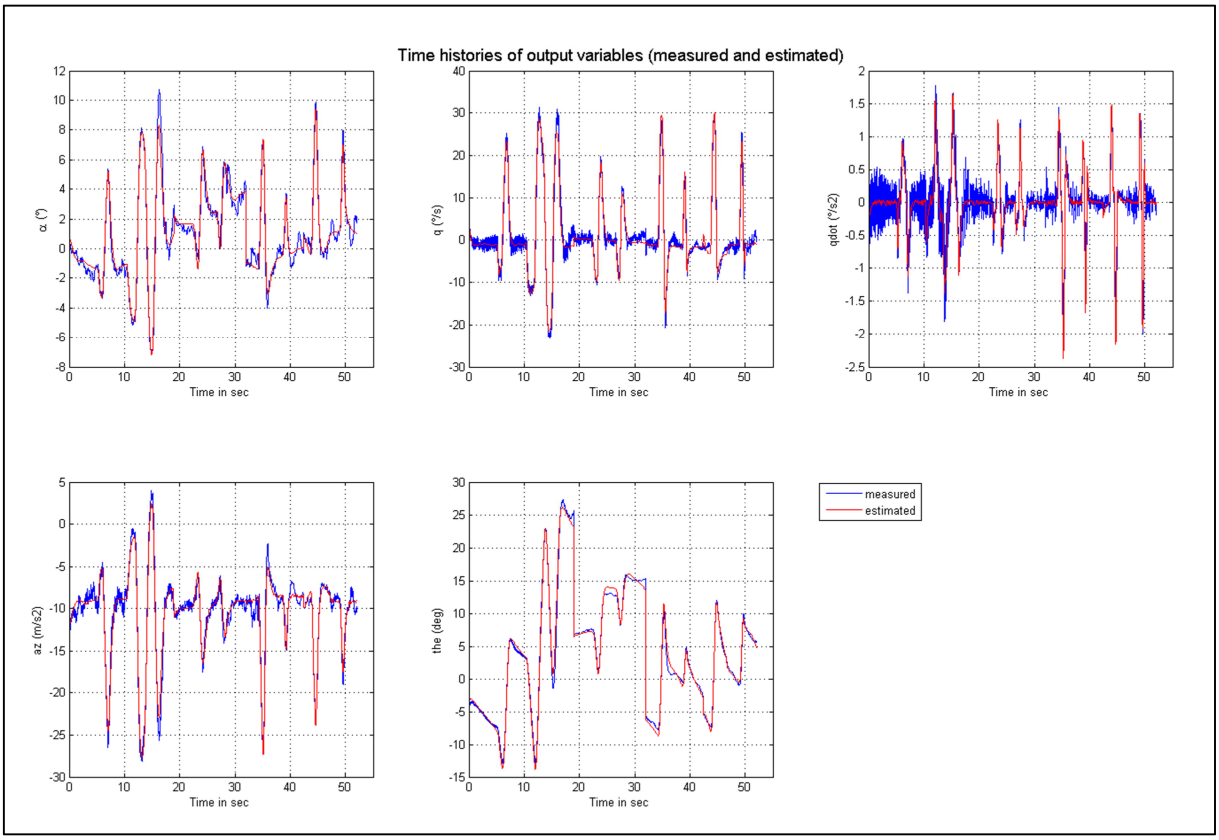

Figure 12: *Time histories of output variables (measured and estimated) – Parameter estimation with output-error method (longitudinal-directional motion)*

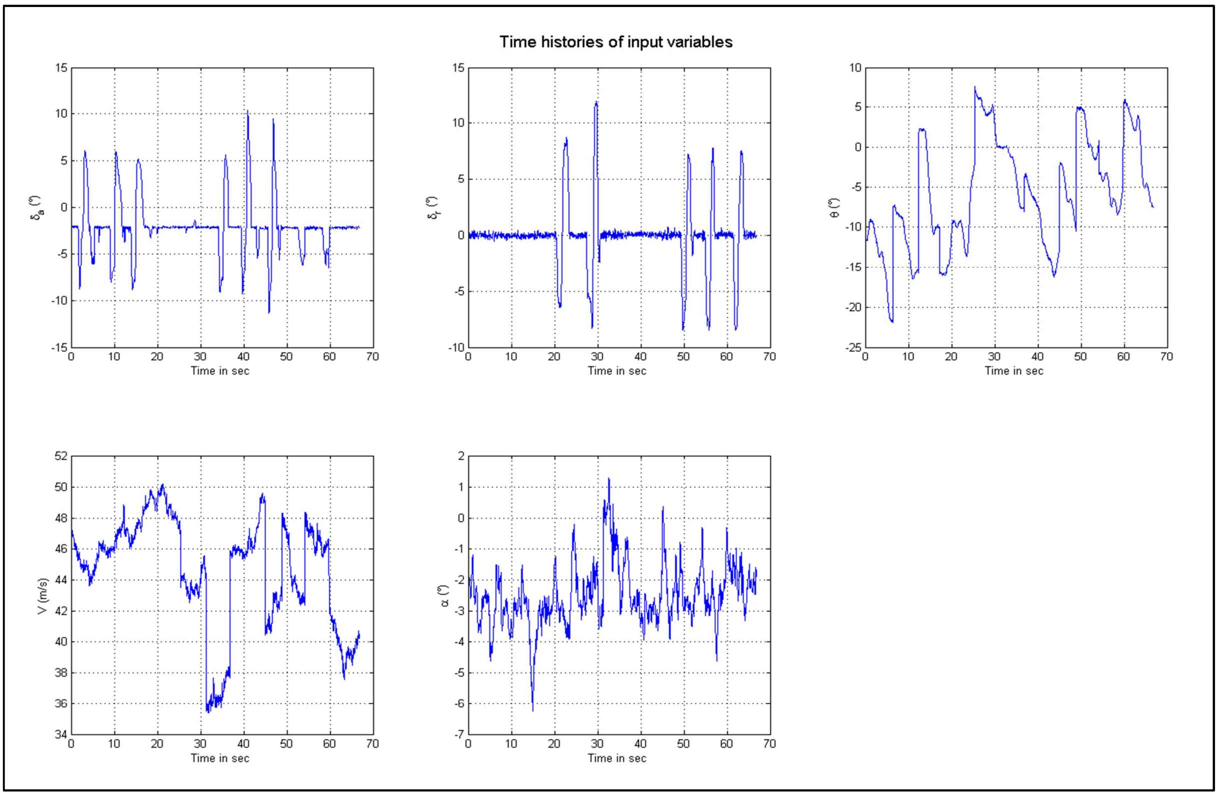

Figure 13: *Time histories of input variables – Parameter estimation with output-error method (lateral-directional motion)*

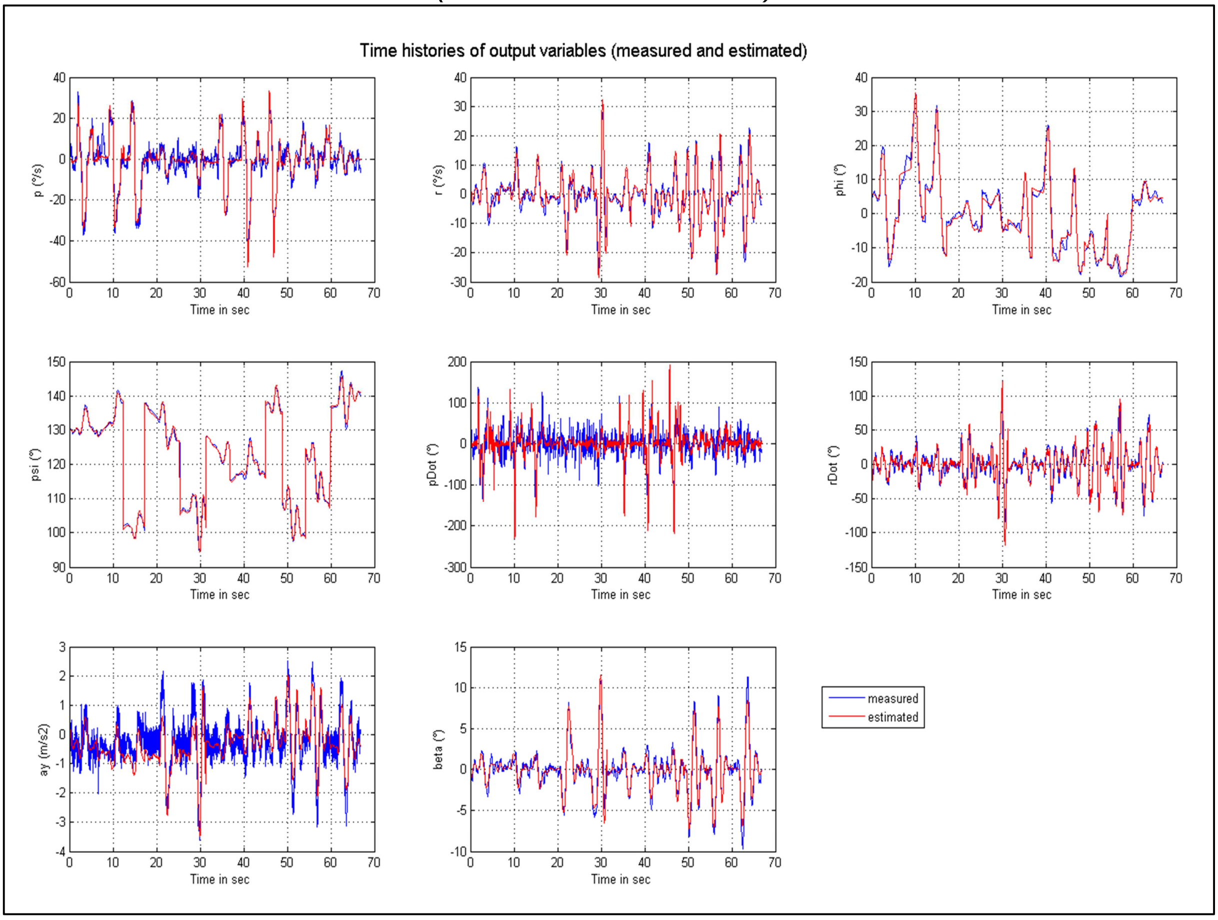

Figure 14: *Time histories of output variables (measured and estimated) – Parameter estimation with output-error method (lateral-directional motion)*

Comparison of Results

Parameter estimation results using output-error method and equation-error method are given below.

As can be seen from Table 3, both equation-error method and output error method provides very close parameter estimates for longitudinal motion. However, for lateral motion, C_l derivatives cannot be estimated by equation-error method. Flight data of Pelikan is noisy, especially in roll axis. Since Pelikan is propelled by two piston-prop engines, motion of propellers affects rolling motion parameters. This effect is not modeled in parameter estimation models. C_n derivatives obtained from two parameter estimation results are similar. C_Y derivatives are not similar because measured a_y is noisy and this affects the accuracy of equation error results.

Table 3: *Comparison of parameter estimation results - longitudinal-directional motion*

	Output-Error Results		Equation-Error Results	
	Parameter Estimation	Relative Std. Deviation (%)	Parameter Estimation	Relative Std. Deviation (%)
C_{L_0}	0.592	0.17	0.595	0.14
C_{L_α}	5.230	0.31	5.357	0.25
C_{m_0}	0.04	0.48	0.043	1.40
C_{m_α}	-0.360	0.79	-0.278	6.51
C_{m_q}	-28.401	0.73	-28.10	3.07
$C_{m_{\delta e}}$	-0.785	0.31	-0.750	1.24

Table 4: *Comparison of parameter estimation results - longitudinal-directional motion*

	Output-Error Results		Equation-Error Results	
	Parameter Estimation	Relative Std. Deviation (%)	Parameter Estimation	Relative Std. Deviation (%)
C_{l_p}	-7.775	0.22	-0.066	18.81
C_{l_r}	3.149	0.95	0.063	12.86
$C_{l_{\delta a}}$	-1.549	0.31	-0.016	15.12
$C_{l_{\delta r}}$	0.017	16.53	0.003	18.83
C_{l_β}	-0.231	3.13	-0.011	9.85
C_{l_0}	-0.047	0.77	-0.001	13.65
C_{n_p}	-0.104	1	-0.115	1.71
C_{n_r}	-0.184	1.01	-0.280	1.01
$C_{n_{\delta r}}$	-0.039	0.4	-0.039	0.83
C_{n_β}	0.131	0.2	0.114	0.45
C_{n_0}	-0.001	3.87	0.002	1.01
C_{Y_r}	0.596	7.63	1.482	3.43
$C_{Y_{\delta r}}$	0.069	5.09	0.034	12.49
C_{Y_β}	-0.785	0.99	-0.663	1.08
C_{Y_0}	-0.011	2	-0.001	21.32

Model Validation

The last step in system identification procedure is model validation. The estimated parameters have physically reasonable values with acceptable accuracy [Klein and Morelli, 2006].

One way of model validation is Theil's inequality coefficient which is the ratio of the square fit error and summation of the root mean square values of the measured and estimated signals [Jategaonkar, 2006].

$$U_i = \frac{\sqrt{1/N \sum_{k=1}^{N} [z_i(t_k) - y_i(t_k)]^2}}{\sqrt{1/N \sum_{k=1}^{N} [z_i(t_k)]^2} + \sqrt{1/N \sum_{k=1}^{N} [(t_k)]^2}} \quad i = 1,2 \dots n_y \tag{66}$$

$U_i = 0$ means perfect fit, $U_i = 1$ is the worst case. The maximum acceptable values of Theil's inequality coefficient is 0.25 – 0.30 [Jategaonkar, 2006].

Theil's inequality coefficients calculated for longitudinal output-error parameter estimations are given Table 5. As can be seen, only coefficient for $\dot{q}$ is greater than 0.3. In parameter estimation model, $\dot{q}$ is derived from measured q values. Because of this derivation process, noise level of $\dot{q}$ is high and this may cause high Theil's coefficient.

Table 5: Theil's inequality coefficient - longitudinal motion

Output Variable	Theil's Coefficient
α	0.18
q	0.17
a_z	0.08
$\dot{q}$	0.42
θ	0.07

Theil's inequality coefficients calculated for longitudinal output-error parameter estimations are given Table 6 which shows that coefficients for p, $\dot{p}, \dot{r}$ *and* a_y is greater than 0.3. The measured p values are noisy because of propeller effect. Since $\dot{p}$ is derived from measured p, noise level in $\dot{p}$ is also high. From flight data, it can be seen that measured a_y values are also noisy. High Theil's coefficient in these variables are due to high noise level of corresponding flight data.

Table 6: Theil's inequality coefficient - lateral motion

Output Variable	Theil's Coefficient
p	0.36
r	0.29
ϕ	0.12
ψ	0.01
$\dot{p}$	0.78
$\dot{r}$	0.37
a_y	0.57
β	0.30

CONCLUSIONS

Aircraft system identification is determination of the aircraft mathematical model using flight data in time domain or in frequency domain. This study aims to obtain stability and control derivatives of Pelikan from flight test data by using time domain system identification approach. Firstly, experiment design to excite longitudinal and lateral dynamics is explained. Then, data compatibility check is discussed. In analysis, lateral motion and longitudinal motion are studied separately. For parameter estimation, equation-error and output-error methods are used. Results obtained from these methods are compared. It is concluded that output-error method is more useful because flight data of Pelikan is noisy, especially in roll axis. Equation error cannot handle measurement noise so that this method is insufficient to identify Pelikan's lateral dynamics. For longitudinal motion both equation error and output error methods give similar results.

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