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DYNAMIC RESPONSE OF THE FGM PLATES TO BLAST LOADING

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ABSTRACT

The dynamic response of the Functionally Graded Material (FGM) plates under explosive blast loading is investigated. In this study, the mechanical properties such as the density, the modulus of elasticity, Poisson's Ratio, the thermal expansion coefficients are all assumed to be temperature dependent. The FGM plate is considered as a clamped thin plate. The Classical Plate Theory (CPT) is used for the structural formulations. The nonlinearity is considered as von Karman type. The FGM plate is subjected to explosive blast loading. Blast pressure is assumed to be uniformly distributed on the top surface of the FGM plate. Dynamic displacement responses of the FGM plate for different loadings, temperature and material parameters are investigated.

INTRODUCTION

The FGMs are new generation advanced composite materials whose mechanical properties are graded in certain directions. They have advantages over the classical laminated composite materials at the interfaces and boundaries since the FGMs have the continuous mechanical properties. They are inhomogeneous, but isotropic. The FGM plates are usually used in a high temperature environment where one constituent (usually ceramic) is used as a thermal barrier and other constituent (usually metal) is used for strength and flexibility. There have been intensive investigations and researches on the FGM plates since they were first introduced in Japan a few decades ago. The extensive studies on the FGM plates based on the different high order plate theories are made by [Shen, 2009]. Reddy and his colleagues presented the analysis of the FGM plates and cylinders based on the first order plate theory and third order plate theory [Reddy, 2000; Praveen, Chin and Reddy, 1999; Praveen and Reddy 1998; Lee, Zhao and Reddy,2010]. Nonlinear bending responses of functionally graded plates subjected to transverse loads and in thermal environments are presented by [Shen, 2002]. Dynamic response of initially stressed functionally graded rectangular thin plates by using the classical plate theory is presented by [Yang and Shen, 2001]. Responses of rectangular laminated composite plates and sandwich plates to explosive blast loadings are investigated by [Librescu, Oh, and Hohe, 2004; Beshara, 1994; Librescu and Nosier, 1990].

In this study, the response of the FGM plates subjected to blast loading is investigated. Temperature dependency of mechanical properties of constituents is taken into consideration. Blast load is assumed to be uniformly distributed on the FGM top surface. The modified Friedlander exponential decay equation is used for blast loading. Nonlinear displacement responses of the FGM plates to blast loadings and temperatures are examined.

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FORMULATION

Equation of Motion

The equation of motion for the FGM plate shown in Figure 1 in terms of the normal deflection $w(x,y,t)$ and the stress function $F(x,y,t)$ can be written as [Dogan, 2013]

$$
\begin{aligned}
& I_0 \frac{\partial^2 w}{\partial t^2} + \left(\frac{I_1^2}{I_0} - I_2 \right) \frac{\partial^2}{\partial t^2} \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) + c \frac{\partial w}{\partial t} \\
& + \left[D_{11}^* \frac{\partial^4 w}{\partial x^4} + 2(D_{12}^* + 2D_{66}^*) \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_{22}^* \frac{\partial^4 w}{\partial y^4} \right] \\
& + \left[B_{21}^* \frac{\partial^4 F}{\partial x^4} + (B_{11}^* + B_{22}^* - 2B_{66}^*) \frac{\partial^4 F}{\partial x^2 \partial y^2} + B_{12}^* \frac{\partial^4 F}{\partial y^4} \right] \\
& + \left[B_{11}^* \frac{\partial^2 N_{xx}^T}{\partial x^2} + B_{21}^* \frac{\partial^2 N_{yy}^T}{\partial x^2} + B_{12}^* \frac{\partial^2 N_{xx}^T}{\partial y^2} + B_{22}^* \frac{\partial^2 N_{yy}^T}{\partial y^2} - 2B_{66}^* \frac{\partial^2 N_{xy}^T}{\partial x \partial y} \right] \\
& - \left[\frac{\partial^2 F}{\partial y^2} \frac{\partial^2 w}{\partial x^2} - 2 \frac{\partial^2 F}{\partial x \partial y} \frac{\partial^2 w}{\partial x \partial y} + \frac{\partial^2 F}{\partial x^2} \frac{\partial^2 w}{\partial y^2} \right] \\
& = p(x,y,t)
\end{aligned} \tag{1}
$$

and the compatibility equation is given as

$$
\begin{aligned}
& \left[A_{11}^* \frac{\partial^4 F}{\partial y^4} + (2A_{12}^* + A_{66}^*) \frac{\partial^4 F}{\partial x^2 \partial y^2} + A_{22}^* \frac{\partial^4 F}{\partial x^4} \right] \\
& - \left[B_{12}^* \frac{\partial^4 w}{\partial y^4} + (B_{11}^* + B_{22}^* - 2B_{66}^*) \frac{\partial^4 w}{\partial x^2 \partial y^2} + B_{21}^* \frac{\partial^4 w}{\partial x^4} \right] \\
& = \left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 - \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2}
\end{aligned} \tag{2}
$$

where the mass inertia terms are

$$(I_0, I_1, I_2) = \int_{-\frac{h}{2}}^{\frac{h}{2}} \rho \cdot (1, z, z^2)\, dz \tag{3}$$

where ρ is the density of the plate, c is the viscous damping coefficient, $p(x,y,t)$ is the pressure on the plate due to explosive blast.

The reduced stiffness matrices are defined as

$$[A_{ij}^*] = [A_{ij}]^{-1} \tag{4}$$

$$[B_{ij}^*] = -[A_{ij}]^{-1}[B_{ij}] \tag{5}$$

$$[D_{ij}^*] = [D_{ij}] - [B_{ij}][A_{ij}]^{-1}[B_{ij}] \tag{6}$$

and where A_{ij} 's are the extensional stiffness, B_{ij} 's are the bending-extensional stiffness, and D_{ij} 's are the bending stiffness, in their usual usage.

All material properties are assumed to vary through the plate thickness only according to a power-law distribution and temperature T as [Shen, 2009, Praveen and Reddy, 1998]

$$P(z,T) = P_b(T) + (P_t(T) - P_b(T))\left(\frac{2z+h}{2h}\right)^n \tag{7}$$

where

$$P(T) = P_0(P_{-1}T^{-1} + 1 + P_1T + P_2T^2 + P_3T^3) \tag{8}$$

where P_0, P_{-1}, P_1, P_2 and P_3 are the coefficients of temperature $T(in\,K)$ in the cubic fit of the material property.

The thermal force and moment resultants are

$$(\{N^T\},\{M^T\}) = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{Bmatrix} (Q_{11}+Q_{12})\alpha \\ (Q_{12}+Q_{22})\alpha \\ 0 \end{Bmatrix} \cdot \Delta T \cdot (1,z)dz \tag{9}$$

where ΔT is the temperature change from a stress free state, α is the thermal expansion coefficient.

It is assumed that plate is clamped along four edges with immovable ends. While in-plane boundary conditions are satisfied on average, the normal boundary conditions are satisfied exactly by the following assumed solution

$$w(x,y,t) = \sum_{m,n} W_{mn}(t)\,(1-\cos(\alpha_m x))\,(1-\cos(\beta_n y)) \tag{10}$$

where $\alpha_m = \dfrac{2\pi m}{a}$ and $\beta_n = \dfrac{2\pi n}{b}$ (11)

As a solution procedure, the stress function is first obtained by substituting Eq. (10) into Eq. (2). The stress function and assumed deflection function then both inserted in Eq.(1), and Galerkin method is employed to obtain the ordinary differential equations in time domain for $W_{mn}(t)$. These coupled nonlinear ordinary differential equations are solved numerically by using Runge-Kutta method.

Explosive Blast Loading

If the detonation center of explosive is far from the plate and the dimensions of the plate are small compared to that distance, the pressure distribution on the top of the plate can be assumed to uniform. The blast overpressure can be written in terms of the modified Friedlander exponential decay equation with both negative and positive phases of the blast.as [Librescu, Oh, and Hohe, 2004; Beshara, 1994; Librescu and Nosier, 1990; Gupta, Gregory, Bitting and Bhattacharya, 1987]

$$p(x,y,t) = p(t) = p_m\,(1 - t/t_p)e^{-\eta t/t_p} \tag{12}$$

where t_p is the positive phase duration of the impulse measured from time of arrival of the blast at the plate surface, p_m is the peak reflected pressure in excess of the ambient, η is the decay parameter, and t is the elapsed time.

NUMERICAL RESULTS AND DISCUSSION

The FGM plate geometry, coordinate system and loading are shown in Figure 1. Following dimensions are used in numerical examples: the plate thickness $h = 2.5\,mm$; the width $a = b = 500\,mm$. The FGM plate composed of titanium alloy Ti-6AL-4V (as metal) and zirconia ZrO_2 (as ceramic) is considered. For explosive blast loading, following values are used:, $p_m = 20\,kPa$, $t_p = 0.005$ sec, $\alpha = 0.3$ (unless otherwise stated). The response is computed at the center of the plate (i.e., $x = a/2$, $y = b/2$).

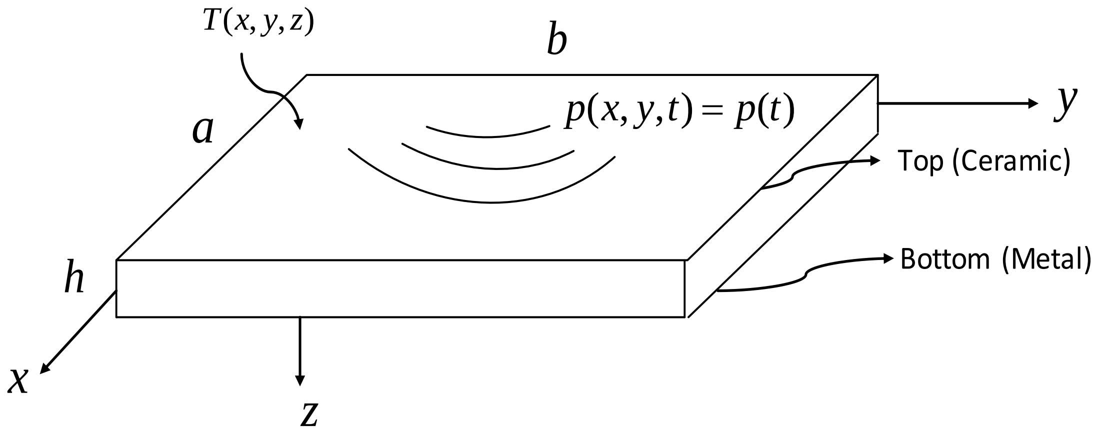

Figure 1: *FGM plate geometry, coordinate system and loading*

Nonlinear displacement history of the FGM plate to blast loading is given in Figure 2. The normalized maximum (peak) value of the displacement is $(w/h)_{\max} = 5.635$ at $t = 0.0013$ s, and the minimum (valley) value is $(w/h)_{\min} = -5.501$ at $t = 0.0165$ s. It is obvious that the response is highly nonlinear reaching the several times the thickness in very early phase of the history, then the response is damped out quickly to the rest position within 0.25 seconds.

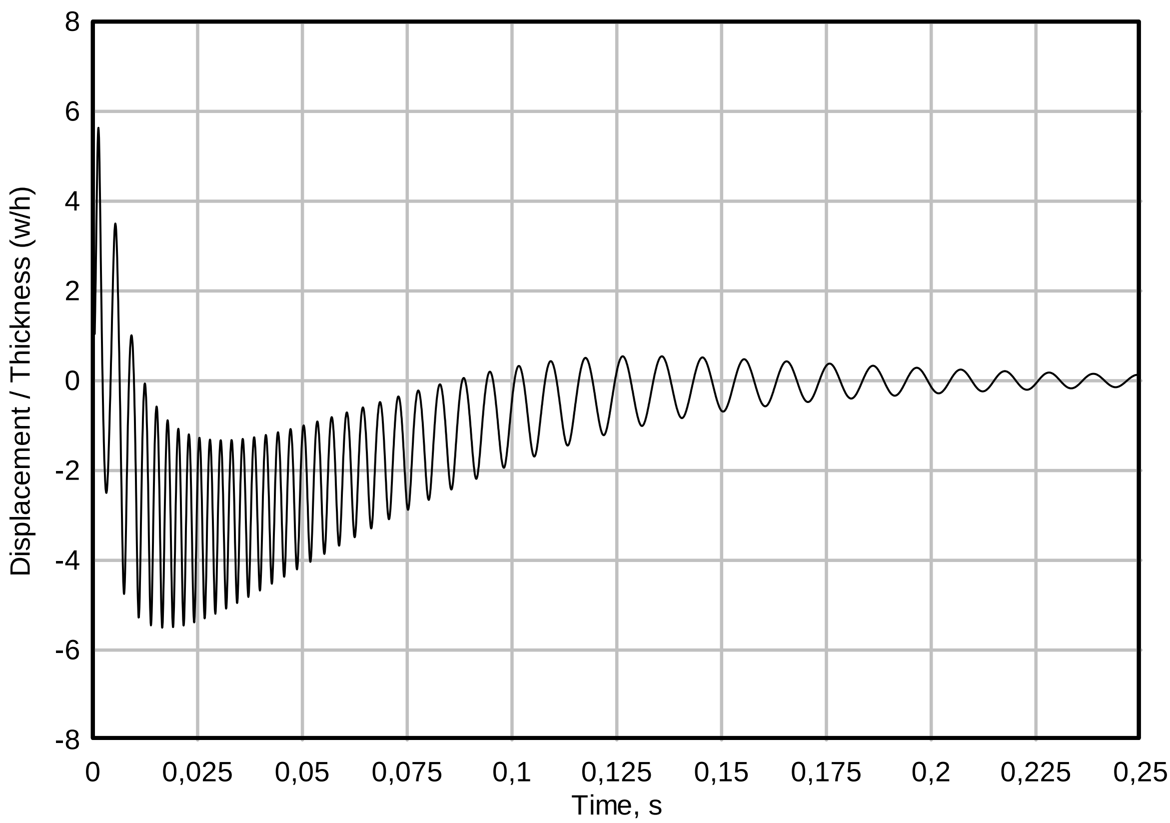

Figure 2: *Displacement-response history ($n = 0.7$)*

The response of the plate at uniformly elevated temperature ($\Delta T = 300\,K$) under the same blast loading is plotted in Figure 3. The stress free reference temperature is assumed to be $T_0 = 300\,K$. In this case, the maximum and minimum responses are $(w/h)_{\max} = 11.504$ at $t = 0.0012\,\text{s}$ and $(w/h)_{\max} = -10.559$ at $t = 0.012\,\text{s}$, respectively. After the FGM plate response reaches its maximum (peak) value in the early stage of the history, it moves to opposite side and continues to vibrate about the buckled position with decreasing amplitude due to damping. The similar response with smaller peak values is obtained for blast loading at linearly varying temperature through the thickness as shown in Figure 4. In this case, the top face temperature is $T_t = 600\,K$ and the bottom face temperature is $T_b = 400\,K$, the variation of temperature from top to bottom is linear. These results indicate that presence and variation of temperature greatly affect the displacement response.

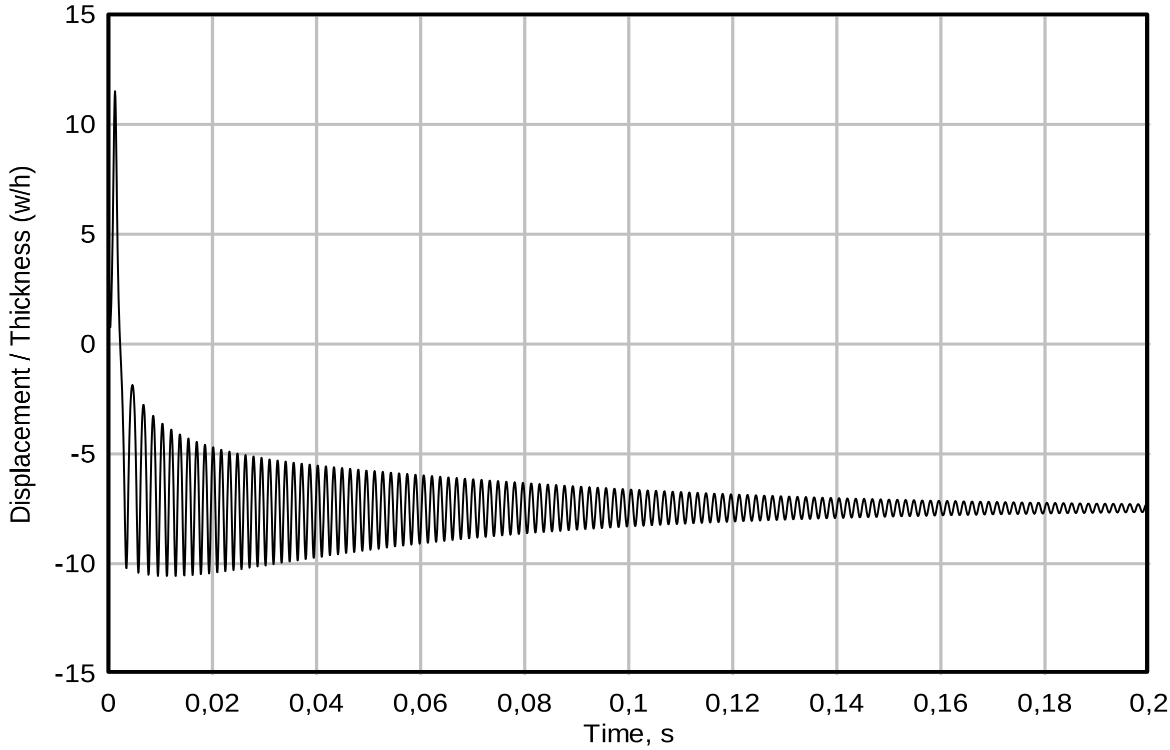

Figure 3: *The displacement response history at uniformly elevated temperature ($\Delta T = 300\,K$, $n = 0.7$)*

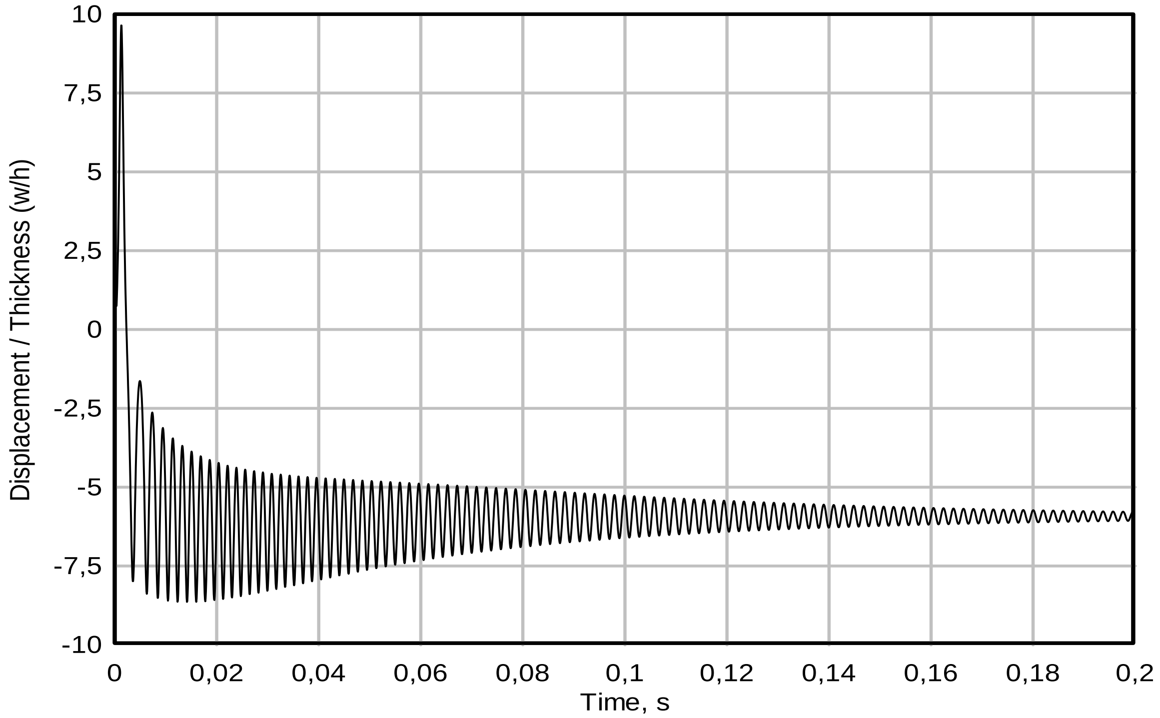

Figure 4: *Displacement response history at linearly elevated temperature* $(T_t = 600\,K\ ,T_b = 400\,K\ ,\ \ n = 0.7\,)$

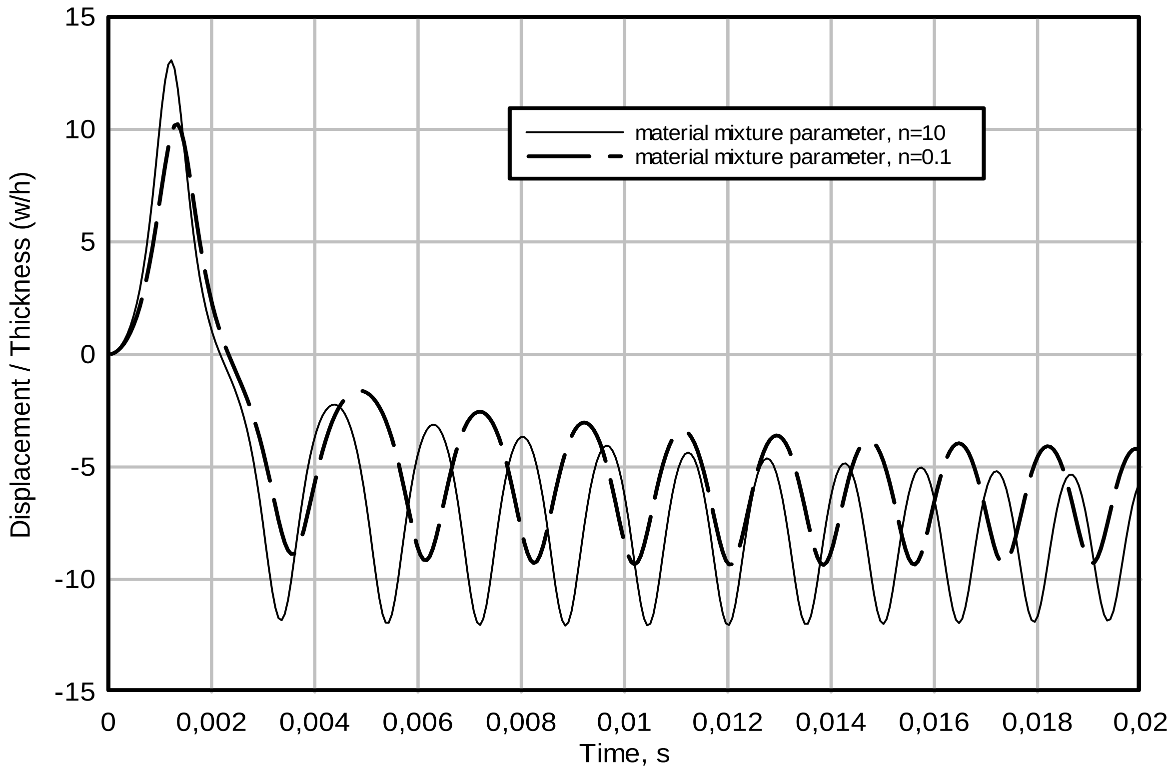

Figure 5: *Displacement response history for different mixture parameters at linearly elevated temperature.* $(T_t = 800\,K\ ,T_b = 400\,K\ )$.

It is well known that the FGM plates are generally used, at thermal environment, for high mechanical loading. Constituents of the FGM mixture can be tailored upon the system requirements. When the material parameter (index) n is large, a metal rich FGM plate is obtained; on the contrary, when n is small, a ceramic rich FGM plate is obtained. Figure 5 illustrates the influence of the material mixture parameter, n , on the response at linearly elevated temperature.

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