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MODELING THE FLIGHT DYNAMICS OF A NEW SUPERSONIC AIRCRAFT DURING PRELIMINARY DESIGN

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ABSTRACT

Nonlinear six-degree-of-freedom flight modeling of a new low boom experimental air vehicle during preliminary design was implemented using MATLAB. Force and moment coefficient data were presented in look up tables as they were available from wind tunnel tests of 1/12 scale model of aircraft. Damping derivatives were obtained using empirical DATCOM methods. Programming routines for trim and numerical integration were included in this investigation. Time history variation of state variables shows aircraft motion for given flight conditions. The modeling procedure outlined in this paper should guide the efforts of new supersonic aircraft dynamic analysis, trajectory studies and control law design.

NOMENCLATURE

a_X, a_Y, a_Z	= linear acceleration along X, Y and Z axis
c	= mean aerodynamic chord (MAC)
C_L, C_D, C_m	= lift, drag and pitching moment coefficients
C_l, C_n, C_Y	= rolling and yawing moment and side force coefficients
C_X, C_Y, C_Z	= non-dimensional X, Y, Z body axis force coefficients
i_h	= horizontal incidence angle [deg]
I_X, I_Y, I_Z	= X, Y, Z body axis moments of inertias [slug-ft^2]
I_{XY}, I_{XZ}, I_{ZX}	= X-Y, X-Z and Z-X body axis product of inertias [slug-ft^2]
m	= aircraft mass [slugs]
M	= Mach number
p, q, r	= roll, pitch and yaw rates [rad/s]
$\bar{q}$	= dynamic pressure [lb/ft^2]
S	= wing area [ft^2]
T	= engine thrust [lb]
u, v, w	= x, y and z velocity components [ft/s]
V_t	= flight velocity [ft/s]
W	= aircraft weight [lb]
x, y, z	= coordinate axis
x_E, y_E, z_E	= X, Y and Z earth axis positions
α	= angle of attack [deg]
β	= sideslip angle [deg]
Θ, Ψ, Φ	= Euler pitch, yaw and roll angle [rad]

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INTRODUCTION

Modeling and simulation using computer tools have become an essential element to mission success of air vehicles. The range of missions involves aircraft operations within the flight envelope, in which the aircraft has to follow a certain flight path. There has to be a reasonable confidence of flying such a flight path before the mission is undertaken.

Modeling the flight dynamics is very useful in planning the aircraft mission and in turn the aircraft design. The simulations based on the modeling provide aircraft response to commands and controls, allowing a careful evaluation of mission flight paths. The value and usefulness of the modeling is closely tied to the reliability and the type of modeling that represents the flight dynamics.

There are analytical and experimental methods commonly used for such flight modeling. Analytical techniques are highly advanced with developments in computer modeling of fluid dynamics. However, these methods have to be correlated to experiments before they can be used in confidence. A combination of these two methods is commonly used in today's applications.

In this article, the methodology outlined in [Stevens B.L. and Lewis, F.L, 2003] is adopted for modeling the flight dynamics of an example aircraft. The example aircraft, as described in [Henne, P. A, Howe, D. C, Wolz, R. R, and Hancock, J. L, 2004] is one of the concept candidates to become a new environmentally friendly supersonic aircraft. With the objective of lowering the sonic boom, as reported in [Howe, D, 2005], CFD methods were used to come up with a low boom configuration, with a geometric view shown in Figure 1. To validate the CFD tools and establish a baseline for design efforts, supersonic wind tunnel tests were conducted to obtain force and moment data on 1/12 scale model of aircraft. Limited wind tunnel data were presented in [Biber, K, 2011] to evaluate cruise flying qualities of the aircraft.

Using the wind tunnel data along with analytical estimates obtained from USAF DATCOM report, [Hoek, D.E., and Fink, R.D., 1960], the nonlinear differential equations for aircraft state vectors presented explicitly in [Garza, F.R, and Morelli, E.A, 2003] is solved numerically for six-degree-of-freedom flight. The MATLAB programming routines include trim and numerical integration routines. For this analysis, the aircraft configuration considered is in level flight and controls fixed. However, an extension could be made to include the effects of control and associated design. This type of work should be considered as a forward step toward the aircraft mission evaluation efforts in preliminary design.

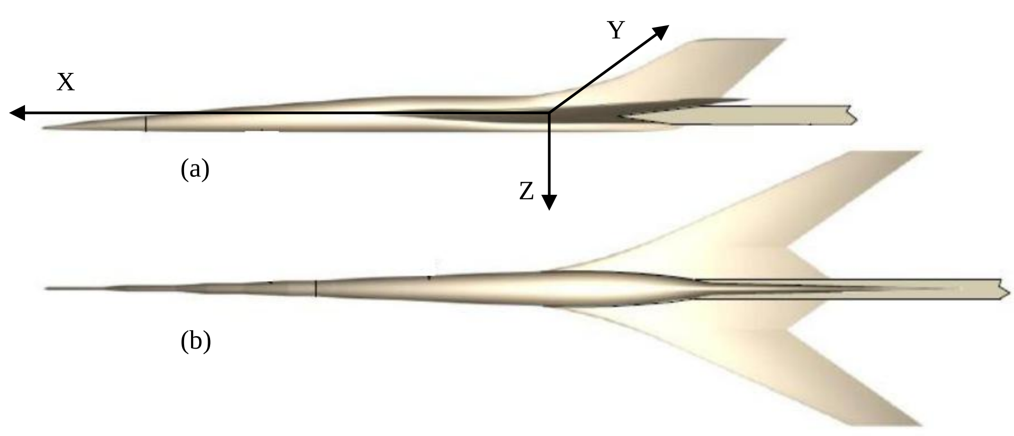

Figure 1: wind tunnel model (horizontal-off) of low-boom experimental vehicle with; a) spike-retracted and b) spike-extended configurations. X, Y and Z body axis with origin at center of gravity.

DESCRIPTION OF THE AIR VEHICLE AND ITS AERODYNAMICS

The flight modeling analysis was implemented for a supersonic aircraft, which has a preliminary aerodynamic database available for cruise configuration. The example aircraft considered is one of the baseline cruise configurations of the low boom experimental air vehicle being developed at Gulfstream Aerospace. Figure 1 shows two views of the vehicle with horizontal-off and controls fixed. Major aircraft data is given below.

Maximum take-off weight		35,000	lb
Wing span		36.3	ft
Wing area		600	ft^2
Mean aerodynamic chord		17.5	ft
Static thrust (sea level)/engine		13,600	lb
Fuselage max diameter		6	ft
Horizontal tail arm length		42.6	ft
Vertical tail arm length		22.17	ft
Fuselage length		172.4	ft
Moments of inertias,	I_{xx}	22458.6	slug-ft^2
	I_{yy}	375115.6	slug-ft^2
	I_{zz}	383021.1	slug-ft^2

To validate the aircraft preliminary design efforts, force and moment data were obtained from NASA Glenn supersonic wind tunnel tests conducted on 1/12 scale model of aircraft. The tunnel test section is 8 ft high by 6 ft wide and 23.5 ft long. The tunnel can operate at very low speeds from 0 to Mach 0.1 and from Mach 0.25 to 2.0. The wind tunnel facility description is given in http://facilities.grc.nasa.gov/8x6/.

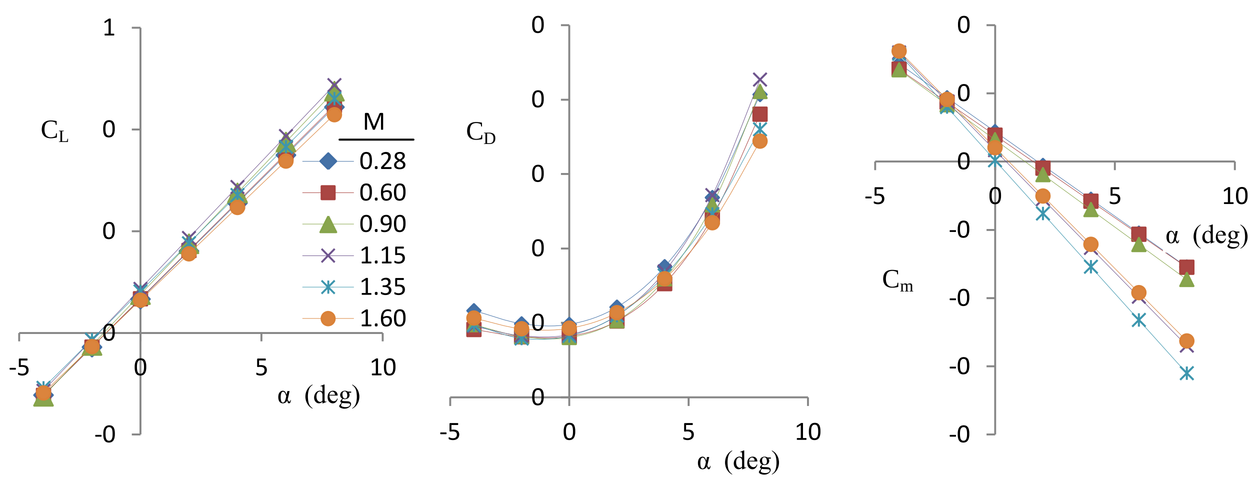

Figure 2: variation of lift, drag, and pitching moment coefficient with angle of attack at Mach numbers of 0.28, 0.60, 0.90, 1.15, 1.35 and 1.60.

The present test data was acquired at Mach numbers ranging from 0.28 to 1.8. Mach 1.6 was considered as supersonic design case at 45000 ft cruising altitude. Baseline test data were obtained for the center of gravity location at 0.35c. The test program included investigating the effects of spike extension and tail setting. Transition trip strips were attached on leading edge section of all surfaces to accommodate the Reynolds number effects. Component build up was made to investigate the effects of each aircraft element on force and moment characteristics.

For tail-on cruise configuration, Figures 2 shows typical variation of lift, drag and pitching moment coefficient with angle of attack. Figure 3 on the other hand shows typical variation of side force, rolling moment and yawing moment coefficients with side slip angle. Sideslip sweep was also made at various angels of attack. Longitudinal data was collected for horizontal incidence angles of 2 and -3 deg in addition to zero setting. The stall region is not modeled here and so the coefficient data is presented only for the linear range of angle of attack.

For the modeling analysis, the coefficient data is presented in two dimensional look up tables at selected Mach numbers of 0.28, 0.60, 0.90, 1.15, 1.35 and 1.60. A linear interpolation routine available in MATLAB was used to compute forces and moments for given angle of attack, side slip angle and Mach number. Additionally, considering the effects of horizontal setting longitudinally, total force and moment coefficients without the effects of flap, elevator, aileron, and rudder were calculated as follows:

Force coefficients

$$C_L = C_L(\alpha,\beta) + \frac{dC_L}{di_h} i_h$$

$$C_D = C_D(\alpha) + \frac{dC_D}{di_h} i_h$$

$$C_Y = C_Y(\beta)$$

Moment coefficients

$$C_l = C_l(\alpha,\beta)$$

$$C_m = C_m(\alpha) + \frac{dC_m}{di_h} i_h$$

$$C_n = C_n(\alpha,\beta)$$

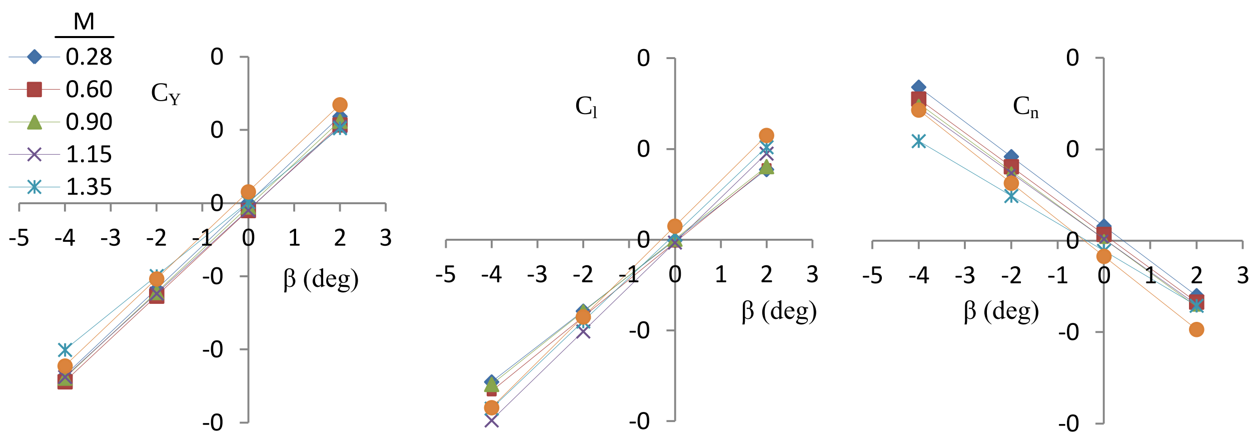

Figure 3: variation of side force, rolling moment and yawing moment coefficient with sideslip angle at Mach numbers of 0.28, 0.60, 0.90, 1.15, 1.35 and 1.60.

The wind tunnel data was obtained in stability axis and transformed into the body axis for use in the modeling analysis. Additionally, damping derivatives were obtained for both subsonic and supersonic range using USAF DATCOM methods as presented in [Hoek, D.E., and Fink, R.D., 1960]. Figure 4 shows variation of these C_{Lq}, C_{Yp}, C_{lp}, C_{np}, C_{Yr}, C_{lr} and C_{nr} derivatives as a function of Mach number. The damping derivative C_{Xq} is considered negligible.

Engine thrust force was assumed to act along the X-body axis through the center of gravity and computed by linear interpolation of engine thrust database available, as a function of altitude, and Mach number,

$$T = T(h, M)$$

The thrust from two engines were considered at 75% of its maximum continuous level. The modeling analysis could be made within the engine flight envelope defined by altitude and Mach number limitations.

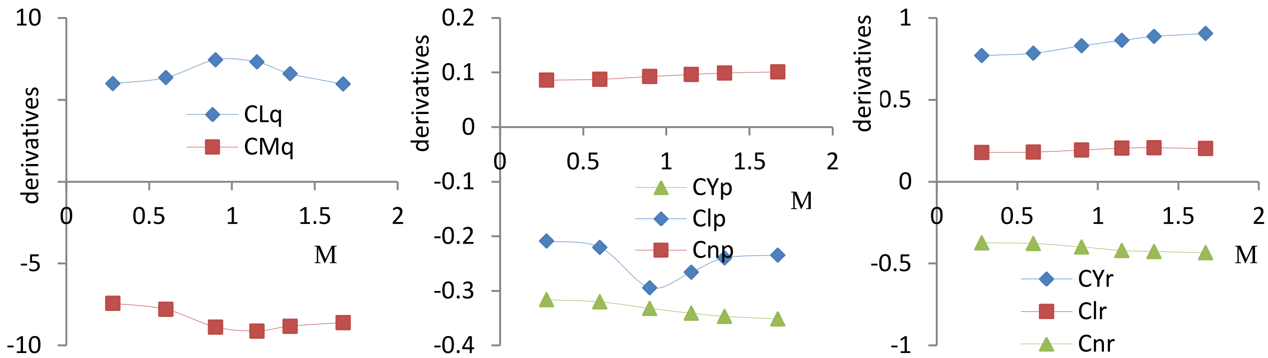

Figure 4: variation of damping derivatives, CLq, CMq, CYp, Clp, Cnp, CYr, Clr, Cnr with Mach number.

ANALYTICAL PROCEDURE FOR AIRCRAFT MODEL

The methodology for developing the nonlinear aircraft equation of motion for six-degree-freedom flight is presented in [Stevens B.L. and Lewis, F.L, 2003]. The aircraft is assumed rigid with constant mass density and symmetry about the X-Z plane in body axes. Forces and moments are acting on aircraft from aerodynamics, propulsion and gravity. The nonlinear aircraft dynamics is modeled in translational and rotational motion. The equations given in literature are transformed into a more suitable form for numerical integration with only one time derivative on the left side of equation. These equations are given as below for translational motion:

$$\dot{u} = rv - qw - g.\sin\theta + (\bar{q}SC_X + T)/m$$

$$\dot{v} = pw - ru + g.\cos\theta.\sin\phi + (\bar{q}SC_Y)/m$$

$$\dot{w} = qu - pv + g.\cos\theta.\sin\phi - (\bar{q}SC_Z)/m$$

The translational equations are expressed in terms of V_T, α, β instead of u, v and w because V_T, α, β can be measured directly on real aircraft and have a more direct relationship to piloting and the aerodynamic forces and moments. The relationships between V_t, α, β and u, v, w are as follows:

$$V_t = \sqrt{u^2 + v^2 + w^2}\,, \qquad \alpha = \tan^{-1}\left(\frac{w}{u}\right), \qquad \beta = \sin^{-1}\left(\frac{v}{V_t}\right)$$

and,

$$u = V_t\cos\alpha\cos\beta\,, \qquad v = V_t\sin\beta\,, \qquad w = V_t\sin\alpha\cos\beta$$

The equations for V_T, α, β can be differentiated with respect to time to give:

$$V_t^{\bullet} = \frac{uu^{\bullet} + vv^{\bullet} + ww^{\bullet}}{V_t}\,, \qquad \alpha^{\bullet} = \frac{uw^{\bullet} - wu^{\bullet}}{u^2 + w^2}\,, \qquad \beta^{\bullet} = \frac{V_t v^{\bullet} - vV_t^{\bullet}}{V_t^2\sqrt{1 - \left(\frac{v}{V_t}\right)^2}}$$

For the rotational motion, equations can be arranged to give time derivatives of roll, pitch and yaw rates as follows:

$$p^{\bullet} = (c_1 r + c_2 p + c_4 h_{eng})q + \bar{q}\,Sb(c_3 C_l + c_4 C_n)$$

$$q^{\bullet} = (c_5 p - c_7 h_{eng})r - c_6 (p^2 - r^2) + \bar{q}\,S\,\bar{c}\,c_7 C_m$$

$$r^{\bullet} = (c_8 p - c_2 r + c_9 h_{eng})q + \bar{q}\,Sb(c_4 C_l + c_9 C_n)$$

where the inertia terms are expressed as follows:

$$c_1 = \frac{(I_Y - I_Z) - I_{XZ}^{\;2}}{I_X I_Z - I_{XZ}^{\;2}} \qquad c_2 = \frac{(I_Y - I_Z + I_Z) I_{XZ}}{I_X I_Z - I_{XZ}^{\;2}} \qquad c_3 = \frac{I_Z}{I_X I_Z - I_{XZ}^{\;2}}$$

$$c_4 = \frac{I_{XZ}}{I_X I_Z - I_{XZ}^{\;2}} \qquad c_5 = \frac{I_Z - I_X}{I_Y} \qquad c_6 = \frac{I_{XZ}}{I_Y}$$

$$c_7 = \frac{1}{I_Y} \qquad c_8 = \frac{(I_X - I_Y) I_X - I_{XZ}^{\;2}}{I_X I_Z - I_{XZ}^{\;2}} \qquad c_9 = \frac{I_X}{I_X I_Z - I_{XZ}^{\;2}}$$

The nonlinear aircraft equations also include rotational kinematic and navigation equations. The rotational kinematic equations, which relate Euler angular rates to body-axis angular rates and are given below:

$$\phi^{\bullet} = p + \tan\theta (q \sin\phi + r\cos\phi)$$

$$\theta^{\bullet} = q\cos\phi - r\sin\phi$$

$$\psi^{\bullet} = \frac{q\sin\phi + r\cos\phi}{\cos\theta}$$

The navigation equations relate aircraft translational velocity components in body axes to earth-axis components, neglecting wind effects. These differential equations describe the time evolution of the position of the aircraft c.g. relative to earth-axis:

$$x_E^{\bullet} = u\cos\psi\cos\theta + v(\cos\psi\sin\theta\sin\phi - \sin\psi\cos\phi) + w(\cos\psi\sin\theta\cos\phi + \sin\psi\sin\phi)$$

$$y_E^{\bullet} = u\sin\psi\cos\theta + v(\sin\psi\sin\theta\sin\phi + \cos\psi\cos\phi) + w(\sin\psi\sin\theta\cos\phi - \cos\psi\sin\phi)$$

$$z_E^{\bullet} = u\sin\theta - v\cos\theta\sin\phi - w\cos\theta\cos\phi$$

Assuming thrust acts along the X body-axis, body-axis accelerations a_X, a_Y, and a_Z are calculated from:

$$a_X = \frac{\bar{q}\,SC_X + T}{mg}, \qquad a_Y = \frac{\bar{q}\,SC_Y}{mg}, \qquad a_Z = \frac{\bar{q}\,SC_Z}{mg}$$

Aircraft state equations can be presented in the state vector as follows:

$$\vec{x} = [V_t, \alpha, \beta, \phi, \theta, \psi, p, q, r, x_E, y_E, z_E]^T$$

For the analysis, control surfaces such as elevator, aileron, and rudder are all "up", that is not deflected; and they could be displayed with a control vector showing the angular movement of three control surfaces as follows;

$$\vec{u} = [\delta_e, \delta_a, \delta_r]^T$$

The set of first order ordinary differential equations described above represent the equations of motion for a rigid aircraft. Solution of such differential equations requires appropriate numerical integration technique.

STEADY-STATE TRIM OF AIRCRAFT

The nonlinear equations of motion can be linearized at an equilibrium flight condition known as trim, which can be used as an initial condition for flight simulation. At the trim, all translational and rotational accelerations are equal to zero. Time derivatives of state equations for V_t, α, β, p, q and r, augmented with a rate of climb and a coordinated turn constraint can make up a total of eight trim equations (see [Stevens B.L. and Lewis, F.L, 2003] for more details). With eight trim equations to find the trim condition, any combination of eight state or control surface deflection variables if available can be free to vary so that the equations can be satisfied.

A convenient method of trim as described in [Stevens B.L. and Lewis, F.L, 2003] is made through a numerical algorithm in which a COST function is formed from the sum of squares of time derivatives of V_t, α, β, p, q and r mentioned above. Initially, the velocity and altitude is user defined, and all other state and control variables are set to zero. A function minimization algorithm called SIMPLEX can then be used to adjust the control variables and the appropriate state variables to minimize this scalar cost.

With trim values of state and control variables defined, the first order nonlinear differential equations are solved numerically using the MATLAB solver "*ode45.*" The solution provides time history of all twelve state variables. For the present analysis, the aircraft is considered at level flight with no controls deflected except horizontal incidence and no control from engine power.

Figures 5 through 8 show typical trends in the time history variation of aircraft state variables at altitude of 45000 ft and Mach number of 1.6.

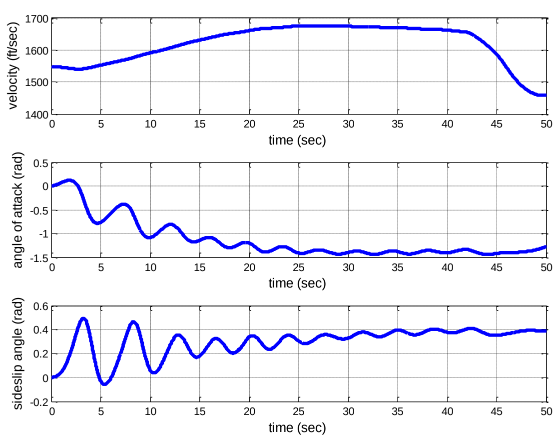

Figure 5: time history of velocity, angle of attack and sideslip

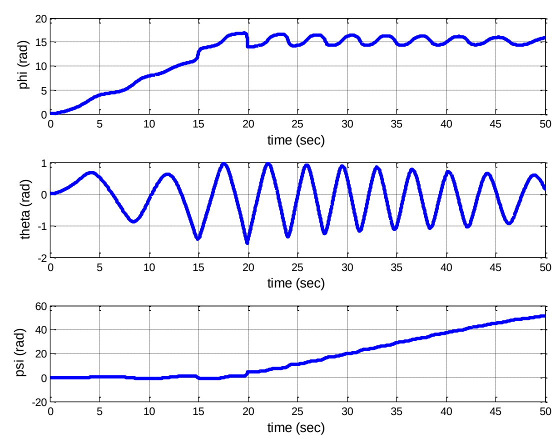

Figure 6: time history of Euler pitch, roll and taw angles.

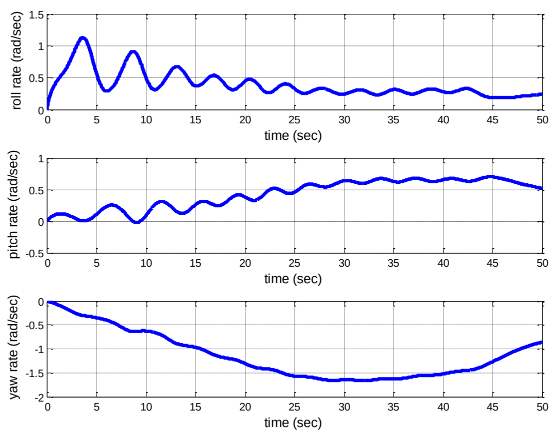

Figure 7: time history of roll, pitch and yaw rates.

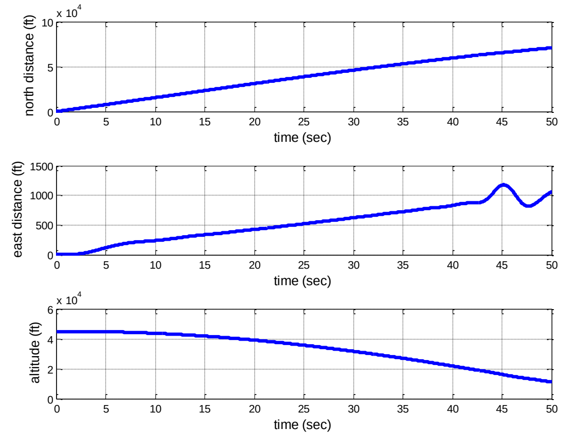

Figure 8: time history of navigation distances.

Within 50 second of time span, state variables show various trends in time. Velocity shows a significant increase initially, but after about 20 seconds it starts decreasing. Angle of attack and sideslip angles both show a shift in zero values and they have oscillations decaying after about 30 seconds. Euler angle in pitch attitude has a rather long oscillation with a decrease in amplitude. Euler angle in yaw has a shift from its zero value at about 20 seconds. Roll and pitch rates have a trend of decaying oscillations with some shift in time. Yaw rate shows a decrease first before it starts increasing at about 30 seconds. Navigation distances also show variation in time. The preliminary data has a view of aircraft dynamics for given flight conditions. Improvement in the data is expected as the aircraft design evolves.

CONCLUSION

Nonlinear six-degree-of-freedom flight modeling of an example supersonic aircraft that has a preliminary aerodynamic database available for level flight was implemented. First order non-linear differential equations were solved for trim condition and time history variations of state variables were obtained. The preliminary data has a view of aircraft dynamics for given flight conditions. Improvement in modeling the flight dynamics is expected as the aircraft design evolves. The procedure outlined in the article should be a forward step for preliminary design efforts of aircraft.

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