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RELIABILITY BASED DESIGN OPTIMIZATION OF SOLAR ARRAY STRUCTURE FOR A LEO NANOSATELLITE

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ABSTRACT

Satellites make use of solar energy to generate the power needed to run their subsystems. Solar arrays are mounted on the surfaces of the satellites to absorb solar rays and to store the accumulated energy in the batteries on board. The orbit of the satellite is extremely important in determining the amount of power that will be generated. This study aims to solve an optimization problem on the solar array design of a Low Earth Orbit (LEO) mission nano-satellite which will be used to investigate yeast bacteria's potential of survival in space conditions for further manned missions. The design objective is maximizing the power output which can be obtained from solar panels in the shortest eclipse time while constraining the solar panel cost for manufacturing and launch expenses within the given budget. In the first step, a deterministic optimization study is achieved, and then uncertainties which may exist in design variables and operating conditions are considered and a reliability based design optimization study is accomplished by using semi-major axis length and orbit inclination of the satellite as random parameters.

NOMENCLATURE

3J	=	triple junction
A	=	area of the solar panel
a	=	semi-major axis length
ADCS	=	attitude determination and control subsystem
A_{nozzle}	=	nozzle area
BOL	=	beginning of life
CDH	=	command and data handling
COM	=	telecommunication subsystem
D_{AM}	=	solar cell assembly and mismatch loss factor
D_C	=	calibration loss factor
D_{fb}	=	solar array fabrication loss factor
D_{fi}	=	solar array flight loss factor
D_{HD}	=	harness and diode loss factor
D_i	=	solar intensity factor
D_{MO}	=	micrometeorites/orbital debris loss factor
D_{op}	=	solar array operational loss factor
D_R	=	solar cell degradation factor
D_{Ra}	=	random loss factor
D_T	=	temperature degradation factor
D_{tc}	=	thermal cycling loss factor

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D_{UV}	=	UV degradation factor
D_V	=	solar cell voltage offset factor
e	=	eccentricity
EOL	=	end of life
EPS	=	electric power system
GaAs/Ge	=	gallium arsenide
$GaInP_2$	=	gallium indium phosphide
h	=	height of the solar panel
i	=	inclination
I	=	albedo
LEO	=	low Earth orbit
N_{Cell}	=	solar cell number
N_c	=	cost of a solar array
N_m	=	cell weight
N_w	=	solar array weight
n_i	=	solar cell type
P_{Cell}	=	solar cell power
P_f	=	packing factor
P_f	=	probability of failure
P_{SA}	=	solar array power
PSLV	=	polar satellite launch vehicle
r	=	radius of circular orbit
Ro	=	radius of the Earth
SCA	=	solar cell assembly area
Si	=	silicon
Si-BSR	=	silicon – with back surface reflector
SJ	=	single junction
SMS	=	structural and mechanical subsystem
T	=	orbit period
t	=	thickness
TCS	=	thermal control subsystem
$T_{Illumination}$	=	illumination time of orbit
T_{Shadow}	=	shadow time of orbit
w	=	width of the solar panel
β	=	Sun – orbit – plane angle
β	=	safety index
ε	=	axial tilt of Earth
η	=	efficiency
μ	=	gravitational constant

INTRODUCTION

The advances in space technology enabled human settlement in space to a certain point, yet the establishment in space is still not suitable for civilized lodging. There are various motives that drive scientists to invest in space travel and accommodation. The further the patrols settle from the Earth, the harder the transfer of supplies will get. For nourishment it may not be possible to cultivate most of the food sources in these patrols but yeast will come in handy for processing the components needed for proper nutrition. Therefore a study about the yeast bacteria's potential of survival in space conditions is extremely essential. The GaLactic Mission is a proposed nano-satellite mission on investigating fermentation process in space conditions. This satellite was designed by the first three authors of this paper for the Mission Idea Contest-2 which was held in Japan [Ozturk et al., 2012]. In case of a possible human settlement in space, bacteria will be needed for nourishment and disease protection. The mission aims to provide artificial heating for the optimum fermentation temperatures [Rosenwald, 1962], different gravitational field alignments, and continuous mixing of the substrates within the containers. The bacteria's fermentation and survival potential under these circumstances are the main study focuses of the mission.

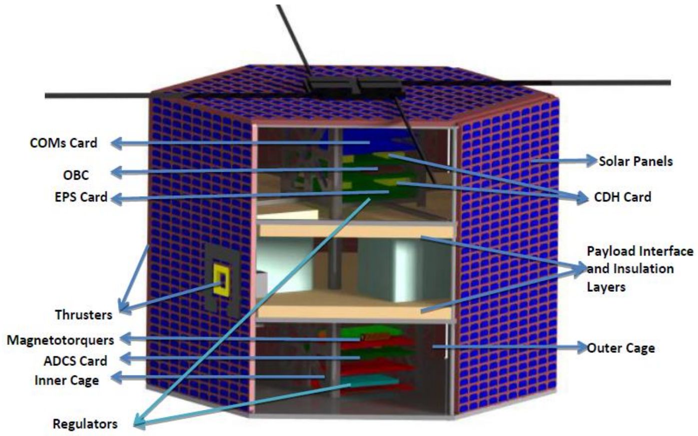

Figure 1: *The structure and subsystem configuration of the GaLactic's initial design*

One of the most important features of a mission is its orbit, since it determines the environment in which the subsystems will function. The Galactic Mission's primary objective is to provide the suitable temperature for fermentation of different types of bacteria in their containers, thus an active thermal control is required. The active thermal control systems include various heaters and sensors. To keep the containers in strict temperature ranges on the orbit, the thermal control system requires the largest portion of the power budget.

The Electric Power System (EPS) of a satellite is responsible for generation, distribution and storage of the necessary electric energy. In space based applications, solar arrays are used to generate the electric energy required by the subsystems of a satellite. The amount of generated power depends on the surface area of the solar panels, type of the solar cells used, the illumination time, the environmental temperature and the solar position angle.

One of the most important constraints in space based applications is the mass budget. The launch cost of a satellite is defined with its mass. In order to generate the adequate power for complicated subsystems a larger surface area or deployable panels are required. However, there is a direct correlation between the cost, mass and surface area, which also limits the generation of electric power to a certain degree. These drawbacks lead a quest for an orbit where the illumination period will be the longest and a solar array design that enables the largest surface area with the most efficient solar cell configuration. The restrictions over the design parameters should be listed and studied carefully while optimizing the solar power generation.

BACKGROUND

The initial step of a mission design is allocation of mass, power and cost budget quantities. The initial designs are made according to the allocated quantity for each subsystem. The nano satellite concept has its own constraints on mass and volume since the nano-satellites are considered as piggybacks and launch vehicle adapters are standardized. The mass and power allocations for the initial design, are configured by using the allocation tables [Brown, 2002], as given in Table-1.

Table 1: *Mass and power allocations for subsystems*

Subsystem on-orbit dry mass allocation	*Allocated mass,kg*	*Allocated power,Wh*
Structure and Mechanisms	7	0
Thermal Control	1	1
Attitude Determination and Control	2,67	2
Power System	7	2
Cabling	1,67	0
Propulsion System	1,67	2
Telecommunication System	1,33	6
Command and Data Handling	1,33	2
Payload	9,67	12

Mass total	33,33	27
Margin	16,67	6
Maximum	50	33

The allocations determine the constraints that will be used in the optimization problem. The preliminary cost budget initiated for the mission is given in Table-2.

Table 2: *Preliminary cost budgets for subsystems*

Subsystems	*Estimated Cost ($)*
SMS	12000
TCS	5000
ADCS	20000
EPS	200000
Propulsion	35000
COM	22000
CDH	7000
Payload	20000
Total	321000

The study aims to achieve an optimum solar array design for EPS subsystem which does not exceed the estimated cost, allocated mass and provides the maximum power output at the same time.

METHOD

The optimization of a solar array design is based on various parameters. The components of the process should be identified and studied with great care. In order to scan most of the commercially available products that will enhance the design, several different cell materials and configurations are taken into account.

In order to provide a basis for the optimization, the initial design is obtained via Trade Studies [Ozturk et al., 2012] by considering power and mass constraints. The power consumption which is defined as the sum of all sensors and actuators' power consumptions is calculated for dayside, eclipse and communication duration separately. The commercially available solar arrays are used in order to check if the power requirements are realistic. It has been experienced that further enhancements in the orbit allows a larger power generation plus the mass and cost budgets can be decreased by using an optimum design. Thus, finding the suitable orbit for this problem is also taken into account as well as the solar array dimension and cell efficiencies.

The enhancements allow power generation by larger quantities however it also increases mass and accordingly cost which is also a very important design criterion. In order to keep the design in a feasible domain, cost and mass budgets are also considered as constraints. There are also other restrictions defined by the other subsystems and mission characteristics. The ADCS of the GaLactic requires spin stabilization around z axis, which requires the inertia around z axis to be the largest, thus the width and height relation should be included in the constraints. The payload of the mission needs to function in a LEO since the atmospheric drag and radiation disruptions are essential for the mission. Therefore the semi-major axis is also restricted with LEO definition. The launch of the satellite to the desired orbit by means of a rocket also limits the allowable dimensions of the solar panels. Furthermore, the uncertainty in the inclination angle of the orbit will be considered and incorporated into the optimization process by using a probabilistic constraint in the criteria. The following sections of this paper will investigate the design parameters and construct the optimization problem with deterministic and probabilistic approaches and then determine the most appropriate optimization algorithm to achieve the solution.

Initial Design Values

The initial design values of the GaLactic Mission are given below [Ozturk et al., 2012]. These values are used as a reference starting point for further calculations and optimization procedure. After the optimum solution is reached the initial design will come in handy for comparison and validation of the efficiency of the new design. The initial design parameters are given in Table-3.

Table 3: *Initial design parameters*

Design Parameters	*Initial Design*
Semi-major axis of the orbit	320km
Inclination of the orbit	96.7471˚
Power generated by one solar array	7.4Watt
Solar cell type	GaAs
Solar cell efficiency	14.90%
Width of one solar panel	200mm
Height of one solar panel	300mm
Mass of one solar panel	0.054 kg
Cost of one solar panel	4000 $
Eclipse time of the orbit	2188.88 sec

Determination of the Orbit Parameters

GaLactic is planned to be in Low Earth Orbit (LEO), where the orbit trajectories are generally assumed circular. However, LEO's height is not constant; it has values of altitude up to 2000 km [Brown, 2002]. Thus, selection of a suitable altitude can take various probabilities since it affects most of the calculations and the required results. It is important to satisfy the minimum required power output by using solar cells for GaLactic to accomplish its mission. The more illumination time the orbit can provide, the more power will be generated. The algorithm used to calculate illumination time and inclination angle is given below.

<u>Algorithm for Illumination Time and Inclination Angle Calculation:</u>

It is important to mention some properties of GaLactic's orbit.

- GaLactic will be in a circular orbit. So its eccentricity (e) will be zero.
$$e = 0$$
- The semi-major axis length (a) is equal to radius of the circular orbit (r).
$$a = r$$
- Radius of the circular orbit is equal to the sum of the radius of Earth (R_0) and altitude (h).
$$r = R_0 + h$$

$$R_0 = 6378.14\ km$$

By using orbit's radius; period of the orbit can be calculated with the equation given below.

$$T = \frac{2\pi}{\sqrt{\mu}} r^{3/2} \tag{1}$$

Where, μ is the gravitational constant of the Earth ($\mu = 398600\ \ km^3/s^2$)

Then, by using the calculated value of orbit's period, it is possible to find the shadow duration of the orbit.

Eclipse time of the orbit can be found by using the equation [Brown, 2002].

$$T_{eclipse} = \left\{\cos^{-1}\left(\frac{\cos B_{max}}{\cos B}\right)\right\}\frac{T}{\pi} \tag{2}$$

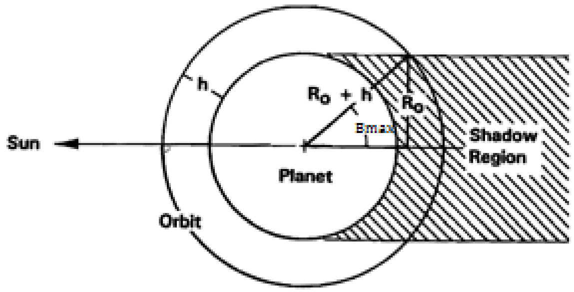

Figure 2: *Maximum Shadow Duration of Circular Orbit [4]*

After calculation of period and eclipse time, illumination duration can be calculated by using the equation given below.

$$T_{Illumination} = T - T_{eclipse} \tag{3}$$

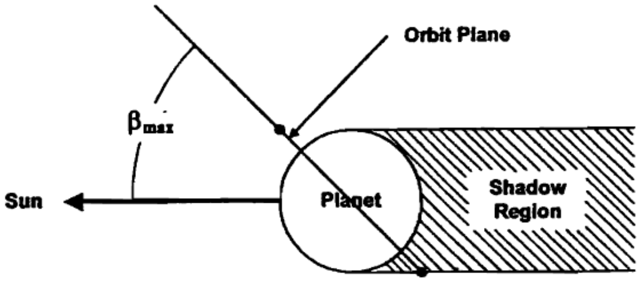

Figure 3: *Maximum B Angle for Shadow Free Orbit*

$$B_{max} = \sin^{-1}\left(\frac{1}{1 + h/R_0}\right) \tag{4}$$

For various B angles, inclination angle (i) of the orbit can be calculated by the equation given below.

$$i = B + \varepsilon \tag{5}$$

Where; ε is axial tilt of the Earth and has a value of 23.44˚.

$$i = B + \varepsilon = B + 23.44^{\circ}$$

Design of a Solar Array

GaLactic will use space solar cells in order to meet the required power generation during its mission time. These solar cells are the only power source of GaLactic, thus selection of solar cell type and designing the optimum solar array is very important. Nowadays, there are various commercially available solar cell types that can be found. In general, single crystalline solar cells are utilized for space applications. Different types of single crystalline solar cells and their some significant properties can be seen in the Table-4.

Table 4: *Single crystalline solar cells and their properties [2]*

Solar cell type	*Solar cell*	*Solar cell*	*Cover glass*	*BOL efficiency*	*Solar cell power*	*Cost*	*Cell Weight*
	thickness (μm)	area (cm^2)	thickness (μm)	(η%)(28°C)	@28°C @BOL (mW/cm^2)	$/$cm^2$	kg/m^2

Si-BSR cells	200	24	100	13.2	18	3.15	0.13-0.50
Thin Si cells	100	24	100	16.7	22.8	5.02	0.13-0.50
GaAs/Ge SJ cells	140	24	100	18.5	25.3	9.49	0.80-1.0
$GaInP_2$/GaAs/Ge 3J cells	140	24	100	27	37	10.55	0.80-1.0

The procedure that is followed for designing a solar array is presented below.

The solar array size calculation is made for the GaLactic considering the end of mission life (EOL) power need by using general solar array sizing formulations. While calculating the solar array size, all loss factors are considered.

The solar array size is determined by using equation (6). In order to calculate solar aray size; the solar cell assembly area, the packing factor and the solar cell power at end of mission is required.

$$Solar\ array\ size\ (m^2) = [P_{SA}(EOL) * SCA\ size]/[P_{cell}(EOL) * P_f] \tag{6}$$

In equation (6) PSA (EOL) refers to the solar array power at the end of mission life in watts. This required value is calculated by using equation (7).

$$P_{SA}(EOL) = P_{cell}(EOL) * N_{cell} \tag{7}$$

The Ncell which is used for calculating PSA (EOL), refers to the number of the solar cells in the solar array. SCA is the solar cell assembly area in m^2, Pcell (EOL) is the solar cell power at end of mission at operating temperature in watts and finally Pf is the packing factor which can be explained as the ratio of the solar cell area to the solar panel area.

Pcell (EOL), the solar cell power at the end of the mission life is calculated by using equation (8).

$$P_{cell}(EOL) = P_{cell}(BOL) * D_R * D_{fb} * D_{op} * D_{f_i} \tag{8}$$

Where D_R, is the solar cell degradation factor due to the particulate radiation and Dfb is the solar array fabrication loss factor which can be calculated by using equation (10).

Pcell (BOL) is the solar cell power at the beginning of its life at 28 °C in watts and can be calculated by using equation (9).

$$P_{cell}(BOL) = I * \eta * SCA\ size \tag{9}$$

Where I is the solar flux value which is 1367 W/m^2 normal to the cell, η is the BOL efficiency of solar cells at 28°C.

$$D_{fb} = D_{AM} * D_C * D_{HD} \tag{10}$$

D_{AM} refers to the solar cell assembly and mismatch loss factor, D_C is the calibration loss factor and D_{HD} is the harness and diode loss factor. D_{OP}, the solar array operational loss factor which is used in equation (8), is calculated by the following equation.

$$D_{OP} = D_i * D_T * D_V \tag{11}$$

Where D_i is the solar intensity factor, D_T is the solar cell temperature degradation factor and D_V is the solar cell voltage offset factor.

D_i can be calculated by using equation (12).

$$D_i = \left(\frac{149.6 * 10^6}{(149.6 * 10^6 - a)}\right)^2 \tag{12}$$

D_T calculation is given below in equation (13).

$$D_T = 1 - 0.005(T - t) \tag{13}$$

In equation (13), T is the temperature of the cells in orbit and t is the temperature at which cells were tested, 28 °C. T values are given below in Table-5.

Table 5: *On-orbit temperatures for different solar cells*

Solar cell type	*Si-BSR cells*	*Thin Si cells*	*GaAs/Ge SJ cells*	*$GaInP_2$/GaAs/Ge 3J cells*
Temperature (°C)	60	70	75	75

D_{fi} is the solar array flight loss factor and is calculated by using equation (14).

$$D_{fi} = D_{UV} * D_{MO} * D_{Ra} * D_{tc} \tag{14}$$

In equation (14), D_{UV} is the UV degradation factor, D_{MO} is the micrometeorites/orbital debris loss factor, D_{Ra} is the random loss factor, and D_{tc} is the thermal cycling loss factor.

The loss factors which are used in solar array size calculations can be seen in Table-6.

Table 6: *Loss factor values for a solar array design [2]*

Loss factor	*Value*
Solar Cell mismatch loss factor (D_M)	0.985
Calibration loss factor (D_C)	0.99
Packing factor (P_f)	0.9
Harness and diodes loss factor (D_{HD})	0.97
UV Degradation loss factor (D_{UV})	0.985
Micrometeorite/orbital debris loss factor (D_{MO})	0.98
Random loss factor (D_{Ra})	0.98
Solar cell degradation factor (D_R)	0.99
Thermal cycling factor (D_{tc})	0.99

To generate power, solar array should receive the solar rays normal to the surface area. Since the satellite is spin-stabilized, its areas that face the Sun proportionally change at every instant. In order to generate the adequate power, three cases are investigated to find area variation.

Case 1:

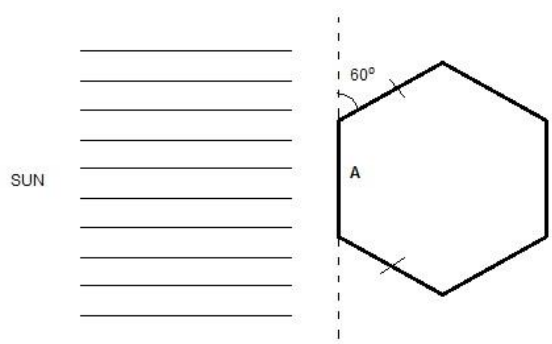

Figure 4: *Solar ray orientation case-1*

$$A_{total} = A + 2 * Acos60°$$
$$A_{total} = 2A$$

Case 2:

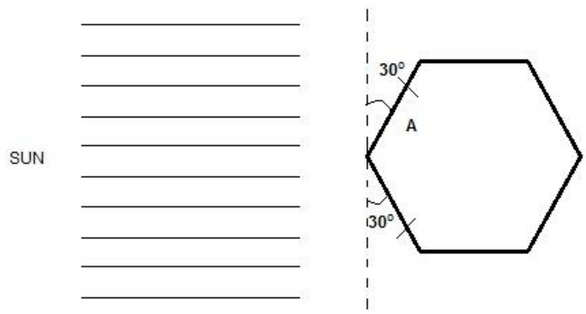

Figure 5: *Solar ray orientation case-2*

$$A_{total} = 2Acos30°$$
$$A_{total} = \sqrt{3}A = 1.732A$$

Case 3:

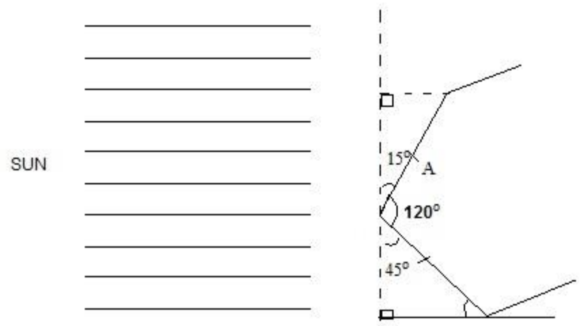

Figure 6: *Solar ray orientation case-3*

$$A_{total} = Acos45° + Acos15° + Acos75° = A\frac{\sqrt{2}}{2} + A\frac{(\sqrt{6}+\sqrt{2})}{4} + A\frac{(\sqrt{6}-\sqrt{2})}{4}$$

$$A_{total} = A\frac{\sqrt{2}}{2} + A\frac{\sqrt{6}}{4} + A\frac{\sqrt{2}}{4} + A\frac{\sqrt{6}}{4} - A\frac{\sqrt{2}}{4} = A\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{6}}{2}\right)$$

$$A_{total} = 1.932A$$

Case 2 is the most critical case and will be considered in optimization. Finally, the optimum design will be compared to the initial design. There is a certain area that is covered with the micro pulsed plasma thruster's nozzle. The area will be lost during the power generation. The area of the nozzle is 5.89 cm^2 and this value has to be considered and subtracted from the total area of one surface to find net area; while making the calculation about the solar array.

DESIGN CONSTRAINTS OF THE SYSTEM

The width and height of the solar panels define both mass and cost of the problem. Thus the restrictions on these parameters are the main constraints. Any property given in parenthesis is special for the solar cell type and can be found in Table-4.

Cost constraints

Cost is defined with solar cell type and mass increment as stated in Advanced Space Transportation Program by NASA. The new design's expenses should not exceed the allocated cost for solar panels.

$$Cost = w.h.(N\_c) + w.h.(N\_w).(22075\ USD/kg) \leq 4000USD \quad (15)$$

The width and height of a solar panel is shown in Figure-7 below. Cost increases with solar array weight by 22075 USD per kilogram.

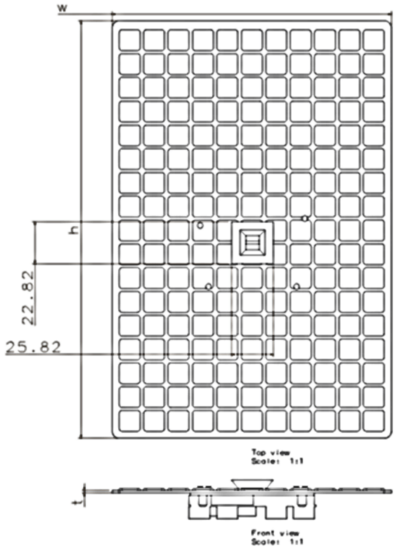

Figure 7: *Solar panel configuration*

Launch constraints

The dimensions given for various Launch Vehicle Adapters restrict the size of a satellite to be launched. General restrictions for satellites under 100 kg are given in Table-7.

Table 7: *Launch vehicle limitations on piggyback satellites [Ramakrishnan et al., n.d.]*

Auxiliary Payload Compartment	
Maximum Mass of One Satellite	100 kg
Maximum Width	600 mm
Maximum Length	600 mm
Maximum Height	800 mm

In order to qualify as a piggyback payload the satellite dimensions are reduced proportionally with the predicted mass.

$$w < 300mm$$
$$h < 300mm$$

ADCS constraints

The spin stabilized ADCS requires the moment of inertia around z to be largest. This inertia can be calculated for the plane passing through the center of gravity of the hexagonal structure and therefore can be assumed as a rectangular shape.

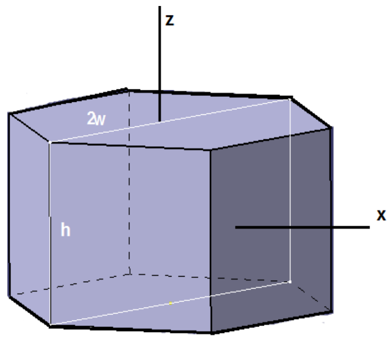

Figure 8: *The inertial plane of the satellite*

$$I_{xx} = \frac{1}{12} 2wh^3$$
$$I_z = \frac{1}{12} 8hw^3 \quad (16)$$

$$I_z = \frac{1}{12} 8hw^3 > I_{x,y} = \frac{1}{12} 2wh^3$$
$$4w^2 > h^2$$
$$2w - h > 0 \quad (17)$$

Orbit constraints

The orbit has its own constraints on inclination and semi-major axis by definition. These are given below.

$$0^\circ \leq i \leq 90^\circ$$
$$200\ km \leq a \leq 1000\ km$$

Mass constraints

The mass of one solar panel should not exceed the initial solar panel design's mass. This property can be found in Table-4 for cm^2 then can be converted to m^2.

$$w * h * (N\_m) \leq 0.054kg \quad (18)$$

Dimension constraints

GaLactic has a two cage structure with an inner and outer cage. The solar arrays of the Galactic will be mounted on the outer cage. The power outcome is related with the dimensions of the solar array, yet they cause a mass increment which is not favorable. The dimensions of the solar arrays are limited by the inner cage of the GaLactic and launch vehicle's auxiliary payload compartment's dimensions. The width and height should be larger than the inner cage. The width and height constraints can also be defined as follows.

$$2w > 174mm$$
$$h > 100mm$$

Battery constraint

The batteries onboard supply continuous power for 2100 seconds as many commercially available batteries do. These batteries are used for keeping the containers in the desired temperature range and communicating with the ground station if it is in the range. Thus the eclipse time should not exceed the duration that batteries can compensate for power. The eclipse time is calculated as follows.

$$T_{eclipse} = \left\{\cos^{-1}\left(\frac{\cos\beta_{max}}{\cos(i-\varepsilon)}\right)\right\}\frac{2}{\sqrt{\mu}}(a+Re)^{3/2} < 2100s \tag{19}$$

RELIABILITY BASED OPTIMIZATION METHOD

Reliability based design optimization (RBDO) is an optimization technique that interprets the functionality of the system under uncertainties. RBDO has been applied to some design problems in aeronautical engineering in the recent years and also to component/system design concerning nano-satellites as well [Nikbay and Kuru, 2013].

In the present work, certain designs may end up in the feasible domain after the deterministic design optimization, however; uncertainties which may result from some long-term space effects or misinterpretations should also be taken into account while improving the performance of a design. The RBDO technique employs the probability of failure concept which helps to express design constraints of the optimization problem with a probabilistic approach.

In comparison to the deterministic methods, the RBDO accounts for uncertainties in design criteria and uses reliability index as a measure of safety. Each probabilistic constraint forms a limit state function in the design space and a different reliability index value can be required for each constraint independently. The limit state function, $g(.)$, and the probability of failure, P_f, can be defined as follows.

$$g(X) = R(X) - S(X) \tag{20}$$

$$P_f = P\,[\,g(X) < 0\,] \tag{21}$$

where *R* and *S* respectively show the resistance and loading of the system in terms of the random variable, *X*.

A sample RBDO problem can be expressed as follows.

$$\min f(s,X)$$

Subject to

$$P_{f_i} = P_i[g_i(s,X) < 0] \le P_{R_i}, \quad i = 1, \dots, m$$
$$g_j^{det}(s) < 0, \quad j = 1, \dots, n \tag{22}$$

The number of the probabilistic constraints is denoted with m, whereas the number of deterministic constraints is expressed with n and s is the vector of the optimization variables. The g_j^{det} shows the deterministic design constraints given in the optimization problem. The probability density function of the $g_i(s,X)$ is given in Figure-9.

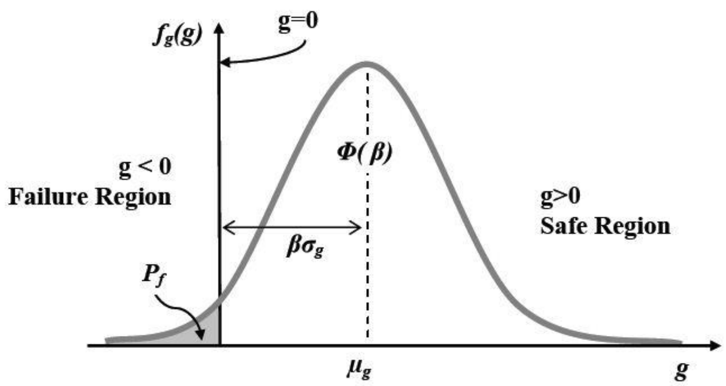

Figure 9: *The probability density function*

The probability of failure, denoted by P_f, can be expressed according to reliability (safety) index, β.

$$P_f = 1 - \Phi(\beta) \tag{23}$$

where Φ is the normal cumulative distribution function of β. Then the probabilistic constraint requires that:

$$\beta_i \geq \beta_{t_i}, \quad i = 1, \ldots, m \tag{24}$$

where the attained reliability index denoted with β_i has to be equal or larger than the target reliability index β_{t_i} for the i^{th} constraint.

MONTE CARLO SIMULATION AND ITS VALIDATION

The launch of a satellite into a desired orbit is a very difficult task. Without an active Attitude Control System the desired inclination can not be acquired most of the times. However the actuators increase the cost, mass and power budgets considerably. For GaLactic the spin stabilization is sufficient to meet Telecommunication requirements, thus in order not to increase the budgets of any kind, actuators for ADCS are not included in the subsystem design. The orbit inclination is defined in a domain. After the optimization process the inclination will converge to a single value, however an uncertainty can be added to the inclination angle to find a more reliable solution.

Reliability Based Design Optimization accounts for uncertainties and thus provides a more reliable design. As an uncertainty propagation algorithm Monte Carlo Simulation (MCS) generates a certain amount of samples using the given variations in the random parameters and then probability of failure is calculated by the ratio of the infeasible designs violationg design constraints to the total samples generated. In the present work, as a Probabilistic Approach, Monte Carlo Simulation will be used.

DEFINITION OF THE OPTIMIZATION PROBLEM

The design of a solar array can be defined with certain constraints and design parameters. There are different solar cell configurations and they have specific values that are listed in Table-4 and defined as a matrix. The uncertain nature of the orbit parameter, inclination, due to launch operations requires an RBDO procedure to be applied in order to simulate the problem realistically. The deterministic approach will also be employed in order to underline the significance of the RBDO, especially in space applications. The optimization variables are defined as width (x_1) and height (x_2) of the solar panel, semi-major axis length (x_3), inclination (x_4), and type of solar cell (x_5). The objective of optimization problem is to maximize power generation while cost, launch, mass, inertia and orbit constraints are also determined. The deterministic optimization problem is given as follows.

Maximize

$$f(\boldsymbol{s}) = P_{SA}(EOL) = A(x_1, x_2) * T(a, i) * N_i \tag{25}$$

Subject to

$$g_1(\boldsymbol{s}) = Cost = x_1 x_2 (N\_c) + x_1 x_2 (N\_s) - 4000USD \leq 0 \tag{26}$$

$$g_2(\boldsymbol{s}) = x_2 - 2x_1 < 0 \tag{27}$$

$$g_3(\boldsymbol{s}) = x_1 * x_2 * (N\_m) - 0.054\,kg \leq 0 \tag{28}$$

$$g_4(\boldsymbol{s}) = \left\{ \cos^{-1}\left(\frac{cos\left(sin^{-1}\left(\frac{1}{1 + x_3/R_0}\right)\right)}{\cos(x_4 - \varepsilon)} \right) \right\} \frac{2}{\sqrt{\mu}} (x_3 + Re)^{3/2} - 2100 < 0$$

$$\boldsymbol{s} = \{x_1, x_2, x_3, x_4, x_5\} = \{w, h, a, i, n_i\} \tag{29}$$

where

$$87mm \leq x_1 \leq 300mm$$
$$230mm \leq x_2 \leq 300mm$$
$$200\ km \leq x_3 \leq 1000\ km$$
$$0^{\circ} \leq x_4 \leq 90^{\circ}$$
$$x_5 \sim \{N_1, N_2, N_3, N_4\}$$

The *N* cluster symbolizes the properties of the solar cells distinctively. These properties can be found in Table-4, 5 and 6. The function *A* is the area function determined with *w* and *h*, *T* is the illumination time on the orbit and expressed with *a* and *i* variables, n_i is the type of solar cell. For input variables the constraints will be given in the optimization driver.

In the second step RBDO is performed. The optimization parameters are summarized in set *s* and the *X* is the set of random variables which are distributed with respect to normal (Gaussian) distribution. And random variables are determined as semi-major axis length (x_3) and inclination (x_4).

$$\boldsymbol{X} = \{a, i\}$$

The RBDO problem can be defined in terms of the safety index, β:

$$f(\boldsymbol{s}, \boldsymbol{X}) = P_{SA}(EOL) = A(w,h) * T(a,i) * N_i \tag{30}$$

Subject to

$$\beta \geq 3.0$$

This means that the probability of failure for the solar array design is 0.006.

$$g_1^{det}(\boldsymbol{s}) = x_1x_2(f(x_5)) + x_1x_2(f(x_5)) - 4000USD \leq 0 \tag{31}$$

$$g_2^{det}(\boldsymbol{s}) = x_2 - 2x_1 < 0 \tag{32}$$

$$g_3^{det}(\boldsymbol{s}) = x_1x_2(f(x_5)) - 0.198\ kg < 0$$

The x_5 is the solar cell type that determines the weight and cost of the system, where $f(x_5)$ is a function of solar cell weight based on the solar cell properties for the chosen type.

(33)

$$g_4^{prob}(\boldsymbol{s}, \boldsymbol{X}) = P\left[2100 - \left\{\cos^{-1}\left(\frac{cos\left(sin^{-1}\left(\frac{1}{1 + x_3/R_0}\right)\right)}{\cos(x_4 - \varepsilon)}\right)\right\}\frac{2}{\sqrt{\mu}}(x_3 + Re)^{3/2} < 0\right] < P_f = 0.006 \tag{34}$$

The constraints denoted as g_i^{det}, (i=1,2,3) are the deterministic constraints that do not contain any uncertainties. The constraint denoted with g_4^{prob} indicates the probabilistic constraint. The reliability index is denoted with β and the probability of failure parameter is denoted with P_f.

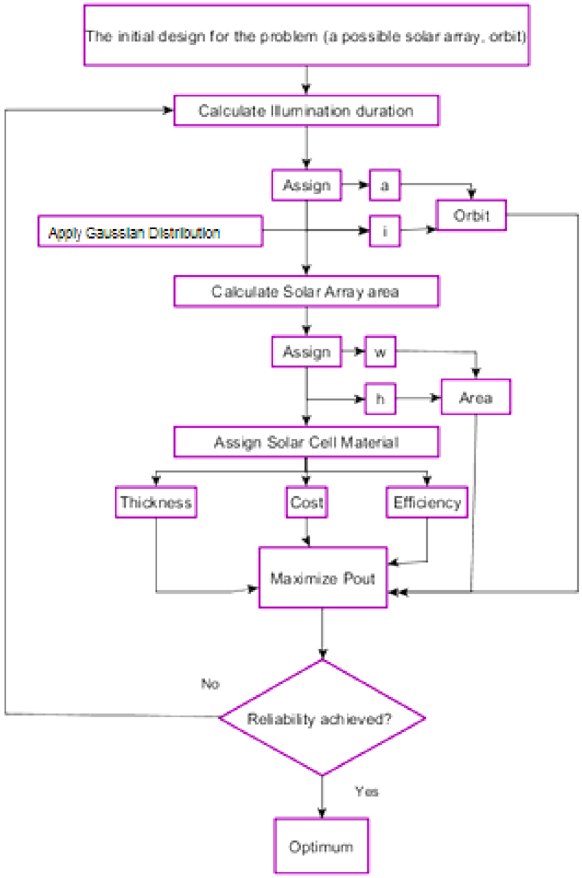

Figure 10: *The Workflow of the design problem*

The flowchart of the optimization procedure is given in Figure-10. The procedure starts with specifying the initial design. The commercial program ModeFrontier will be employed in configuring the optimization problem with deterministic and reliability based approaches.

OPTIMIZATION PROCESS

NLPQLP-Nonlinear Programming with Non-Monotone and Distributed Line Search is used as the optimization algorithm.The NLPQLP is an algorithm developed for solving smooth nonlinear programming problems with a gradient based approach [Schittkowski, 2004]. The code is employed from the ModeFrontier Scheduler Library. The method is chosen for its special focus on satisfying the linear constraints and boundary values.

Latin Hypercube Sampling

The design space created by the ModeFrontier should be sampled for design of experiments (DOEs) since optimization algorithms use these samples as initial design points. There are various algorithms to sample DOEs. The Unfiorm Latin Hypercube Sampling (LHS) is chosen for sampling the DOEs. The bias and extreme values are prevented with the LHS method since this technique can cover all the design space by generating equal number of design samples for equal interval lengths.

The Workflow in Optimization Driver

In order to solve the optimization problem, a commercial optimization driver, ModeFrontier will be used. The workflow of the optimization procedure is given in the Figure-11.There are four design variables and one design matrice that contains the values of the solar cell specifications. There are four different solar cell types that are included in the optimization problem. Their efficiency, thickness, cost and operational temperatures all effect the resulting power output. The other four design parameters are divided into two categories, first one being the orbital properties and the second being the solar array dimensions. The inputs are defined with upper and lower bounds in the workflow.

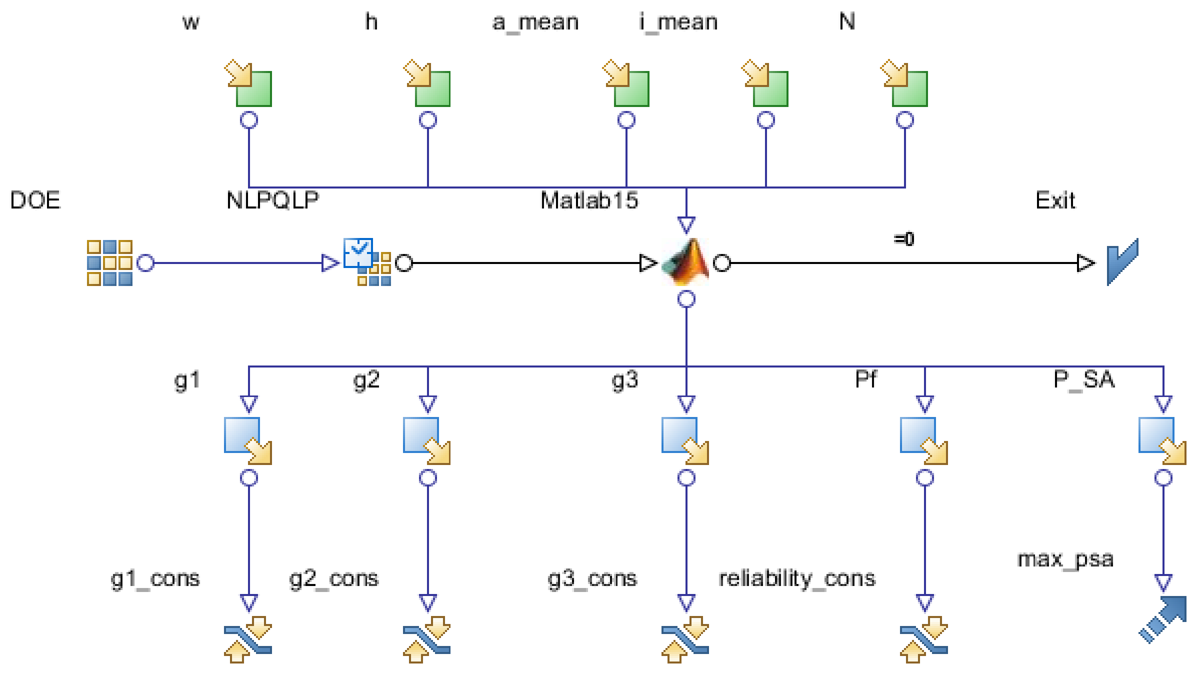

Figure 11: *The Workflow of the design problem defined in modeFRONTIER*

The inputs are linked with MATLAB, where the derived algorithm calculates the values for the constraints and the cost function. The flowchart for the MATLAB algorithm is given in Figure-12.

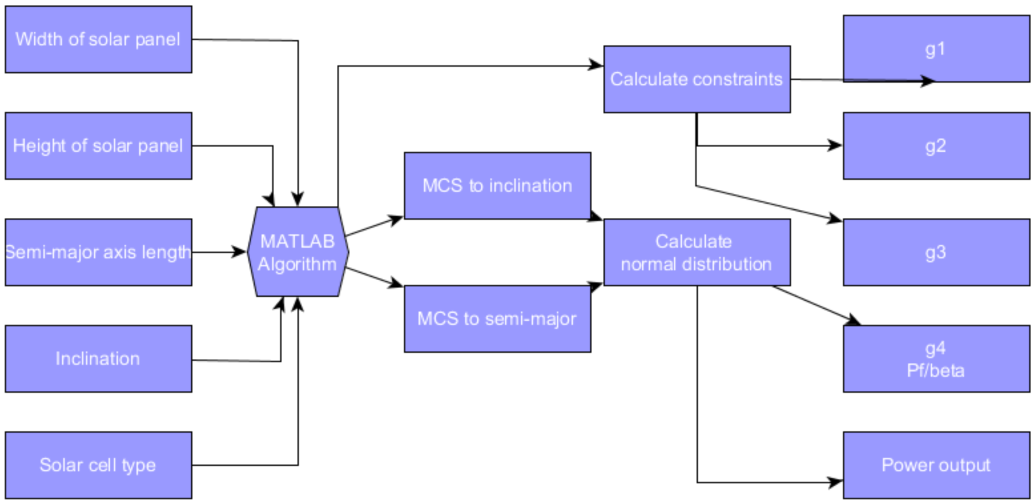

Figure 12: *The Workflow of the design problem defined in MATLAB*

The DOEs are sampled with LHS and 100 DOEs are generated. Both deterministic optimization and RBDO are performed in a platform which has Intel(R) Xeon(TM) CPU 2 E5645@2.40 GHz processor and 12GB of RAM in Win-7 64-bit operating system. In RBDO, 10% of random variation is considered for random parameters, inclination and semi-major axis length, by using 1000 design samples with MCS method.

There are five outputs from the MATLAB algorithm. Four of them are the design constraints and the fifth is the solar array power output calculated. The first three constraints are deterministic constraints the latter term is the probabilistic constraint in RBDO.

Result

The optimum solutions have been obtained after 100 design evaluations. The resulting deterministic and reliable optimum designs are given in Table-9.

Table 9: *The optimum design properties*

General Information	*Deterministic Optimization*	*RBDO*
Number of DOEs	100	100
Number of total designs	1000	1000
Unfeasible design	717	830
Computational time	4 minutes	32 hours

The distribution of feasible solar power output with solar cell type is given in Figure-13. The first and second types of solar cells dominate the feasible solutions.

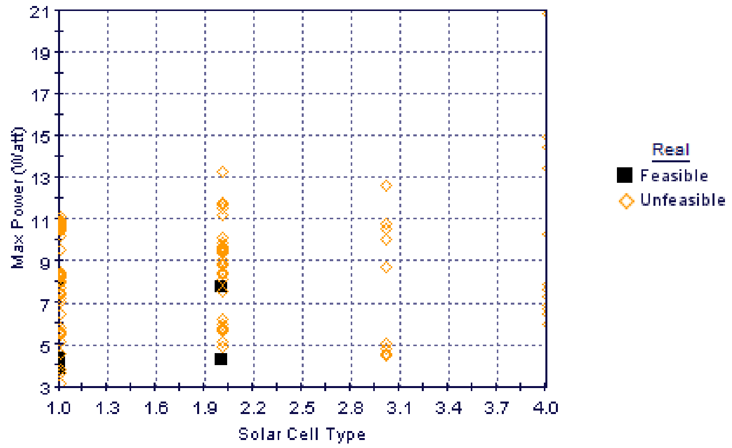

Figure 13: *The distribution of solar cell type*

The width of the solar array size varies does not necessarily contribute to the solar array power output. The distribution can be seen in Figure-14.

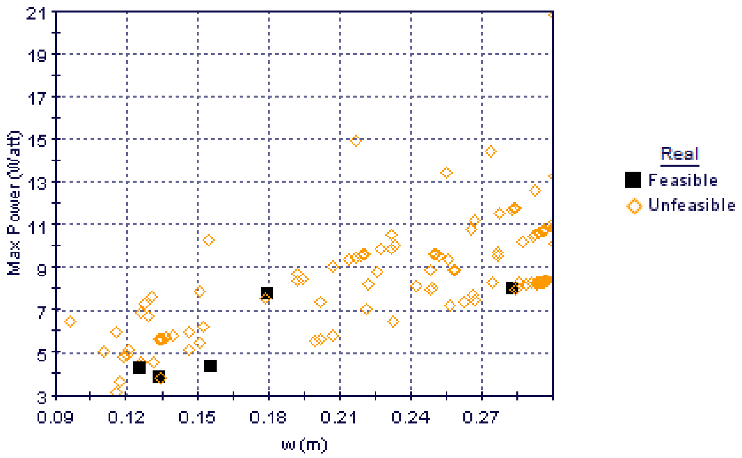

Figure 14: *The distribution of solar array width*

The distribution of the solar array height is seen in Figure-15.

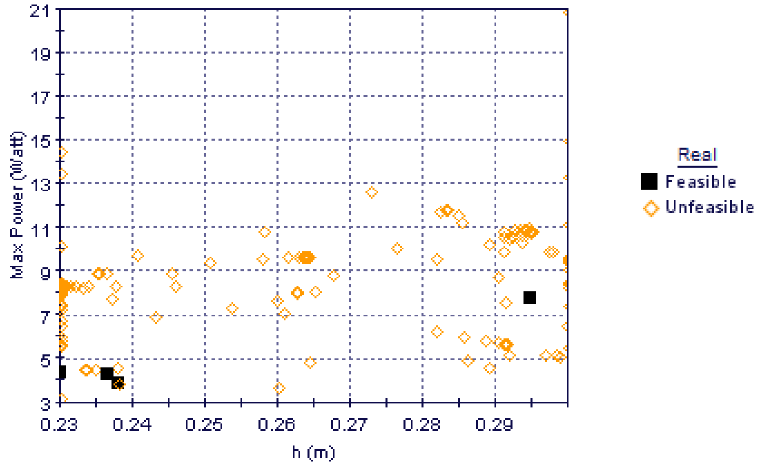

Figure 15: *The distribution of solar array height*

The semi-major axis of the orbit varies greatly in between 200 and 1000 km. However most of the feasible solutions are located in between 700 and 1000 km. The distribution of the semi-major axis length with solar power output are shown in Figure-16.

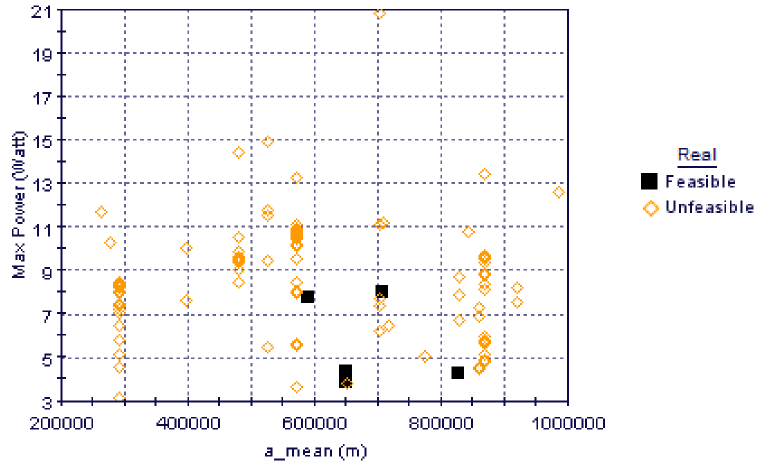

Figure 16: *The distribution of semi-major axis length*

Table-10 shows the initial and optimum design parameters for the GaLactic's solar array. The semi-major axis length has increased which results in a decrease in eclipse time, thus the power output is maximized by means of orbit. The initial and optimum inclination angles are complimentary angles thus the inclination component of the orbit does not improve the initial design however it proves for the necessity of employing uncertainty in optimizing the design. More efficient solar cells are used in the new design. The resultant solar array is lighter and cheaper, wider but shorter. The power output is 0.5 Watts larger than the initial design.

Table 10: *The optimum and initial design values*

Design Parameters	*Initial*	*Deterministic*	*Reliable*
Altitude of the orbit	320km	915.83 km	704.70 km
Inclination of the orbit	96.7471˚	21.478 ˚	57.159 ˚
Power generated by one solar array	7.4Watt	8.5508 Watt	8.0966 Watt
Solar cell type	GaAs	Thin-Si	Si-BSR
Solar cell efficiency	14.90%	16.7%	13.2%
Width of one solar panel	200mm	195.74 mm	282.88 mm

Height of one solar panel	300mm	294.92 mm	230 mm
Mass of one solar panel	0.054 kg	0.019 kg	0.021 kg
Cost of one solar panel	4000 $	3500 $	2728 $
Eclipse time of the orbit	2188.88 sec	2099.58 sec	2025.67 sec
Reliability index	1.6546	0.1282	3.0902
Probability of failure	4.9 %	44.9 %	0.1 %

The study shows that the design in which the reliability is taken into account is a successful solution both by means of low probability of failure and high performance. The deterministic approach provides the highest power output, yet the probability of failure is exceptionally high to assume that the solar arrays will function with full-capacity in space environment. The reliability of the deterministic approach is even lower than the initial design since the solution was found by scanning the solution through the borders of the design space. The probabilistic constraint determines whether the system can operate during severe Space Weather activity or not. The design genereated by the required reliability level in RBDO produces more power with lower mass and cost criteria when compared to the initial design.

CONCLUSIONS

A design optimization problem on the solar array design of a Low Earth Orbit (LEO) mission nano-satellite which will be used to investigate yeast bacteria's potential of survival in space conditions for further manned missions is investigated. The design objective is maximizing the power output which can be obtained from solar panels in the shortest eclipse time while constraining the solar panel cost for manufacturing and launch expenses. In the first step, a deterministic optimization study is achieved to optimize for the width, height of the solar panels, semi-major axis length and orbit inclination of the satellite, and the solar cell type. Next considering the randomness of space environment, uncertainties which may result in design variables and operating conditions are considered and a reliability based design optimization study is accomplished by using semi-major axis length and orbit inclination of the satellite as random parameters. The satellite analysis code and reliability computation code are developed in Matlab and as optimization driver a commercial code is used for it NLPQLP gradient based optimization algorithm. A reliable design with improved power generation with lower satellite mass and cost is generated as the result of RBDO study.

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