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CONVERGENCE ACCELARATION OF LATTICE BOLTZMANN METHOD VIA FINITE-DIFFERENCE BASED IMPLEMENTATION ON UNIFORM GRIDS

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ABSTRACT

In this paper, finite-difference based Lattice Boltzmann Method (LBM) is implemented to perform fluid flow analysis on uniform grids. With the approach followed, the known limitation of the LBM on Courant-Friedrichs-Lewy (CFL) number being equal to one is eased and convergence to steady-state solution is accelerated. 2D lid-driven cavity problem is selected as a benchmark and solved on grids of various resolutions at low Reynolds number conditions.The results are compared against the results from the literature. Very close agreement is obtained regarding the velocity profiles along the vertical and horizontal centerlines of the square cavity. The convergence rates are compared for different CFL numbers. The improvement of the convergence rate is significant and presented in this manuscript.

INTRODUCTION

The LBM is a fairly new numerical method, which is originated from the Lattice-Gas Automata (LGA) method. The LGA method simulates the behavior and interaction of gas particles in a simple way, thus can be considered as a type of Molecular Dynamics method. In this method, the gas is modeled as a cluster of solid spheres moving along a uniform lattice [Chopard, B. and Droz, M. 2005]. Each solid sphere has a discrete set of possible velocities and the collision between separate particles is handled by a set of elastic collision rules. Macroscopic quantities, such as particle density and velocity at each lattice node, can be computed using the microscopic quantities, making it possible to study the macroscopic behavior of a fluid flow. Even though the method is based on a simple molecular dynamics model unlike the continuum assumption that the Navier-Stokes equations are derived, it still shows the same physical behavior. Beyond this, it has several advantages over the conventional Computational Fluid Dynamics (CFD) methods such as low memory requirement and high parallelization capability. However, numerically, the LGA method suffers the statistical noise caused by the averaging procedure to obtain the macroscopic properties from the microscopic properties.

To remedy the statistical noise that the LGA method suffers, the Lattice Boltzmann Method (LBM) was developed. As being a derivate of the LGA method, the LBM basically relies on the same idea. However, instead of handling single particles, the LBM handles particle distributions. This removes the need for averaging to obtain the macroscopic properties from the microscopic properties, so the statistical noise is also removed. Still, it retains the same advantages as LGA method [Nourgaliev, R.R. et al 2003]. This makes the LBM an attractive method, and there is an increasing interest in the LBM in the CFD community. This provides a rapid progress towards development and employment of the LBM. Even, there are various LBM based commercial flow solvers available on the market, like PowerFLOW [Yu, D.Z., Mei, R.W., and Shyy, W. 2002] developed by Exa Corporation and XFlow [Botella, O. and Peyret, R. 1998]

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developed by Next Limit Technology. However, the method has restrictions such as the uniformity of the computational grid to be used and the unit CFL number, which the LBM inherited from the LGA method. These shortcomings are to be the major handicap on widespread use of LBM in engineering problems. Therefore, a lot of research is going on to improve these aspects of LBM.

In this paper, LBM is implemented using the finite difference approach. Following this way, the known limitation of the LBM on CFL number being equal to one is eased and convergence to steady-state solution is accelerated. A generic viscous flow problem, 2D lid-driven cavity, is solved on grids of various resolutions at low Reynolds number conditions and the results are compared against the results from the literature. The convergence rates are also compared for different CFL numbers.

NUMERICAL METHOD

In the LBM, one solves the kinetic equation of particle distribution function. A kinetic model widely used in the literature is Bhatnagar-Gross-Krook (BGK) model [Bhatnagar, P.L., Gross, E.P., and Krook, M. 1954], which has the following form;

$$\frac{\partial f}{\partial t}+\vec{e}\cdot\vec{\nabla}f=-\frac{1}{\tau}(f-f^{0}) \tag{1}$$

where is the particle distribution function, in which is the position vector, is the particle velocity vector, t is the time, is the Maxwell- Boltzmann distribution function, and is the relaxation time. To solve numerically, Equation (1) is discretized in the velocity space using a set of velocities, [He, X.Y. and Luo, L.S. 1997];

$$\frac{\partial f_{\alpha}}{\partial t}+\vec{e}_{\alpha}\cdot\vec{\nabla}f_{\alpha}=-\frac{1}{\tau}(f_{\alpha}-f_{\alpha}^{0}) \tag{2}$$

In the above equation, is the distribution function associated with the th discrete velocity, . For 2D problems, one might use D2Q9 model [He, X.Y. and Luo, L.S. 1997] of which the discrete velocities are shown in Figure 1 below;

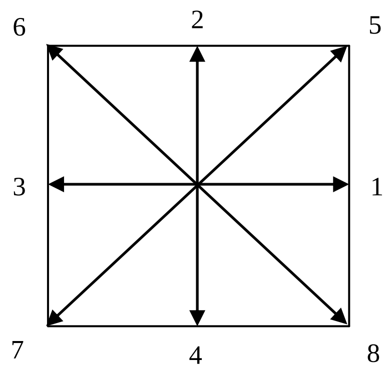


Figure 1 Discrete velocities of D2Q9 model.

The discrete velocities of D2Q9 model are presented by:

$$e_0=0 \qquad \text{for } \alpha=0$$

$$e_{ax,ay}=c\left(\cos\left(\left(\alpha-1\right)\frac{\pi}{2}\right),\sin\left(\left(\alpha-1\right)\frac{\pi}{2}\right)\right) \qquad \text{for } \alpha=1,2,3,4 \tag{3}$$

$$e_{ax,ay}=\sqrt{2}c\left(\cos\left(\left(\alpha-5\right)\frac{\pi}{2}+\frac{\pi}{4}\right),\sin\left(\left(\alpha-5\right)\frac{\pi}{2}+\frac{\pi}{4}\right)\right) \qquad \text{for } \alpha=5,6,7,8$$

where is the lattice velocity. The macroscopic properties can be calculated using the particle distribution functions using:

$$\rho = \sum_{\alpha} f_{\alpha}$$
$$\vec{u} = \frac{1}{\rho}\sum_{\alpha} f_{\alpha}\vec{e}_{\alpha} \qquad (4)$$

The equilibrium function [Kandhai, D. et al 2000] for each direction can be written as follows:

$$f_{\alpha}^{0} = w_{\alpha}\rho\left[1 + \frac{\vec{e}_{\alpha}\cdot\vec{u}}{e_s^2} + \frac{1}{2}\frac{(\vec{e}_{\alpha}\cdot\vec{u})^2}{e_s^4} - \frac{1}{2}\frac{\vec{u}}{e_s^2}\right] \qquad (5)$$

The convection term in Equation (2) might be discretized using any central or upwind schemes. The central scheme is less dissipative compared to upwind schemes, but it is known to produce unphysical wiggles in the solution field. To remedy this, a mix of central and upwind schemes is used for the present study and 2nd order accuracy is obtained in spatial discretization. The time integration of Equation (2) is done using 2nd order Implicit-Explicit Runge Kutta time discretization scheme [Pareschi, L. and Russo, G. 2005]. The implementation of boundary conditions is done using the extrapolation method giving in [Guo, Z.L., Zheng, C.G., and Shi, B.C. 2002].

RESULTS

2-D lid-driven cavity problem for Reynolds number of 100, 400, and 1000 are solved on different resolutions of grids and the results are compared with the data from the literature [Ghia, U., Ghia, K.N., and Shin, C.T. 1982] to show the validity of the current method. The solutions are obtained on uniform Cartesian meshes.

The calculated streamlines for the 128x128 grid and Reynolds numbers of 100, 400, and 1000 are shown in Figure 2, Figure 3 and Figure 4 respectively.

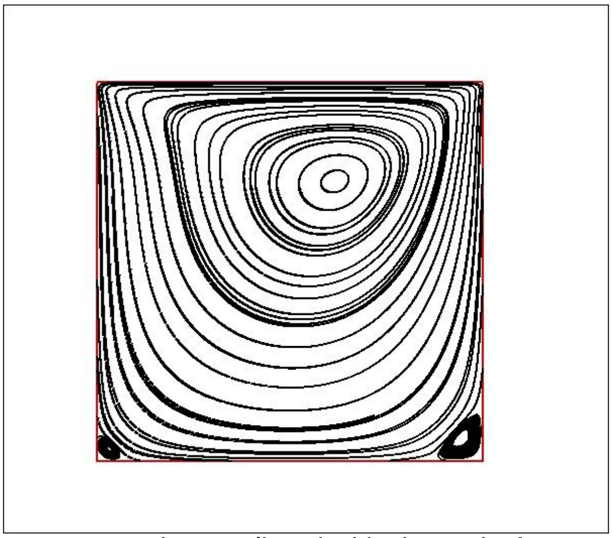


Figure 2 Computed streamlines inside the cavity for *Re* = 100.

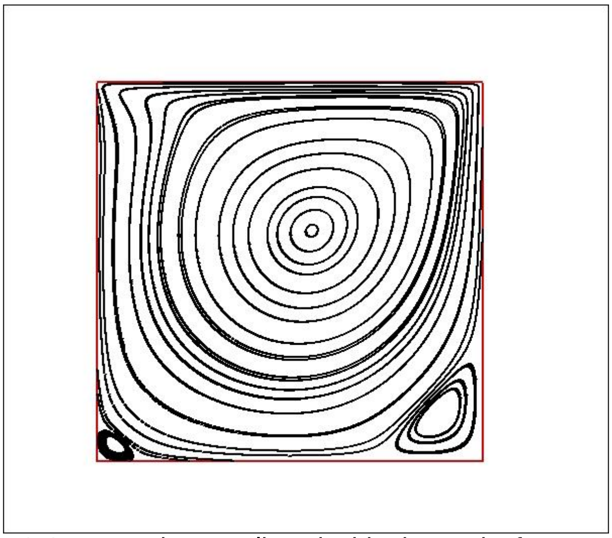


Figure 3 Computed streamlines inside the cavity for *Re* = 400.

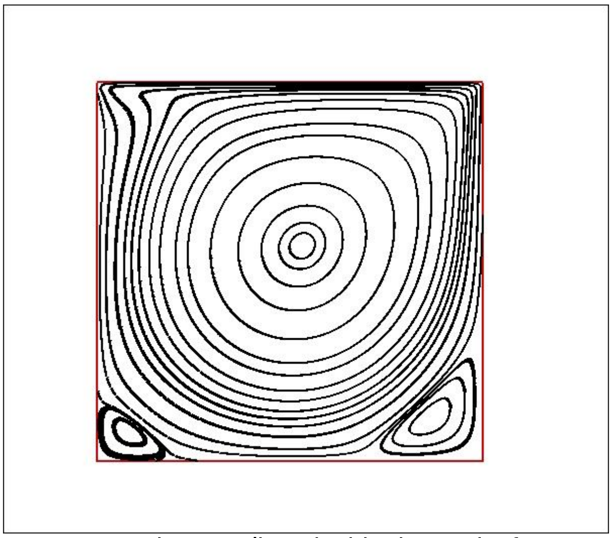
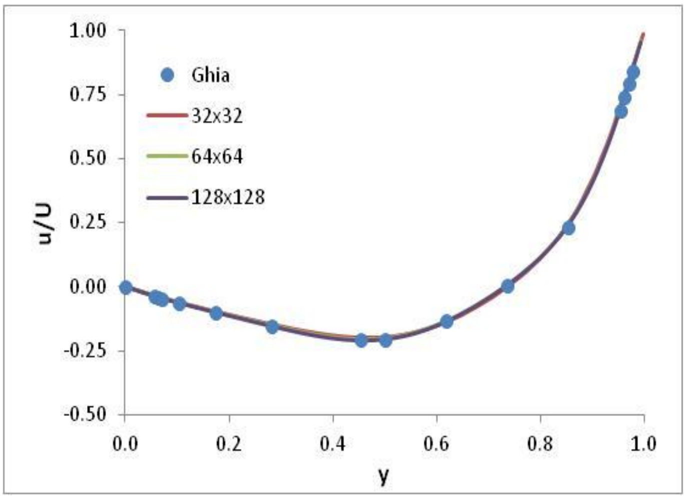


Figure 4 Computed streamlines inside the cavity for *Re*= 1000.

Comparisons of the velocity profiles along the vertical and horizontal centerlines of the square cavity for different grid resolutions and Reynolds numbers of 100, 400, and 1000 are shown in Figure 5, Figure 6 and Figure 7, respectively.



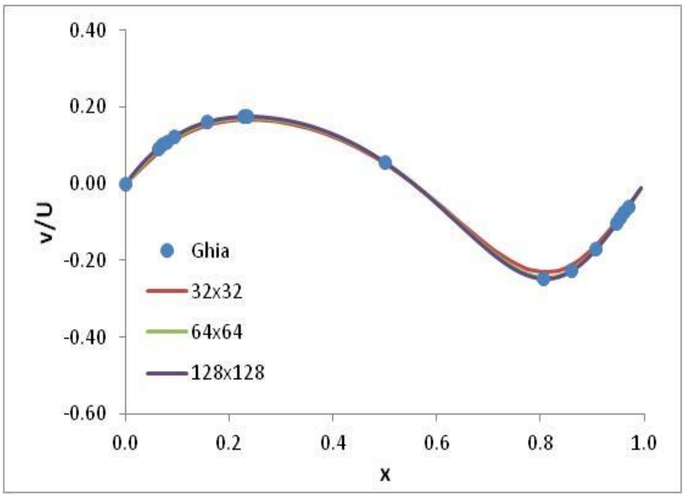


Figure 5 Comparison of velocity components in x and y directions, respectively, with the results from [Ghia, U., Ghia, K.N., and Shin, C.T. 1982], for *Re* = 100.

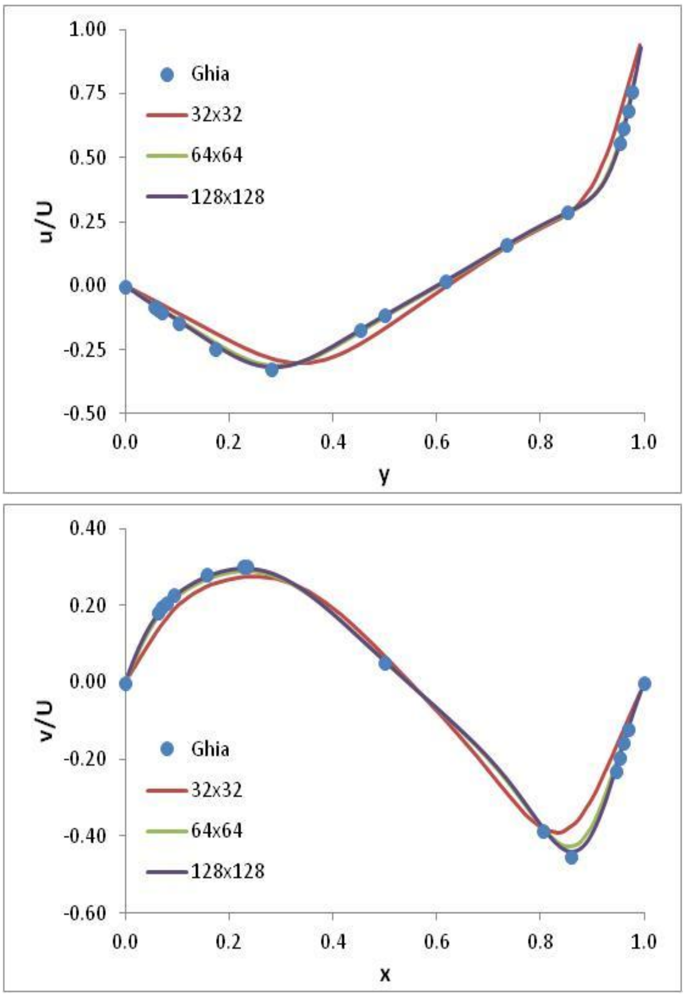
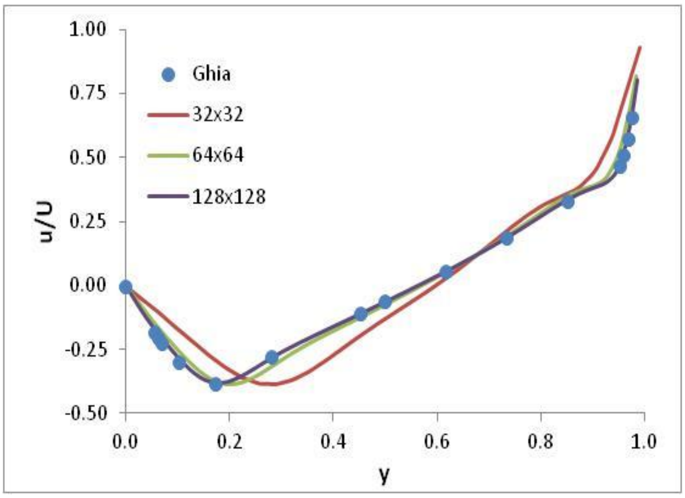


Figure 6 Comparison of velocity components in x and y directions, respectively, with the results from [Ghia, U., Ghia, K.N., and Shin, C.T. 1982], for *Re* = 400.



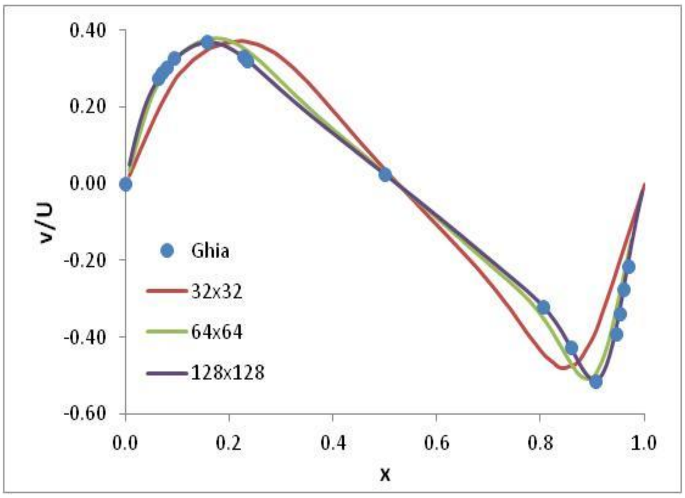


Figure 7 Comparison of velocity components in x and y directions, respectively, with the results from [Ghia, U., Ghia, K.N., and Shin, C.T. 1982], for Re=1000.

The convergence history of different CFL numbers for 64x64 grid is given in Figure 8

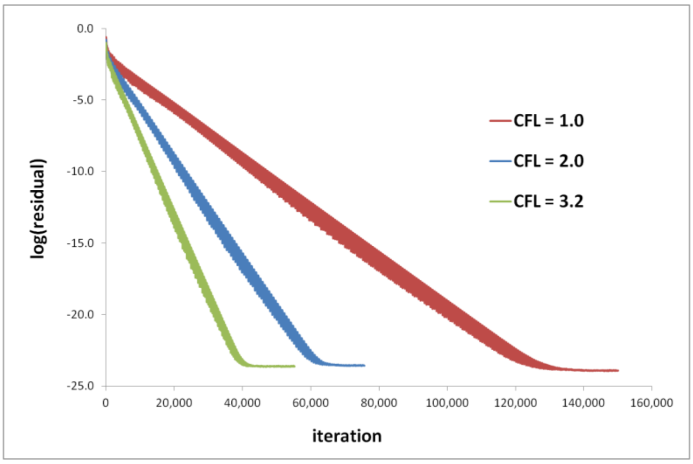


Figure 8 Comparison of convergence history for different CFL numbers at Re=400

References

Bhatnagar, P. L., Gross, E. P., and Krook, M.(1954)A Model for Collision Processes in Gases .1. Small Amplitude Processes in Charged and Neutral One-Component Systems,94p:511-525, 1954

Botella, O. and Peyret, R.(May 1998)Benchmark spectral results on the lid-driven cavity flow,27p:421-433, May 1998

Chopard, B. and Droz, M.(2005)Cellular Automata Modeling of Physical Systems,Cambridge University Press, 2005

Ghia, U., Ghia, K. N., and Shin, C. T.(1982)High-Re Solutions for Incompressible-Flow Using the Navier Stokes Equations and A Multigrid Method,48p:387-411, 1982

Guo, Z. L., Zheng, C. G., and Shi, B. C.(Jun 2002)An extrapolation method for boundary conditions in lattice Boltzmann method,14p:2007-2010, Jun 2002

He, X. Y. and Luo, L. S.(Dec 1997)Theory of the lattice Boltzmann method: From the Boltzmann equation to the lattice Boltzmann equation,56p:6811-6817, Dec 1997

Kandhai, D., Soll, W., Chen, S., Hoekstra, A., and Sloot, P.(Jul 2000)Finite-Difference Lattice-BGK methods on nested grids,129p:100-109, Jul 2000

Nourgaliev, R. R., Dinh, T. N., Theofanous, T. G., and Joseph, D.(Jan 2003)The lattice Boltzmann equation method: theoretical interpretation, numerics and implications,29p:117-169, Jan 2003

Pareschi, L. and Russo, G.(Oct 2005)Implicit-explicit Runge-Kutta schemes and applications to hyperbolic systems with relaxation,25p:129-155, Oct 2005

Yu, D. Z., Mei, R. W., and Shyy, W.(May 2002)A multi-block lattice Boltzmann method for viscous fluid flows,39p:99-120, May 2002