

## LAUNCH VEHICLE TRAJECTORY ROBUSTNESS MODIFICATION

Ali Asghar Bataleblu<sup>1</sup>, Masoud Ebrahimi<sup>2</sup> and Jafar Roshanian<sup>3</sup>  
K.N. Toosi University of Technology  
Tehran, Iran

### ABSTRACT

Optimal trajectory generation is a major part of launch vehicle design. A robust trajectory can improve launch vehicle reliability, safety and operational cost. In this paper, robustness of a trajectory of a two stage expendable Launch Vehicle is modified. The Three-degree-of-freedom trajectory simulation program with appropriate atmosphere and earth models are used. Some important uncertainties such as uncertainty in launch vehicle dry mass, engine's thrust force, aerodynamic force coefficients and engine's burn time is considered. Assuming normal distribution for parameters with uncertainty, Monte Carlo simulation method is used to calculate probability density function of output parameters for predetermined optimal pitch angle program. In this study, mission constraints (final orbit components) and constraints that appear during the flight (such as separation height, fall down position of the stages, Angle Of Attack (AOA) when Mach number is close to 1 and maximum amount of dynamic pressure multiply in AOA) are considered as output parameters. Afterwards, pitch angle program is modified to reduce the sensitivity of output parameters. Again, Monte Carlo simulation method is used to calculate probability density function of output parameters for modified pitch angle program. Results show that some modification in pitch angle program can improve the trajectory robustness.

### INTRODUCTION

One of the most important disciplines in launch vehicle design is performance or trajectory. Some of guidance methods are based on nominal trajectory tracking. Therefore, finding a nominal trajectory is very important. Simulation and optimization of trajectories within sixty years has a long-standing study on exterior ballistic. Almost all the previous researches have been done on the deterministic trajectory optimization.

In real-world applications, uncertainties exist in every stage of the design process which has significant impact on the design solution. A little change in launch vehicle (such as mass, thrust force, and geometry) or environmental parameters may cause big change in objectives and constraints (reduce performance or violate constraints). Besides, in this condition methodic errors appear in guidance algorithm and more energy is required to return to the nominal path. As a result, trajectory design in presence of uncertainties is very important [4,7].

Two major classes of uncertainty-based design problems, robust design problems and Reliability-based design problems have been proposed. The robust design method is essential for improving the engineering productivity. The basic definition of robust design is described as a product or process which is insensitive to the effects of sources of variability; even through the sources themselves have not been eliminated. In other words a robust design is 'less sensitive' to variation in uncontrollable design parameters than the traditional design point. Robust design has found many successful applications in engineering and is successively being expanded to design phases [3,5].

In this paper, only parametric uncertainties are studied. The parametric uncertainties can be clearly described by the way of interval bounds, membership functions, or probability density functions.

<sup>1</sup> Msc Student, Email: ali.batalebloo@gmail.com

<sup>2</sup> PhD Student, Email: ebrahimi\_k\_m@yahoo.com

<sup>3</sup> Associate Prof, Email: roshanian@kntu.ac.ir

The most well developed methods to uncertainty analysis are based on parametric uncertainties indicated in terms of probability density functions (PDFs) [6]. In this paper probability density functions are used and the trajectory robustness is modified by small variation in predetermined optimal pitch angle program. This procedure is typically implemented by using some type of Monte Carlo method. The simplest approach (the fundamental Monte Carlo method) is used in this study.

In the following the flight simulation program and Monte Carlo method is discussed briefly. The manuscript continues with the computational results for predetermined optimal pitch angle and the modified one. Finally, conclusions are given.

### FLIGHT SIMULATION

To analyze the flight-path, a three degree-of-freedom trajectory (3DOF) model is developed and simulated in C++. Within the present study, the vehicle is treated as a point-mass and flight in 3D over the spherical rotating earth was assumed. The aerodynamic code generates tabulated aerodynamic coefficients. These tables present coefficient values relative to Mach number, Reynolds number and angle of attack. The vehicle is modeled as a two stage (each stage has its own aerodynamic propulsion inputs). Environment routines use Standard atmosphere model, this model is the most accurate and perfect model that can be used for conceptual design.

The equations of motion (equations 1) are numerically integrated (4th order runge-kutta) from initial state conditions to the injection condition.

$$\begin{cases} \dot{u} = \left(\frac{1}{m}\right)(T + q s c_x) + g_x - (\omega_y w - \omega_z v) \\ \dot{v} = \left(\frac{1}{m}\right)(q s c_y) + g_y - (\omega_z u - \omega_x w) \\ \dot{w} = \left(\frac{1}{m}\right)(q s c_z) + g_z - (\omega_x v - \omega_y u) \end{cases} \quad (1)$$

Where  $u$ ,  $v$  and  $w$  are components of velocity,  $m$  is LV mass,  $T$  is thrust,  $q$  is dynamic pressure,  $s$  is reference area,  $g_x$ ,  $g_y$  and  $g_z$  are gravity components and  $\omega_x$ ,  $\omega_y$  and  $\omega_z$  are components of angular velocity.

### TRAJECTORY CONSTRAINTS

There are different constraints in a trajectory design problem (mission constraints and those that appear in duration of flight trajectory) which some of them are mentioned below. The first constraint is to keep the angle of attack near zero in transonic flight, because the aerodynamic forces change dramatically and the resulting perturbations affect the vehicle undesirably.

It is known that the aerodynamic forces that act on the vehicle structure are proportional to the product of dynamic pressure and angle of attack; hence it should be considered in trajectory design.

Reliable separation is an essential part of multi-stages flight vehicles. Angular speed of the vehicle during the separation phase as well as the product of dynamic pressure and angle of attack will endanger the separation process. As a result they should be confined.

The next problem is the location of the impact of the separated parts, since they cannot fall anywhere; so in the trajectory generation this constraint should be considered.

Finally, mission constraints are considered, it means that the position and the velocity of the payload at the injection point should have determined values. In other words, orbital parameters have to equal to desired value.

### LAUNCH VEHICLE PARAMETRIC UNCERTAINTY

Some degree of uncertainty in characterizing any real engineering system is inevitable. In many cases, the objective function and constraints may be highly sensitive to these uncertainties which will lead to constraints violation or performance reduction.

To use uncertainty-based design methods, the various uncertainties associated with the design problem must be characterized and managed. Two complementary categories of uncertainties are parameter uncertainties and model form uncertainties. In this paper only parameter uncertainties are used.

One common reason of uncertainty occurs as a result of uncontrollable probabilistic deviations in the values of design parameters. There are a lot of parameter uncertainties in launch vehicle trajectory generation that some of them are mentioned. Because of uncertainty in geometry, aero coefficients

calculation, and atmospheric condition, aero forces are uncertain. Uncertainty in structural material properties (such as density) leads to uncertainty in dry mass. Finally, Uncertainty in propellant material properties (such as thermo-chemical characteristic) leads to uncertainty in thrust force and engine burn time.

So four major uncertainties which have direct effect on mass and energetic characteristics of launch vehicle (include uncertainties in launch vehicle dry mass, engines thrust force, aerodynamic force coefficients and engines burn time) with typical value for their standard deviation are chosen and used in the trajectory robustness analysis.

### MONTE CARLO SIMULATION

Simulation methods like Monte Carlo simulation (MCS) have been intensively used in uncertainty analysis since their introduction in the 1940s. The underlying principle is frequentist: probability distribution of the output of a process induced by the probability distribution of stochastic inputs is obtained by performing  $m$  repetitions of the process. Each time one sampling point of the input space is drawn according to the known (or assumed) distribution of the inputs, each individual analysis is considered one simulation. These analyses are repeated until output distributions of suitable accuracy are generated. This accuracy is based on the desired probability of constraint satisfaction. One major disadvantage of MCS with respect to other uncertainty analysis methods is that the number of samples required for a sufficiently accurate estimate of the mean and the variance can be very large, thus requiring a large computational effort. Albeit, Monte Carlo analysis is computationally expensive but it is simple to implement and the most accurate probabilistic methods until now. [1, 2].

### RESULTS

Monte Carlo simulation is applied to a Two-Stage launch vehicle with predetermined characteristic and optimal pitch angle program. Figure 1 indicates deterministic and Monte Carlo simulation for ascent flight and first stage fall down trajectories.

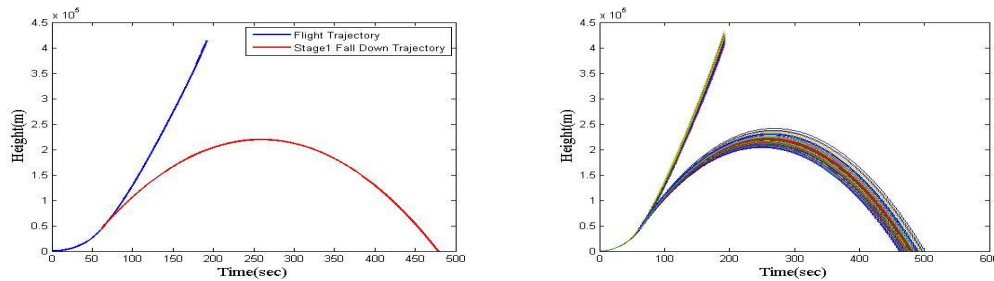


Figure 1: Ascent flight and first stage fall down trajectory(left: deterministic simulation-right: Monte Carlo simulation)

Figure 2 illustrate nominal destination orbit and destination orbits in presence of uncertainties.

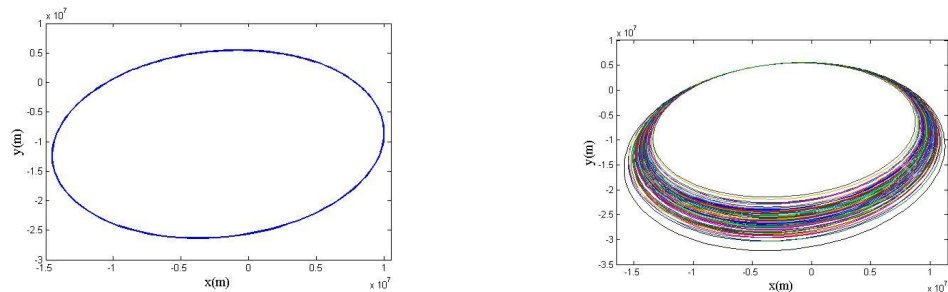


Figure 2: Destination orbit (left: deterministic simulation-right: Monte Carlo simulation)

As seen in figures 1-2 and table 1, there are large variations in some trajectory parameters because of uncertainty in launch vehicle parameters. These variations may lead to violate desirable values. So, the pitch angle program is modified to reduce these variations. Figure 3 shows the predetermined and modified pitch angle program.

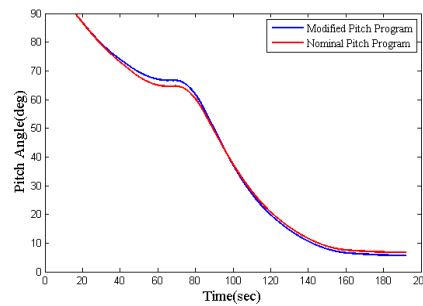


Figure 3: Nominal and Modified pitch angle program

Table 1 demonstrates mean and variance of some important parameters.

Table 1: Parameters Distributions of Monte Carlo Simulation

Parameters	Nominal	Predetermined pitch angle program		Modified pitch angle program	
		Mean	Sigma	Mean	Sigma
Final Height	413570 m	412445	5572.5	412430	2544.23
Velocity at injection point	9624.1 m/s	9610.61	78.0964	9532.7	37.6913
Seperation Height of First Stage	40795.82 m	40612.2	4773.492	40805.6	753.323
First Stage Fall Down Position	X = 6.36e6 m	6.36071e6	752.458	6.36187e6	693.08
	Y = 47192 m	471011	10138	455020	5669.93
	Z = -3.7e-10 m	-3.70256e-10	5.93022e-13	-3.71418e-010	5.58942e-13
$(q.\alpha)_{\max}$	-4276.96 pa.rad	-4236.27	283.264	-4028.21	167.623
$(\alpha)_{\max}^{Mach=1}$	-2.31944 deg	-2.32568	0.0509	-2.3011	0.0207
Eccentricity	0.662385	0.659059	0.019421	0.637235	0.017043
Perigee Radius	5.4362e6 m	5.4318e6	33849.2	5.45433e6	25040.9
Apogee Radius	2.67673e7 m	2.65443e7	1.97399e6	2.47075e7	1.72704e6

### CONCLUSION

The design of launch vehicles trajectory is a particularly challenging design problem. Presence of uncertainties in different parts of design can be lead to design failure, hence, pay attention to that is very important and design must be robust toward uncertainties. The results reveal that although modified trajectory is more robust than nominal trajectory but it isn't robust enough. Due to significant importance in reach to mission goals, it should be improved. Robust Optimization method is then used to deal with this situation which is under work.

### References

- [1] M. Ebrahimi, J. Roshanian, *Reliability Modification in Multidisciplinary Design Optimization of a Solid Propellant Launch Vehicle*, 61st International Astronautical Congress, 2010.
- [2] John Marc Zentner, *A Design Space Exploration Process for Large Scale, Multi-objective Computer Simulations*, Georgia Institute of Technology, August 2006.
- [3] Saqlain Akhtar and He Linshu, *An Efficient Evolutionary Multi-Objective Approach for Robust Design of Multi-Stage Space Launch Vehicle*, 11th AIAA/ISSMO Multidisciplinary Analysis and Optimization Conference, September 2006.
- [4] Andy J.. Keane, Prasanth B. Nair, *Computational Approaches for Aerospace Design The Pursuit of Excellence*, John Wiley & Sons Ltd, 2005.

- [5] C. Zang, M.I. Friswell and J.E. Mottershead, *A review of robust optimal design and its application in dynamics*, Computers and Structures, p 315–326, 2005.
- [6] Thomas A. Zang, Michael J. Hensch, Mark W. Hilburger, Sean P. Kenny, James M. Luckring, Peiman Maghami, Sharon L. Padula, and W. Jefferson Stroud, *Needs And Opportunities For Uncertainty-Based Multidisciplinary Design Methods For Aerospace Vehicles*, Langley Research Center, July 2002.
- [7] David Jeremy McCormick, *Distributed Uncertainty Analysis Techniques for Conceptual Launch Vehicle Design*, Georgia Institute of Technology, July 2001.