6. ANKARA INTERNATIONAL AEROSPACE CONFERENCE
14-16 September 2011 - METU, Ankara TURKEY

AIAC-2011-030

SERVICE LIFE PREDICTION OF SOLID ROCKET PROPELLANTS CONSIDERING RANDOM THERMAL ENVIRONMENTS

Okan Yılmaz[1] and Bayındır Kuran[2]
Roketsan A. S.
Ankara, Turkey

Gökhan Özgen[3]
Middle East Technical University
Ankara, Turkey

ABSTRACT

Solid propellant rocket motor is the primary propulsion technology used for tactical missiles. Its widespread usage gives rise to diversity of the environments which it is handled and stored. These uncontrolled thermal environments induce random stresses and strains in the propellant of a rocket motor which provoke mechanical damage along with chemical degradation. In this study, a service life prediction technique which is based on response surface method is used and explained. A time dependent random function is used for the temperature model. Solid rocket propellant is modeled using linear viscoelastic material model. Mechanical properties of the propellant corresponding different temperatures and loading rates are found from the mechanical tests performed on test samples. A three dimensional finite element model is used to predict stresses and strains induced on the propellant. A cumulative damage model is used since during the storage and the deployment stresses in the propellant accumulate with time. Aging behavior of the propellant is taken into consideration as well and Layton model is used for this purpose. Response surface method is used to construct surrogate models in terms of parameters associated with the material properties and the propellant temperature. Latin Hypercube Sampling (LHS) is used for the generation of multivariate samples. Limit state functions are used for failure modes of the propellant. The instantaneous reliability indexes and the probability of failure for thermal loading are predicted by means of the First Order Second Moment (FOSM) Method. The progressive reliability of the propellant is illustrated on a rocket motor.

NOMENCLATURE

T_M	=	Mean temperature on the outer surface of the motor case
T_Y	=	Yearly temperature amplitude
T_D	=	Daily temperature amplitude
t	=	Time (hours)
t_0	=	Yearly temperature phase
t_1	=	Daily temperature phase
C_1	=	First constant of WLF shift function
C_2	=	Second constant of WLF shift function
T_0	=	Reference temperature
D	=	Cumulative damage factor
S_0	=	Mechanical property of the property after curing
k	=	Aging rate of a mechanical property

[1] Engineer, System Design Department, Email: okyilmaz@roketsan.com.tr
[2] Manager, System Design Department, Email: bkuran@roketsan.com.tr
[3] Asst. Prof., Mechanical Engineering Department, Email: gozgen@metu.edu.tr

A	=	Arrhenius constant
E_a	=	Activation energy
R	=	Ideal gas constant
G_∞	=	Equilibrium shear modulus of the propellant
α	=	Thermal expansion coefficient of the propellant
$g(x)$	=	Approximated limit function
A_{strain}	=	Allowable strain of the propellant
P_{strain}	=	Maximum principal strain on the inner bore of the propellant
A_{stress}	=	Allowable stress of the propellant
P_{stress}	=	Maximum principal stress on the inner bore of the propellant
$A_{intstress}$	=	Allowable stress for the propellant-insulation interface
$P_{intstress}$	=	Maximum principal stress for the propellant-insulation interface
$f(t)$	=	Probability density function
$F(t)$	=	Cumulative distribution function
λ	=	Time dependent hazard rate
β	=	Safety index
R	=	Time dependent reliability
φ	=	Standardized normal distribution for cumulative distribution function
P_f	=	Probability of failure for any failure mode
μ_g	=	Mean value of g function
$Var(g)$	=	Variance of g function

INTRODUCTION

Aim of this study is to assess the service life of the solid rocket propellant which is exposed to random thermal loads during storage and deployment. Since material properties are temperature and loading-rate dependent and have variabilities and uncertainities like environmental loads, probabilistic methods are to be used in predicting the service life.

The induced stresses and strains as well as material capability parameters have inherent uncertainties. In addition, material capability parameters such as propellant modulus, allowable stress and allowable strain capability may change with increasing age. In probabilistic service life prediction methods, uncertainties along with the degradation mechanisms are taken into consideration. Distributions for the material capability and induced stresses/strains are utilized to calculate the so called probability of failure [4].

Cumulative damage and aging are two important degradation mechanisms of solid propellants. To simulate aging behavior, Layton model [3,15] is used. And a linear cumulative damage model is used as stresses and strains accumulate in the propellant with time.

To construct surrogate mathematical models for the stress and strain states of the propellant, Response Surface Method (RSM) is used. Then, limit state functions are developed and reliability indexes and probability of failure values are calculated by means of First Order Second Moment (FOSM) method.

In literature, several studies for assessing the service life of the propellants have been published over the years [2,5-7,9-14,16]. In this study, preliminary service life analysis for solid propellants considering random thermal loads is discussed.

METHOD

Determination of Storage Conditions

The loads induced during the storage phase of the life cycle are represented by random variables. Time dependent thermal environment which the solid rocket motor is subjected to is defined in terms of long term mean temperature, yearly and daily temperature amplitudes. In literature thermal loads in the storage conditions in which large percentage of rocket life is spent, are considered. Change in temperature of the motor case outer surface can be taken as a harmonic function of time [4].

$$T(t) = T_M + T_Y \sin\left\{\frac{2\pi}{8760}(t - t_0)\right\} + T_D \sin\left\{\frac{2\pi}{24}(t - t_1)\right\} \quad (1)$$

If the change in temperature of the store is known or stored in a database, the parameters given above will be determined easily. However, hourly changes in temperature may not be kept during long years so that such uncertainties should be considered in calculation of the service life. In this study, wind and solar radiation effects are not considered.

Propellant Behavior

The basic behavior of the solid rocket propellant is viscoelastic. This kind of material behavior depends on time, temperature and strain rate. Two characteristic phenomena revealing the viscoelastic material response are creep and relaxation. Creep represents the deformation behavior of the material under constant load; relaxation represents the load change under constant deformation. In order to characterize viscoelastic material behavior, creep or relaxation modulus is utilized.

Time and temperature dependent behavior of viscoelastic materials can be defined by master curves by using linear viscoelastic material theory. The master curves are obtained by shifting the parameters or curves found at different temperature tests. To shift the parameters, a reference temperature is selected. This temperature is generally chosen as glass transition temperature of the propellant or room temperature. WLF (Williams, Landel and Ferry) shift function is used for shifting and this function can be written as:

$$\log(a_T) = \frac{C_1(T - T_{ref})}{C_2 + (T - T_{ref})} \quad (2)$$

When all curves are shifted to the reference curve, a single curve called as "master curve" is obtained. The horizontal axis of the master curve is expressed by reduced parameters. Reduced strain rate parameter, ($R.a_T$) or reduced time parameter (t^*/a_T) can be used as reduced parameter. R is strain rate, a_T is shift factor and t* is time to reach maximum stress or rupture.

The shifting of the curves along time axis implies a superposition between time and temperature. By shifting to the right or left, one assumes that low temperature has the similar effect with shorter time and high temperature has the similar effect as longer time. This phenomenon is frequently called as time-temperature equivalence and the materials for which this time-temperature superposition is valid are called as thermorheologically simple materials. Since, temperature in the store has variation; temperature induced in the propellant grain has also variation. In addition, the master curve of the propellant is illustrated in Figure 1.

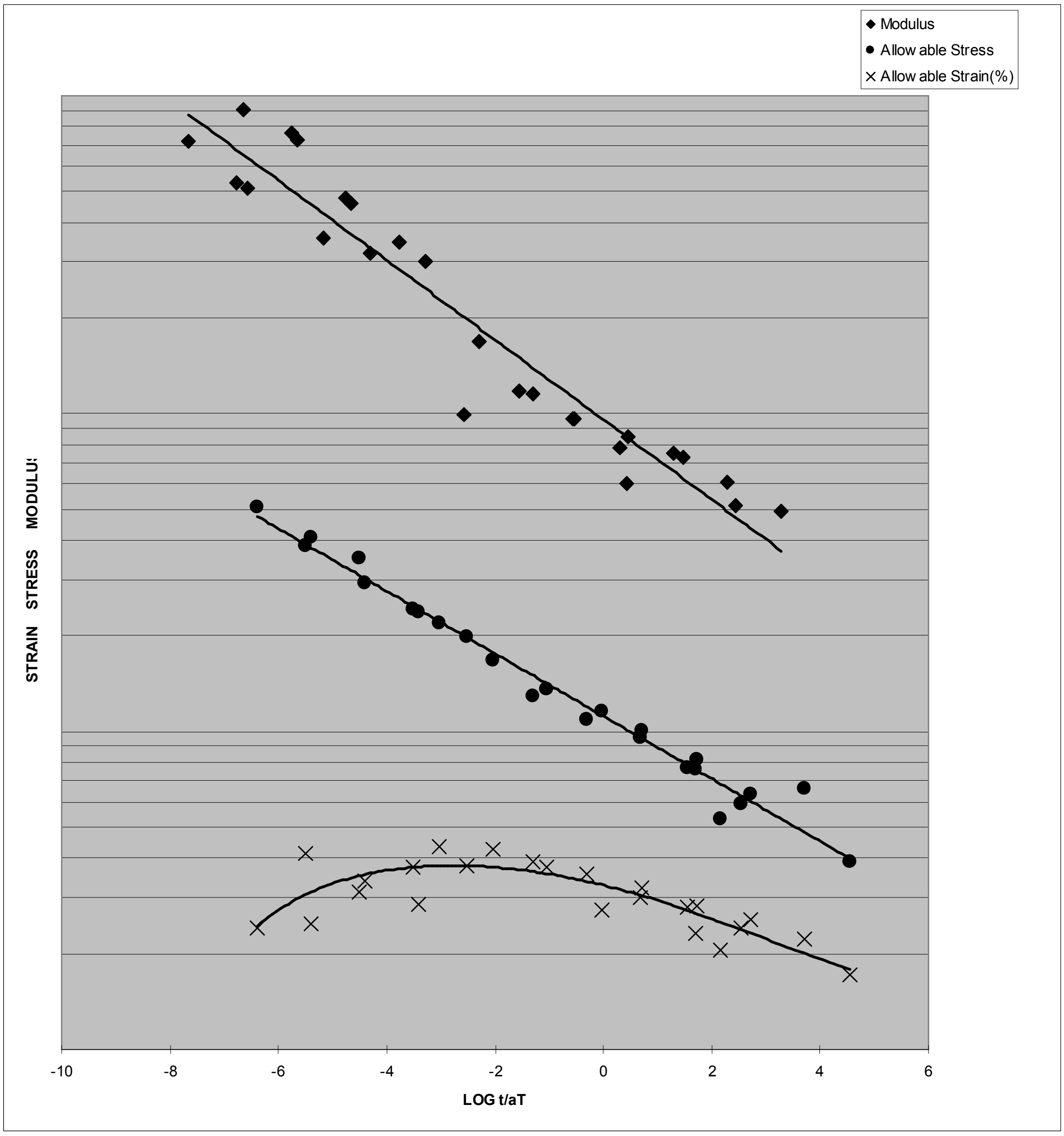

Figure 1: Master curve for the propellant obtained from mechanical tests (Allowable strain-allowable stress and modulus vs. reduced time)

Reliability prediction method is based on the assumption that several variables have uncertainties. In this study, equilibrium shear modulus and coefficient of thermal expansion of the propellant are the mechanical properties that have uncertainties. Considering the storage conditions, long term mean for storage temperature, yearly amplitude of storage temperature and daily amplitude of storage temperature have uncertainty as well. Moreover, uncertainties for allowable strain and allowable stress of the propellant and allowable stress for the propellant-case interface taken into consideration along with the degradation mechanisms which are cumulative damage and aging factors of the propellant.

Cumulative Damage Function

The loads on the propellant during the storage and the deployment cause the accumulation of the stresses on the propellant. The linear cumulative damage model can be utilized for the determination of the service life of rocket motors [9,12]. Parameters of this damage model can be calculated from log(failure stress) versus log(failure time) graph obtained from constant stress or constant strain rate tests [1]. In constant stress tests, the propellant specimens are subjected to a constant stress loading at different temperatures and failure times for different stress levels and temperatures are stored. An example of the log(failure stress) versus log(failure time) graph can be seen in Figure 2.

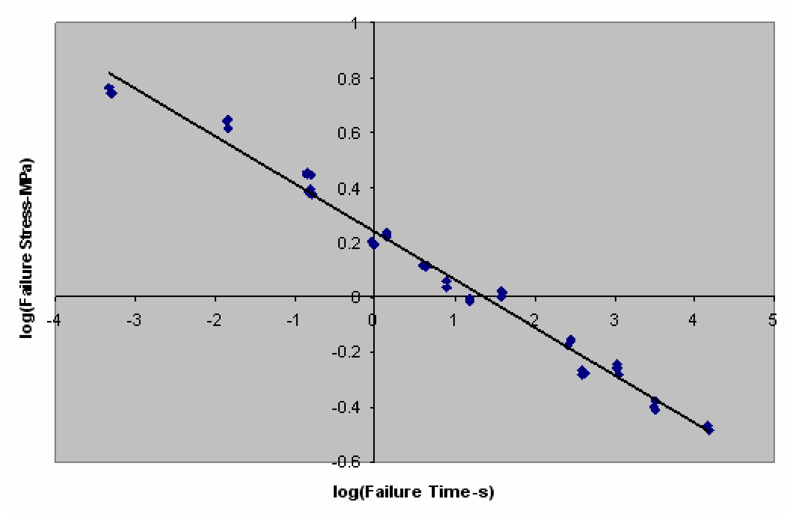

Figure 2: Sample graph obtained from tests to determine damage factors

In this study, cumulative damage function is used in the reduction of the propellant strength with time for thermal loading[11]. If the propellant strength at t=0 is $S(t_0)$, the strength of propellant S at time t is found as:

$$S(t) = S(t_0)(1 - D) \tag{3}$$

where D is the cumulative damage factor, which is found using Miner's rule.

Aging Model

The solid rocket propellant shows a significant aging behavior with time. For this reason, propellant specimens taken from accelerated aged at high temperature or naturally aged rocket motors are used in determining aging model of the propellant. The service life of solid rocket propellant is usually found by measuring the changes in mechanical properties of the propellant. For this aim, Layton model [3,15] is used to estimate the service life of solid rocket propellants. Any mechanical property at time t can be expressed as:

$$S(t) = S_0 + k\log(t) \tag{4}$$

Change in mechanical properties at different temperatures is expressed as the change in k parameter and k is given in Arrhenius equation as

$$k = A\exp\left(\frac{-E_a}{RT}\right) \tag{5}$$

Aging tests can be performed by using test specimens stored in a store (naturally aged). In this case, the aging parameters can be calculated from the specimens taken from the youngest, medium aged and the oldest motors. For every property such as relaxation modulus, rupture stress and rupture strain, different aging parameters are obtained. After the changing model of k parameter according to temperature is found, change in any mechanical property with time can be predicted.

Surrogate Model Construction Using Response Surface Method

Response Surface Method (RSM) is a design of experiments technique in order to determine the behavior of a complex system [8]. Relations between inputs and high order effects can be found with this method. System outputs are calculated from predetermined input parameters to find response surface of the system. For this aim, computer simulations and experiments are performed and the surfaces are fitted to between inputs and outputs of system. Latin Hypercube Sampling (LHS) is used for the generation of multivariate samples.

If the output of system is g and the factors affecting output are X_i (input parameters), the output can be defined as a second order mathematical model shown below:

$$g = a_0 + \sum_{i=1}^{k} a_i X_i + \sum_{\substack{i=1,\, j=2 \\ i<j}}^{k} a_{ij} X_i X_j + \sum_{i=1}^{k} a_{ii} X_i^2 \qquad (6)$$

Using this technique, surrogate models are established for mean, yearly and daily amplitudes for stress and strain in the inner bore and stress at the propellant-insulation interface.

Variable Parameters

It is discussed previously in the text that reliability prediction method is based on the assumption that several parameters have uncertainities. Table 1 shows which parameters have uncertainity. Furthermore, maximum and minimum values of these parameters are illustrated in Table 2.

Table 1: Fixed and variable parameters

Parameter	**Uncertainity (Yes/No)**
Geometric Parameters of the Grain and the Motor	No
Mechanical/Thermal Parameters of the Motor Case	No
Mechanical/Thermal Parameters of the Insulation	No
Propellant Equilibrium Shear Modulus (G_∞)	**Yes**
Propellant Thermal Expansion Coefficient (α)	**Yes**
Propellant Thermal Conductivity	No
Propellant Heat Capacitance	No
Propellant Density	No
Propellant Poisson Ratio	No
Yearly Amplitude of Storage Temperature (T_Y)	**Yes**
Daily Amplitude of Storage Temperature (T_D)	**Yes**
Yearly/Daily Temperature Phase	No
Long Term Mean for Storage Temperature (T_M)	**Yes**
Allowable Strain of the Propellant	**Yes**
Allowable Stress of the Propellant	**Yes**
Allowable Stress for the Propellant-Case Interface	**Yes**
Cumulative Damage	No
Aging Factors	No

Table 2: Maximum and minimum values of variable parameters

Parameter	Minimum Value	Maximum Value
G_∞ (MPa)	0.189	0.630
α (μm/mm/°K)	75	115
T_M (°C)	9	19
T_Y (°C)	10	35
T_D (°C)	2	5

The Limit State Functions

Three limit state functions can be given for the stress induced in propellant grain, the strain induced in the propellant grain and the stress at the propellant-insulation interface.

$$g_1 = A_{stress} - P_{stress} \tag{7}$$

$$g_2 = A_{strain} - P_{strain} \tag{8}$$

$$g_3 = A_{\text{int}\,stress} - P_{\text{int}\,stress} \tag{9}$$

They comprise amplitudes of stresses and strains which are calculated through Fast Fourier Transform. The method described in this study handles the limit state functions separately depending on the type of loading. For thermal loading, stresses and strains predicted from the finite element analysis are harmonic functions of time. Hence, response surface models are constructed using response surface method (RSM). Established response surface models can easily be used to generate stress and stress values in time domain for reliability analysis.

Calculation of Instantaneous Probability of Failure

Failure condition can be defined as:

$$g_1 \leq 0 \text{ or } g_2 \leq 0 \text{ or } g_3 \leq 0 \tag{10}$$

Failure condition is defined in terms of limit state functions. Dissatisfaction of any of the limit state function means failure of the propellant grain. Probability of failure and reliability calculations have been performed using this failure condition. A safety (reliability) index, β, is used in calculating the probability of failure and reliability values of the propellant[13]. The safety index can be written as:

$$\beta = \frac{\mu_g}{\sqrt{Var(g)}} \tag{11}$$

Variance of g can be calculated by using the equation below :

$$Var(g) \approx \sum_{j=1}^{m} \left(\frac{\partial g}{\partial x_j} \right)^2 Var(x_j) \tag{12}$$

If the variables are normally distributed, probability of failure can be found as:

$$P_f = \Phi(-\beta) \tag{13}$$

In the case of non-normally distributed variables, probability of failure can be calculated by proper transformation [13]. The found probability of failure and reliability values are instantaneous. Time dependent reliability values of the propellant are calculated with conditional probability theory [5,11,14].

Instantaneous reliability/probability of failure values can also be found by using Monte Carlo simulations. However, this method takes longer time in spite of its simplicity.

Calculation of Failure/Reliability

Thermal loads can be expressed as daily and yearly changes as mentioned before. Time dependent reliability values of the propellant can be calculated with conditional probability theory [5,11,14]. Time dependent reliability can be expressed as:

$$R(t) = e^{-\int_0^t \lambda(\xi) d\xi} \tag{14}$$

Hazard rate, λ, is defined as a component that has survived to time t, and is probability of failure at time interval dt.

Hazard rate can be calculated using the formulas below

$$\lambda(t) = \frac{f(t)}{1 - F(t)}, \quad \lambda(t) = \frac{P_f(t_i)}{1 - \sum_{j=1}^{i-1} P_f(t_i - 1)} \tag{15}$$

Case Study

Three dimensional finite element model which is prepared for the case study is shown in Figure 3. The model consists of 20354 hexahedral elements and 26029 grid nodes. Only 1/7 of the rocket motor is modeled due to symmetry.

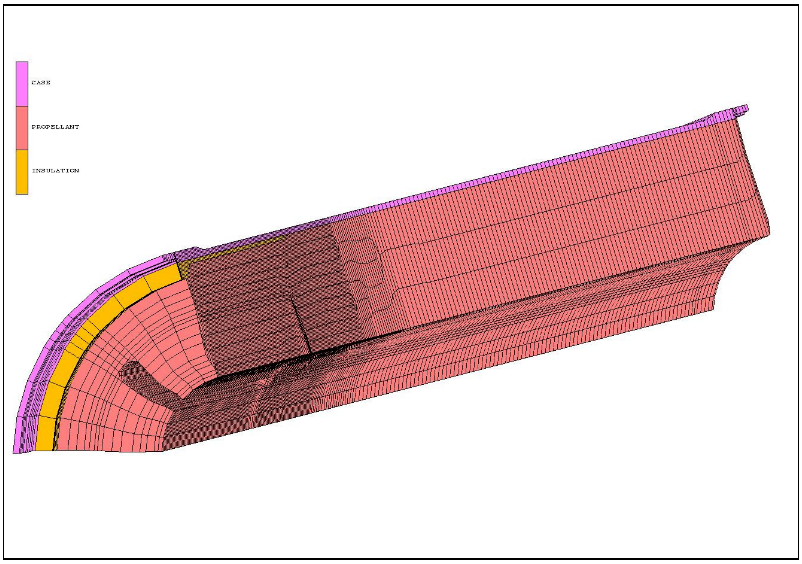

Figure 3: Finite element model

This methodology gives a reliability index, hazard rate and probability of failure for the solid propellant grain with time which yields an assessment of the service life. Reliability index, hazard rate and probability of failure values with respect to time can be seen in Figure 4,5 and 6 respectively. With a probabilistic approach that has been carried out throughout the study, uncertainties and contradicting effects (e.g. at low temperatures propellant grain induced higher strains and stresses but aging effect is reduced) are taken into account and assessment of service life is done accordingly. Since the most critical limit state function comes out to be the stress induced in the propellant grain, end results are presented accordingly.

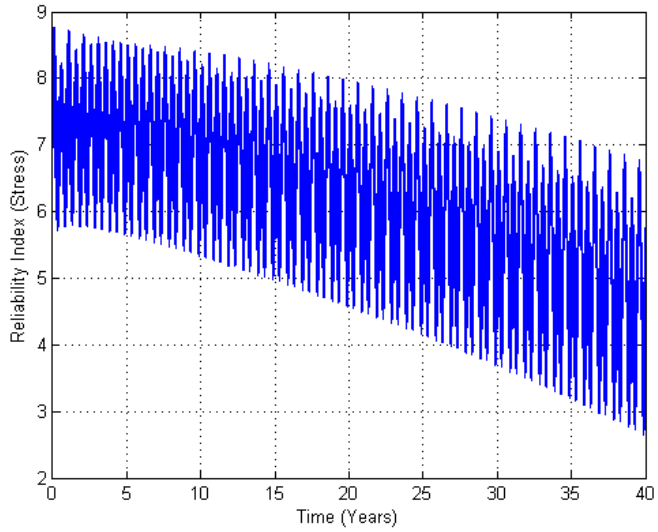

Figure 4: Reliability index vs. time

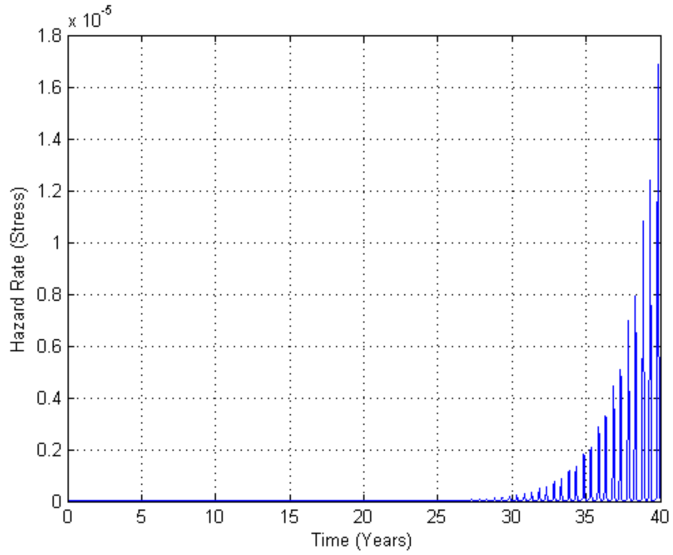

Figure 5: Hazard rate vs. time

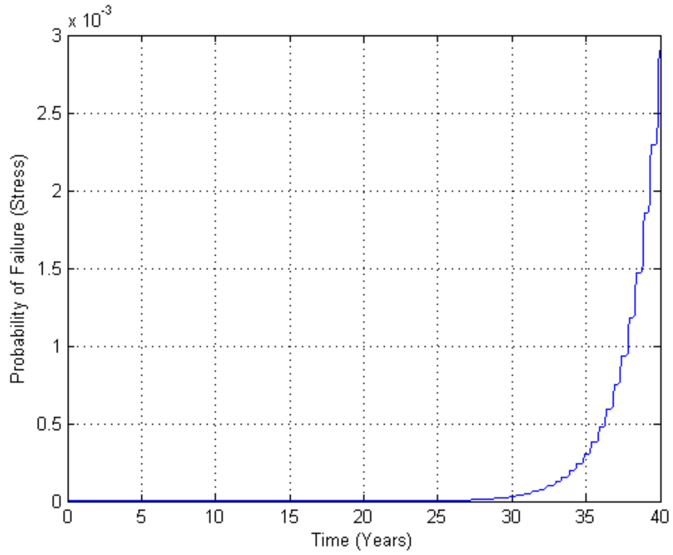

Figure 6: Probability of failure vs. time

Conclusions

The methodology presented in this study yield an assessment of the service life of the solid rocket propellant which is critical considering the diversity of environments that the propellant is exposed to. Considering the effect of uncertainties in the material properties, random thermal environment conditions, linear viscoelastic material model, aging behavior and cumulative damage factors, the progressive reliability of the propellant for different storage scenarios are illustrated on a rocket motor. As it is seen in Figure 6, degradation mechanisms tend to dominate the probability of failure more and more as time passes. Using this approach, with defining an acceptable risk of failure for design, a specific service life time can be estimated.

References

[1] Kunz , R. K., Characterization of Solid Propellant for Linear Cumulative Damage Modeling, 45th AIAA/ASME/SAE/ASEE Joint Propulsion Conference & Exhibit, Denver, Colorado, AIAA 2009-5257, Aug 2009.

[2] Hasanoğlu, M. S., Storage Reliability Analysis of Solid Rocket Propellants, M.Sc. Thesis, Middle East Technical University, Aug 2008.

[3] M. Kivity, G. Hartman, A. M. Achlama, Aging of HTPB Propellant, AIAA/ASME/SAE/ASEE Joint Propulsion Conference, AIAA 2005-3802, 2005.

[4] Marotta, S. A., Kudiya, A., Ooi, T. K., Toutanji, H. A., Gilbert J. A., Predictive Reliability of Tactical Missiles Using Health Monitoring Data and Probabilistic Engineering Analysis, 5th International Workshop, Structural Health Monitoring, Stanford University, 2005.

[5] Akpan, U. O, Dunbar, T. E., Wong, F. C., Probabilistic Risk Assessment of Solid Propellant Rocket Motors, Journal of Spacecraft and Rockets, Vol. 40, No. 3, 2003.

[6] Biggs, G. L., Forecasting Structural Reliability of Rocket Solid Propellant over Time, RTO-MP-091, Sep 2002.

[7] Akpan, U. O, Wong, F. C., The Role of Probabilistic Sensitivity Analysis in Assessing the Service Life of Solid Rocket Motors, 43rd AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conference, Apr 2002.

[8] Myers, R. H., Montgomery, D. C., Response Surface Methodology, John Wiley & Sons Inc., 2002.

[9] Structural Assessment of Solid Propellant Grains, AGARD 350, 1997.

[10] Margetson, J, Wong, F. C., Service Life Prediction of Solid Rocket Propellant Motors Stored in a Random Thermal Environment, AGARD CP-586, No.36, 1996.

[11] Heller, R. A., Thangjitham, S., Janajreh, I. M., Probabilistic Service Life Prediction for Solid Propellant Motors Subjected to Environmental Thermal Loads, AGARD CP-586, No.34, 1996.

[12] Jensen G. E., Netzer, L. S., Tactical Missile Propulsion, Progress in Astronautics and Aeronautics, AIAA V-170, 1996.

[13] Janajreh, I. M., Heller, R. A., Thangjitham, S., Safety Index Approach to Predicting the Storage Life of Rocket Motors, Journal of Spacecraft and Rockets, Vol. 31, No.6, pp. 1072-1078, 1994.

[14] Heller, M. P., Singh, M. P., Thermal Storage Life of Solid Propellant Motors, J. Spacecraft, Vol. 20, No.2, p:144-149, 1983.

[15] A. G. Christiansen, L. H. Layton, R. L. Carpenter, HTPB Propellant Aging, Journal of Spacecraft and Rockets, Vol. 18, p:211, 1981

[16] Heller, R. A., Kamat, M. P., Singh, M. P., Probability of Solid Propellant Motor Failure Due to Environmental Temperatures, J. Spacecraft, Vol. 16, No.3, p:140-146, 1979.