6. ANKARA INTERNATIONAL AEROSPACE CONFERENCE AIAC-2011-026
14-16 September 2011 - METU, Ankara TURKEY

DETERMINING OPTIMUM NUMBER OF STRUCTURAL ELEMENT TESTS FOR WING, HORIZONTAL AND VERTICAL TAILS

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ABSTRACT

Aircraft structural design is still performed using a deterministic design framework, within which the current practice to choose the number of structural tests for aircraft components is based on experience. This paper focuses on element tests among many different types of structural tests, and explores determination of the optimum number of structural element tests that must be performed for wing, horizontal tail and vertical tail by probabilistic methods. A representative system composed of a wing, a horizontal tail and a vertical tail is considered. It is assumed that the design of the wing and the tails are driven by their most critical components, which can be represented by a small region characterized by a width and a thickness. The widths of the critical components are kept constant and the thicknesses of these components are designed together with their corresponding number of structural element tests. The number of structural element tests and the additional company knockdown factors (that alter the design thicknesses) for each component are selected as design variables to perform system reliability-based design optimization (RBDO) for minimum direct operating cost.

INTRODUCTION

Aircraft structural design is still performed with a deterministic design philosophy, but there has been growing interest in applying probability methods to aircraft structural design [1–5]. In traditional reliability-based optimization, all uncertainties that are available at the design stage are considered in calculating the reliability of the structure [6-10]. However, the effects of post-design measures (e.g., structural tests, health monitoring activities) that can effectively reduce uncertainties are usually not included. It has been shown that it would be beneficial to include the effects of these uncertainty reduction measures (URMs) in the design process [11-14].

Among the wide variety of URMs, structural tests have been taken attention of many researchers. There are few papers in the literature that address the effect of tests on structural safety. Jiao and Moan [15] explored the effect of proof tests on structural safety using Bayesian updating. They showed that the proof testing reduces the uncertainty in the strength of a structure, thereby leading to substantial reduction in probability of failure. Jiao and Eide [16] explored the effects of testing, inspection and repair on the reliability of offshore structures. Beck and Katafygiotis [17] addressed the problem of updating a probabilistic structural model using dynamic test data from structure by utilizing Bayesian updating. Similarly, Papadimitriou et al. [18] used Bayesian updating within a probabilistic structural analysis tool to compute the updated reliability of a structure using test data. They found that the reliabilities computed before and after updating were significantly different.

The aforementioned studies focused on the tests that have already been performed. Acar et al. [19] extended these works in simulating all possible outcomes of future tests, which would allow the designer to design the tests together with the structure. In a following work, assuming that the structural design of the aircraft has been driven by the most critical component, Acar et al. [20] showed that structural tests can be designed together with the structure trading off the cost of more weight against the cost of additional tests. More recently, system reliability considerations were taken

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into account by Acar [21], who performed RBDO of a representative wing and tail system together with structural tests.

The two main contributions of this paper are the followings. First, the representative system in the earlier work [21], composed of a wing and a horizontal tail, is extended by incorporating the vertical tail into the system. Second, the reliability estimations are performed by using two alternative techniques (i) separable Monte Carlo simulations (MCS) and (ii) tail modeling, and their efficiency are compared. The paper is structured as follows. The next section presents the RBDO problem for minimum direct operating cost. The following section provides a brief description of the reliability estimation using the guided tail modeling method. Finally, the preliminary results and the concluding remarks are given in the last two sections of the paper, respectively.

RBDO OF STRUCTURE AND TESTS FOR MINIMUM LIFECYCLE COST

In this section, problem formulation of RBDO of a representative system composed of wing, horizontal tail and vertical tail together with corresponding number of structural element tests. The design variables of the optimization problem are chosen as the additional company knockdown factor k_f and the number of element tests n_e. Since these variables may change from the wing to the tails, there exist six design variables $(k_f)_W, (n_e)_W, (k_f)_{HT}, (n_e)_{HT}, (k_f)_{VT}, (n_e)_{VT}$, where the subscripts *W, HT* and *VT* correspond to the wing, the horizontal tail and the vertical tail, respectively. The RBDO for minimum direct operating cost (DOC) can be performed by solving the following optimization problem:

$$\text{Find} \quad (k_f)_W, (n_e)_W, (k_f)_{HT}, (n_e)_{HT}, (k_f)_{VT}, (n_e)_{VT} \tag{1.1}$$

$$\text{Min} \quad \mathrm{DOC}\left[(k_f)_W, (n_e)_W, (k_f)_{HT}, (n_e)_{HT}, (k_f)_{VT}, (n_e)_{VT}\right] \tag{1.2}$$

$$\text{S.t.} \quad P_{FS}\left[(k_f)_W, (n_e)_W, (k_f)_{HT}, (n_e)_{HT}, (k_f)_{VT}, (n_e)_{VT}\right] \le (P_{FS})_{nom} \tag{1.3}$$

$$0.8 \le (k_f)_W, (k_f)_{HT}, (k_f)_{VT} \le 1.0; \quad 1 \le (n_e)_W, (n_e)_{HT}, (n_e)_{VT} \le 5 \tag{1.4}$$

The $(P_{FS})_{nom}$ term in the constraint is taken as the value of P_{FS} when the design variables take their nominal values (i.e., $(k_f)_W = (k_f)_{HT} = (k_f)_{VT}$ = 0.95, $(n_e)_W = (n_e)_{HT} = (n_e)_{VT}$ = 3).

System failure probability

The failure probabilities of the wing, the horizontal tail, and the vertical tail are denoted by $(P_f)_W$, $(P_f)_{HT}$*, and* $(P_f)_{VT}$, respectively. In this study, the failures of the components are taken correlated by assuming the error in load calculation, the error in failure stress calculation, and the error geometric properties are assumed to be the same for all components. That is,

$$(e_P)_W = (e_P)_{HT} = (e_P)_{VT}, \; (e_f)_W = (e_f)_{HT} = (e_f)_{VT}, \text{ etc.} \tag{2}$$

For the purpose of demonstration, a simplifying assumption is made that the wing and the tail structures are stressed equally for the nominal case. Thus, the nominal failure probabilities of all components are equal, that is $\left[(P_f)_W\right]_{nom} = \left[(P_f)_{HT}\right]_{nom} = \left[(P_f)_{VT}\right]_{nom}$. Note that the failure probabilities are dependent of the number of element tests as well as the company knockdown factors as follows:

$$(P_f)_W = P_f\left[(k_f)_W, (n_e)_W\right]; \quad (P_f)_{HT} = P_f\left[(k_f)_{HT}, (n_e)_{HT}\right]; \quad (P_f)_{VT} = P_f\left[(k_f)_{VT}, (n_e)_{VT}\right] \tag{3}$$

The failure probabilities of the components are calculated by using two alternative techniques: (i) separable MCS, and (ii) tail modeling as described in the next two sections.

Cost model

The cost model used in this study is taken from an earlier work [20], which is based on the paper by Kaufmann et al. [22]. The direct operating cost of the aircraft structure is defined as

$$\mathrm{DOC} = p\,\mathrm{W} + \mathrm{C}_{\text{test}} + C_{other} \tag{4}$$

Here, p is the total cost saving attained by reducing the structural weight W by one unit, and C_{test} is the cost of tests. Since this study focuses mainly on the element tests, and the constant cost terms do not affect the optimization, the DOC can be reformulated as

$$\mathrm{DOC} = p\ \mathrm{W} + \mathrm{C_e} \tag{5}$$

where C_e is the cost of element tests. The total direct operation cost of the whole system is

$$\mathrm{DOC}\left[(k_f)_W,(n_e)_W,(k_f)_{HT},(n_e)_{HT},(k_f)_{VT},(n_e)_{VT}\right] = \mathrm{DOC_W}\left[(k_f)_W,(n_e)_W\right] + \mathrm{DOC}_{HT}\left[(k_f)_{HT},(n_e)_{HT}\right] + \mathrm{DOC}_{VT}\left[(k_f)_{VT},(n_e)_{VT}\right] \tag{6}$$

where DOC_W, DOC_{HT} and DOC_{VT} are the direct operation costs of the wing, the horizontal tail and the vertical tail, respectively, and they can be formulated as

$$\mathrm{DOC_W}\left[(k_f)_W,(n_e)_W\right] = p\ \mathrm{W_W}\left((k_f)_W,(n_e)_W\right) + \mathrm{C_e}\left[(n_e)_W\right] \tag{7}$$

$$\mathrm{DOC}_{HT}\left[(k_f)_{HT},(n_e)_{HT}\right] = p\ \mathrm{W}_{HT}\left((k_f)_{HT},(n_e)_{HT}\right) + \mathrm{C_e}\left[(n_e)_{HT}\right] \tag{8}$$

$$\mathrm{DOC}_{VT}\left[(k_f)_{VT},(n_e)_{VT}\right] = p\ \mathrm{W}_{VT}\left((k_f)_{VT},(n_e)_{VT}\right) + \mathrm{C_e}\left[(n_e)_{VT}\right] \tag{9}$$

where W_W , W_{HT} and W_{VT} are the structural weights of the wing, the horizontal tail, and the vertical tail. Details of each term in the cost equation are provided below.

The weight penalty

Curran et al. [23] proposed that the economical value of weight saving is 300 $/kg. Similarly, Kim et al. [24] stated that a 1 lb weight reduction amounts to a total saving of $200 for a civil transport aircraft. Jenkinson et al. [25] noted that operating cost of carrying an additional 1 lb over the lifetime of a 300-600 seat civil aircraft is around $1,000. In this study, the weight penalty is varied between $200/lb and $1000/lb and its effect on the optimum values of the design variables is explored.

The structural weights of the wing and the tail

Jenkinson et al. [25] provided component weight estimations for civil aircraft normalized, and noted that the structural weight of a typical wing was about 10 to 12% of the MTOW, and the structural weight of a tail is about 1.5 to 3% of the MTOW. In this paper, the ratio of the structural weights of the components to the MTOW are taken as $\frac{W_W}{MTOW} = 10\%$, $\frac{W_{HT}}{MTOW} = 2\%$, and $\frac{W_{VT}}{MTOW} = 1\%$, respectively.

In this paper, a typical civil transport aircraft with an MTOW of 300,000 lbs is considered, so the structural weights are then taken as W_W=30,000 lbs, W_{HT}=6,000 lbs, and W_{HT}=3,000 lbs, respectively. Since the test costs can be attributed to fleet of aircraft rather than a single one, total structural weight of the fleet is considered. Therefore, the structural weight formulas can be written as

$$\mathrm{W_W}\left[(k_f)_W,(n_e)_W\right] = \frac{A\left[(k_f)_W,(n_e)_W\right]}{A_{nom}} \times N_a \times 30{,}000 \tag{10}$$

$$\mathrm{W}_{HT}\left[(k_f)_{HT},(n_e)_{HT}\right] = \frac{A\left[(k_f)_{HT},(n_e)_{HT}\right]}{A_{nom}} \times N_a \times 6{,}000 \tag{11}$$

$$\mathrm{W}_{VT}\left[(k_f)_{VT},(n_e)_{VT}\right] = \frac{A\left[(k_f)_{VT},(n_e)_{VT}\right]}{A_{nom}} \times N_a \times 3{,}000 \tag{12}$$

Here A is the load carrying area, and A_{nom} is the value of A when the design variables take their nominal values. It is assumed that a typical airliner has a production line of 1,000 aircraft before it is discontinued or substantially redesigned, so N_a=1,000.

The cost of structural element tests

The costs of the tests are taken as $150,000 for each element test based on an earlier study [20]. Hence, the C_e[ ] term in Eqs. (7-9) can be written as

$$C_e\ (n_e) = 150{,}000 \times N_{elem} \times n_e \quad \text{(in \$)} \tag{13}$$

where N_{elem} is the number different types of structural elements tested (taken as N_{elem}=100).

The flowchart of the RBDO problem is given in Fig. 1. The optimization starts with determining the design variables of the problem. Here, the design variables are $(k_f)_W, (n_e)_W, (k_f)_{HT}, (n_e)_{HT}, (k_f)_{VT}$, and $(n_e)_{VT}$. Then, the lower and upper limits for the design variables are specified. Here, these bound are $0.8 \le (k_f)_W, (k_f)_{HT}, (k_f)_{VT} \le 1.0$, $1 \le (n_e)_W, (n_e)_{HT}, (n_e)_{VT} \le 5$. The calculation of the objective function and the constraint requires evaluations of the load carrying areas and the probabilities of failure of the components that are computationally expensive. Therefore, response surface approximations are constructed. For that purpose, first design of experiments methodology is used to specify the training points in the design variable space. Then, the responses (load carrying areas and the probabilities of failure) are computed at the training point. The training points and the corresponding responses constitute the training set. The training set is then used to generate the response surfaces. The cost parameters and the constructed response surfaces are used in solving the optimization problem for the minimum direct operating cost to find the optimum values of the design variables.

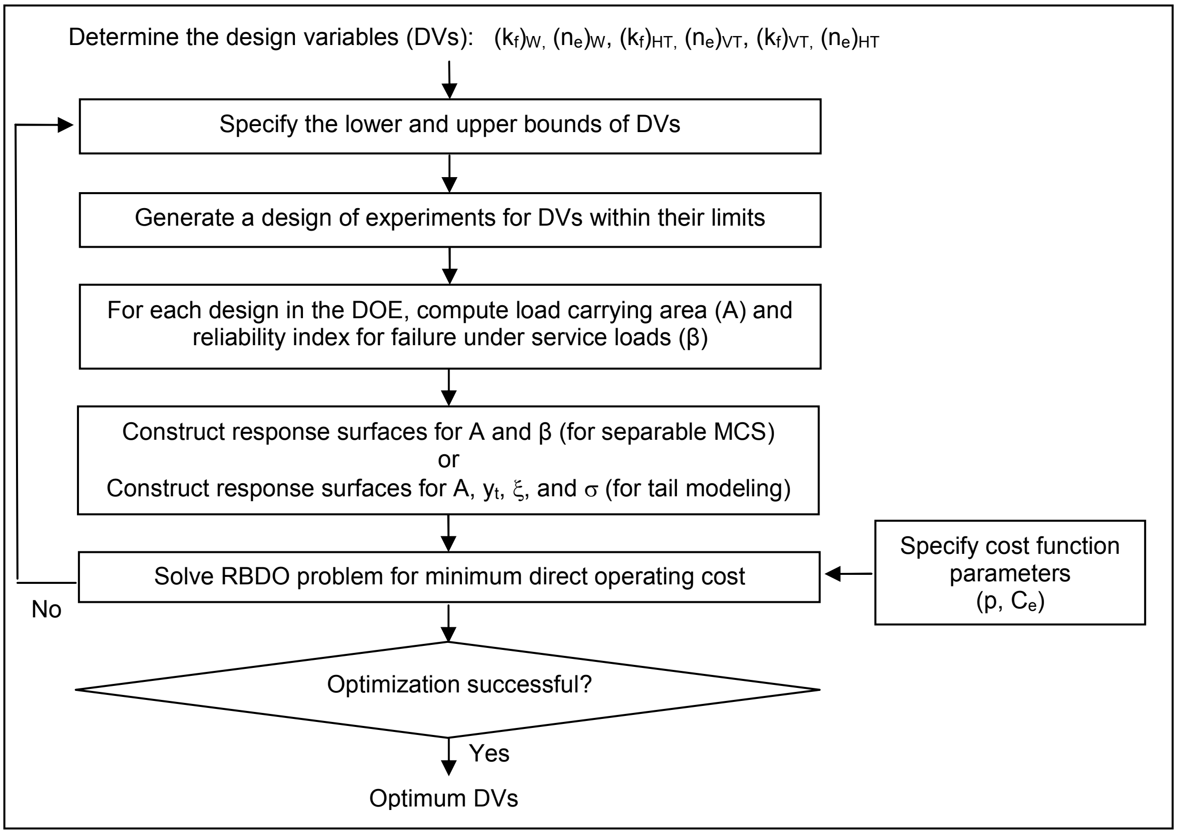


Figure 1. RBDO flowchart

RELIABILITY ESTIMATION USING SEPARABLE MONTE CARLO SIMULATIONS

The prediction of probability of failure using conventional MCS requires trillions of simulations for level of 10^{-7} failure probability. To reduce the computational cost, separable Monte Carlo procedure can be used. The reader is referred to Smarslok and Haftka [26] for more information on the separable Monte Carlo procedure. This procedure applies when the failure condition can be expressed as $g_1(x_1)>g_2(x_2)$, where x_1 and x_2 are two disjoint sets of random variables. To take advantage of this procedure, we need to formulate the failure condition in a separable form, so that g_1 will depend only on variabilities and g_2 only on errors. The common formulation of the structural failure condition is in the form of a stress exceeding the material limit. This form, however, does not satisfy the separability requirement. For example, the stress depends on variability in material properties as well as design area, which reflects errors in the analysis process. To bring the failure condition to the right form, the failure condition is formulated as the required cross sectional area A'_{req} being larger than the built-average area A. So, the failure condition can be defined in terms of the built area and the required area as:

$$A < \frac{A_{req}}{(1+v_t)(1+v_w)} \equiv A'_{req} \tag{14}$$

where A_{req} is the cross-sectional area required to carry the actual loading conditions for a particular copy of an aircraft model, and A'_{req} is what the built area needs to be after allowing for variabilities.

$$A_{req} = P/\sigma_f \tag{15}$$

Notice that the required area depends only on variability, while the built area depends only on errors. The separable MCS procedure is summarized in Fig. 2.

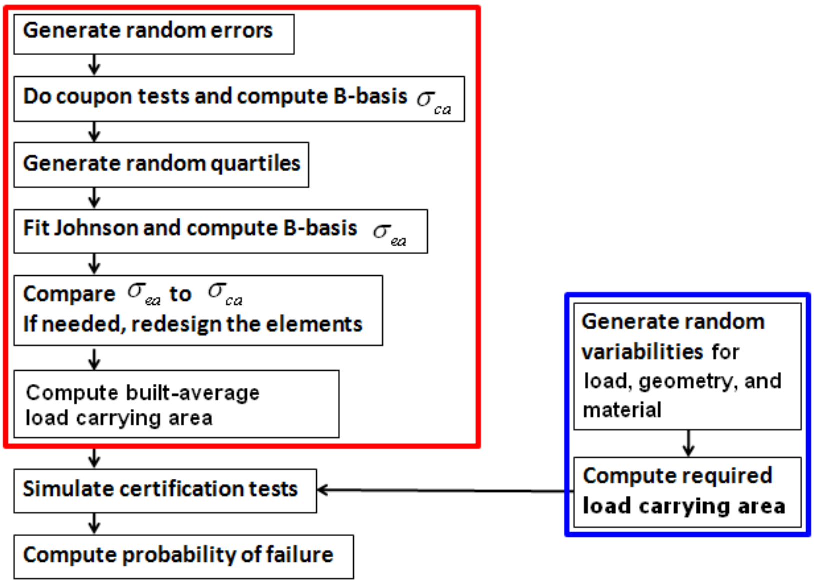


Figure 2. Flowchart for separable Monte Carlo simulations

RELIABILITY ESTIMATION USING TAIL MODELING

The fundamental idea of the tail modeling technique is based on the property of tail equivalence. Two distribution functions *F*(*x*) and *G*(*x*) are called tail equivalent if the following condition is satisfied [27].

$$\lim_{x\to\infty} \frac{1-F(x)}{1-G(x)} = 1 \tag{16}$$

Here, the tail model of *F*(*x*) is used to approximate the upper (or lower) tail of *G*(*x*). This approach does not take into account the central behavior of the distribution. Rather, it focuses on the upper or lower tail behavior, which fits for the purpose of reliability analysis of highly safe mechanical systems.

Now, consider the limit-state function y(**x**), where **x** is the vector of random variables. For a large threshold value of y_t (see Fig. 3), the region above the threshold (i.e., the tail portion) can be approximated using generalized Pareto distribution (GPD). The GPD approximates the conditional excess distribution of $F_z(z)$, where $z = y - y_t$, via

$$F_z(z) = \begin{cases} 1 - \left\langle 1 + \frac{\xi}{\sigma} z \right\rangle_+^{-\frac{1}{\xi}} & if\ \xi \neq 0 \\ 1 - \exp\left(-\frac{z}{\sigma}\right) & if\ \xi = 0 \end{cases} \tag{17}$$

where $\langle A \rangle_+ = \max(0, A)$, $z \geq 0$, and $F_z(z)$ is the GPD with shape and scale parameters ξ and σ, respectively, which must be found.

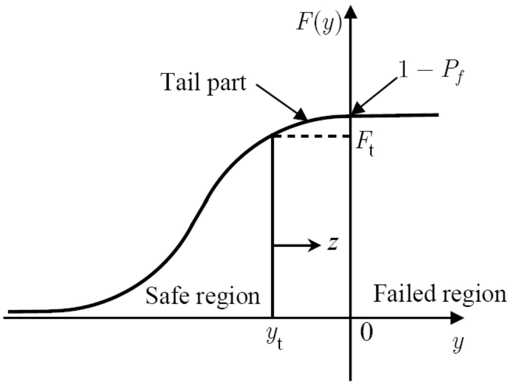


Figure 3. Tail modeling concept

The conditional excess distribution can be related to the cumulative distribution *F*(y) through

$$F_z(z) = \frac{F(y) - F(y_t)}{1 - F(y_t)} = \frac{F(y) - F_t}{1 - F_t} \tag{18}$$

Then, *F*(*y*) above the threshold (i.e., $y \geq y_t$) is expressed in terms of the conditional excess distribution, $F_z(z)$, via

$$F(y) = F_t + (1 - F_t) F_z(y - y_t) \tag{19}$$

Once the cumulative distribution function *F*(*y*) is obtained, the probability of failure, P_f, can be estimated from [28]

$$P_f = 1 - F(y = 0) = (1 - F_t) \left\langle 1 - \frac{\xi}{\sigma} y_t \right\rangle_+^{-\frac{1}{\xi}} \tag{20}$$

The reliability index can be calculated from

$$\beta = \Phi^{-1}(1 - P_f) \tag{21}$$

where Φ is the cumulative distribution function of a standard normal random variable.

The classical tail modeling methods are based on the following three step procedure.

1. *N* samples of the limit-state function *y*(**x**) are generated through Monte Carlo simulation (or Latin Hypercube Sampling). In mechanics problems, the samples of the input random variables, **x**, are first generated from the given distribution types, and the corresponding *y* values are then calculated through the structural analysis (usually through computationally expensive finite element analysis).
2. A threshold value for CDF (F_t) is determined and the corresponding limit-state value (y_t) is found. The specification of the threshold has been the subject of extensive research, and empirical values for it have been proposed [29-31]. In this study, three different values are tried for the threshold: F_t = 0.95, F_t = 0.97, and F_t = 0.99. The use of F_t = 0.99 leads to the best results.
3. The shape and scale parameters in the GPD (i.e. ξ and σ), are estimated by fitting the tail model with the empirical CDF. In this study, the maximum likelihood method is used to find these parameters.

There are also more efficient models than the classical tail modeling (e.g., guided tail modeling (GTM) [32]). However, in this study, there exist uncertainties in many layers (coupon level, element level, and structural level). The GTM requires tracking uncertainty in those levels, so it would be very difficult to use GTM in this study. The classical tail modeling, on the other hand, does not require track uncertainty in different layers; thus, the classical tail modeling is used for reliability estimation in this study.

COMPARING THE ACCURACIES OF SEPERABLE MCS AND TAIL MODELING

As noted earlier, response surface approximations are utilized in solving the RBDO problem. When reliability calculations are performed using separable MCS, response surfaces are generated for the load carrying area A and the reliability index β. When reliability calculations are performed using tail modeling, on the other hand, response surfaces are generated for the load carrying area A and the tail model parameters y_t, ξ, and σ. Since the accuracies of the response surfaces directly affect the accuracy of the RBDO solution, the separable MCS and tail model techniques are compared based on the accuracies of the response surfaces.

The input variables of the response surface models and their bounds are provided in Table 1. The full factorial design (FFD) with five levels (equally spaced between their bounds) for each variable is utilized to generate the training points. So, overall 2^5=25 training points are generated. The accuracies of the constructed response surface models are evaluated by using leave-one-out cross-validation errors. That is, response surface models are constructed 25 times, each time leaving out one of the training points. The difference between the exact response at the omitted point and that predicted by each variant response surface model defines the cross-validation error. Table 2 provides the root mean square error (RMSE), the mean absolute error (MAE), the maximum absolute error (MAXE) as well as the mean of the response. For separable MCS, Table 2 shows that the response surfaces generated for *A* and *β* have acceptable accuracies. For tail modeling, Table 2 shows that the response surfaces generated for *A*, y_t and σ have acceptable accuracies, while the error in response surface generated for ξ is very large. Alternatively, for tail modeling, response surface is also generated for the reliability index, and it is seen that the errors are large. Based on these findings, the separable MCS technique is decided to be used. The constructed response surfaces for *A* and *β* of the separable MCS are also depicted in Fig. 4.

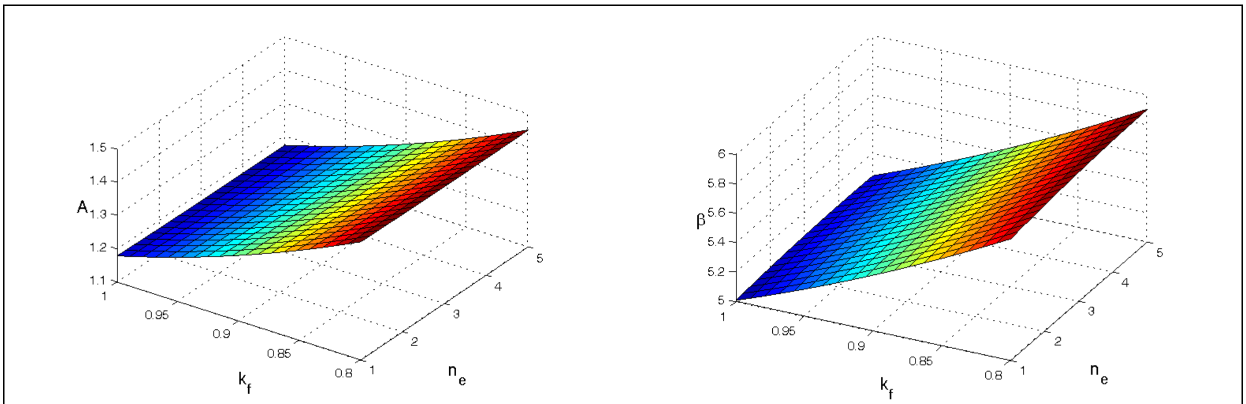
Table 1. Input variables of the response surface models and their bounds

Variable	k_f	n_e
Lower bound	0.8	1
Upper bound	1.0	5

Table 2. Evaluating accuracies of response surface models using leave-one-out cross validation errors

Response	*Mean of response*	*RMSE*[(a)]	*MAE*[(b)]	*MAXE*[(c)]
Separable MCS				
A	1.30	0.0016	0.0012	0.0039
Rel. Index, *β*	5.43	0.0074	0.0059	0.0172
Tail modeling				
A	1.30	0.0014	0.0012	0.0031
y_t	-0.5549	0.0039	0.0031	0.0082
ξ	0.0027	0.0405	0.0307	0.0956
σ	0.0445	0.0049	0.0037	0.0126
Rel. Index, *β*	5.43	0.4087	0.3284	0.8961

[(a)] RMSE: root mean square error; [(b)] MAE: mean absolute error; [(c)] MAXE: maximum absolute error



(a) for *A* (b) for *β*

Figure 4. Constructed response surfaces for (a) the average load carrying area and (b) the reliability index of the critical regions of the wing and the tails

RESULTS

Using the response surfaces constructed, the RBDO problem in Eq. (1) is solved by using *fmincon* function of MATLAB® based on the sequential quadratic programming algorithm. The solution of the optimization problem yields real numbers for the number of tests, but they should be integer numbers. To resolve this issue, the following approach is followed. After the optimum solution is obtained as real numbers for the design variables, the nearest two integers are considered for the number of element tests. For instance, if the optimum numbers element tests for the wing and the tail are found as $(n_e)_W$=4.21, $(n_e)_{HT}$=3.67, and $(n_e)_{HT}$=2.35, respectively, then the following eight [$(n_e)_W$, $(n_e)_{HT}$, $(n_e)_{VT}$] combinations are considered: (4,3,2), (4,3,3), (4,4,2), (4,4,3), (5,3,2), (5,3,3), (5,4,2), and (5,4,3). Then, for each of these combinations, the optimization problem in Eq. (1) is reduced to a three-variable optimization problem (in terms of $(k_f)_W$, $(k_f)_{HT}$ and $(k_f)_{VT}$ only), and their optimum values are calculated. Finally, the combination with the best performance (i.e., with minimum direct operating cost) is declared as the optimum.

The RBDO results for *p*=200 $/lb are provided in Table 3. The optimum values of company knockdown factors for the wing and the tails are found as $(k_f)_W$=0.9945, $(k_f)_{HT}$=0.9059 and $(k_f)_{VT}$=0.8768, respectively. The optimum number of element tests for the wing and the tails are found as $(n_e)_W$=4, $(n_e)_{HT}$=1 and $(n_e)_{VT}$=1, respectively. That is, it is found that larger company knockdown factors must be used for the heavier components than the lighter components. To compensate for that, it is found that higher number of structural element tests must be performed for the heavier components than the lighter components. Table 3 shows that 299 lbs of the wing material is moved to the horizontal tail, 251 lbs of the wing material is moved to the horizontal tail, and meanwhile 689 lbs is saved through optimization. This redesign reduces the wing thickness by about 4%, increases the horizontal tail thickness by about 5% and increases the horizontal tail thickness by about 8%. Consequently, the reliability of the wing is increased and the reliabilities of the tails are increased, while the system reliability is maintained. Overall, the optimization results in weight saving of 689 lbs (per airplane) and cost saving of 183 million dollars (for the entire fleet with N_a=1,000 airplanes).

The RBDO results corresponding to the penalty parameters p=200 $/lb, p=300 $/lb, p=500 $/lb, p=750 $/lb, and p=1,000 $/lb are provided in Tables 4 through 7. In addition, the summary of RBDO results for various penalty parameter values is given in Table 8. The variation of the optimum number of element tests as well as the optimum values of the knockdown factors for the wing and the tail are depicted in Figs. 5(a-b), and the overall weight and cost saving with respect to the weight penalty parameter is shown in Figs. 5(c-d).

The general observations obtained from the RBDO results given in Tables 3 through 8 and Fig. 5 can be summarized as follows:

(1) The optimum reliability allocation for minimum cost is obtained by moving a small fraction of the wing material to the tails. This operation increases the wing failure probability, decreases probabilities of failure of the tails while the system failure probability is maintained. The optimum value of the probability of failure of the wing is larger than those of the tails.

(2) Since the wing material is moved to the tail during optimization, the optimum company knockdown factor for the wing is larger (i.e., the safety factor is smaller) than those of the tails.

(3) Since the probability of failure of the wing is larger than those of the tails, the optimum number of element tests for the wing is larger than or equal to those of the tails to compensate for that.

(4) As the weight penalty parameter p increases, the economical value of the structural weight increases; hence, the structural weight reduction of the system and the overall cost saving increase.

Table 3. RBDO results when the penalty parameter is taken as *p* = 200 $/lb.

	k_f	n_e	*Weight* (*lb*)	P_f (10^{-7})	*p W* ($M)	C_e ($M)	DOC ($M)
Nominal values							
Wing	0.95	3	30,000	0.899	6,000	45	6,045
H.Tail	0.95	3	6,000	0.899	1,200	45	1,245
V.Tail	0.95	3	3,000	0.899	600	45	645
System	---	---	39,000	2.697	7,800	135	7,935
Optimum values							
Wing	0.9945	4	28,761	2.039	5,752	60	5,812
H.Tail	0.9059	1	6,299	0.437	1,260	15	1,275
V.Tail	0.8768	1	3,251	0.220	650	15	665
System	---	---	38,311	2.566	7,662	90	7,752
Overall weight saving ≈ 689 lbs (per airplane)					**Overall cost saving ≈ 183 $M**		

Table 4. RBDO results when the penalty parameter is taken as *p* = 300 $/lb.

	k_f	n_e	*Weight* (*lb*)	P_f (10^{-7})	*p W* ($M)	C_e ($M)	DOC ($M)
Nominal values							
Wing	0.95	3	30,000	0.899	9,000	45	9,045
H.Tail	0.95	3	6,000	0.899	1,800	45	1,845
V.Tail	0.95	3	3,000	0.899	900	45	945
System	---	---	39,000	2.697	11,700	135	11,835
Optimum values							
Wing	0.9946	4	28,759	2.043	8,628	60	8,688
H.Tail	0.9118	2	6,247	0.433	1,874	30	1,904
V.Tail	0.8769	1	3,251	0.221	975	15	990
System	---	---	38,256	2.697	11,477	105	11,582
Overall weight saving ≈ 744 lbs (per airplane)					**Overall cost saving ≈ 253 $M**		

Table 5. RBDO results when the penalty parameter is taken as *p* = 500 $/lb.

	k_f	n_e	*Weight* (*lb*)	P_f (10^{-7})	*p W* ($M)	C_e ($M)	DOC ($M)
Nominal values							
Wing	0.95	3	30,000	0.899	15,000	45	15,045
H.Tail	0.95	3	6,000	0.899	3,000	45	3,045
V.Tail	0.95	3	3,000	0.899	1,500	45	1,545
System	---	---	39,000	2.697	19,500	135	19,635
Optimum values							
Wing	0.9947	4	28,758	2.044	14,379	60	14,439
H.Tail	0.9157	3	6,212	0.431	3,106	45	3,151
V.Tail	0.8771	1	3,250	0.222	1,625	15	1,640
System	---	---	38,220	2.697	19,110	120	19,230
Overall weight saving ≈ 780 lbs (per airplane)					**Overall cost saving ≈ 405 $M**		

Table 6. RBDO results when the penalty parameter is taken as *p* = 750 $/lb.

	k_f	n_e	*Weight* *(lb)*	P_f (10^{-7})	*p W* ($M)	C_e ($M)	DOC ($M)
Nominal values							
Wing	0.95	3	30,000	0.899	22,500	45	22,545
H.Tail	0.95	3	6,000	0.899	4,500	45	4,545
V.Tail	0.95	3	3,000	0.899	2,250	45	2,295
System	---	---	39,000	2.697	29,250	135	29,385
Optimum values							
Wing	0,9947	4	28,756	2.047	21,567	60	21,627
H.Tail	0,9158	3	6,211	0.432	4,659	45	4,704
V.Tail	0,8825	2	3,225	0.218	2,418	30	2,448
System	---	---	38,192	2.697	28,644	135	28,779
Overall weight saving ≈ 808 lbs (per airplane)				**Overall cost saving ≈ 606 $M**			

Table 7. RBDO results when the penalty parameter is taken as *p* = 1000 $/lb.

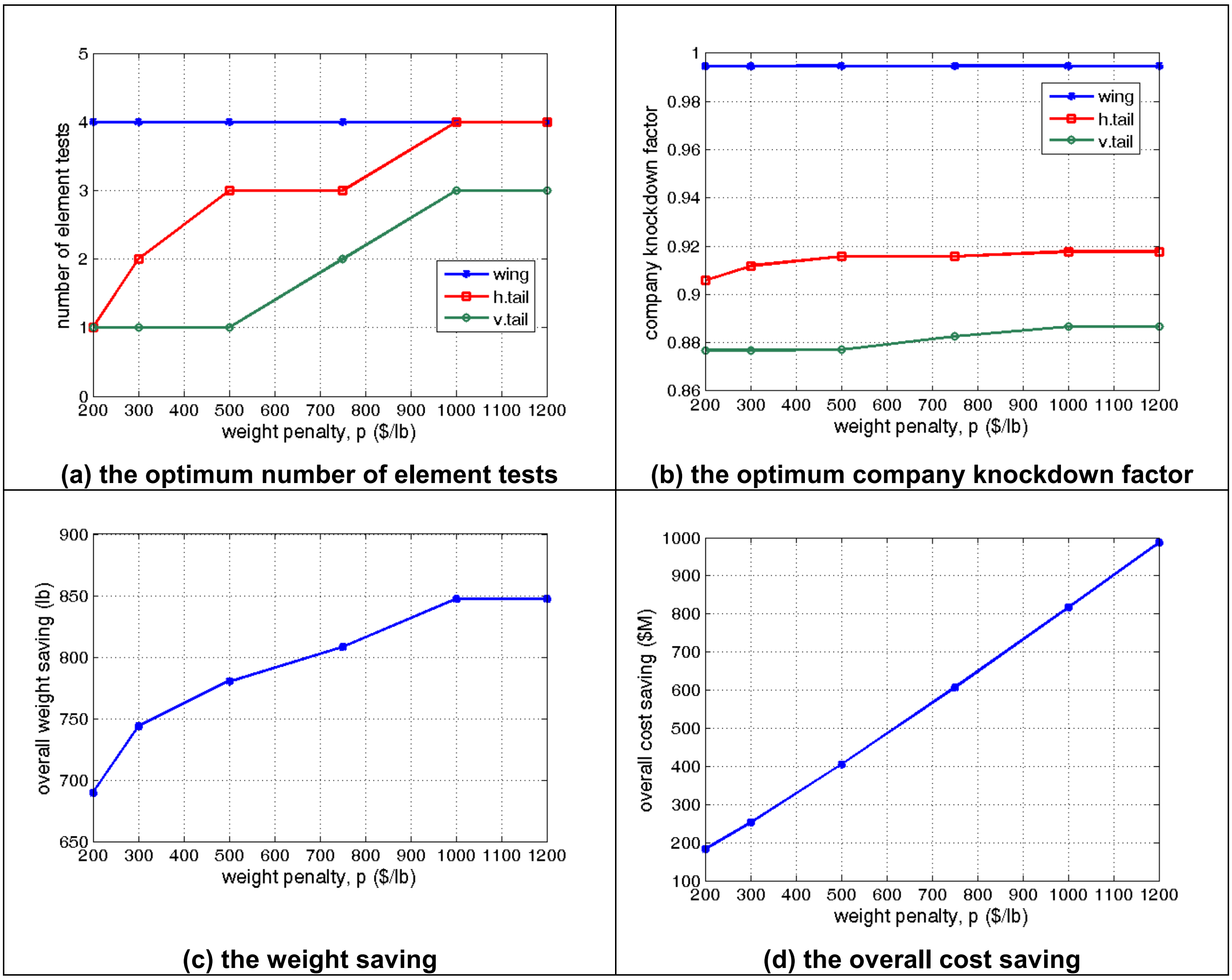
	k_f	n_e	*Weight* *(lb)*	P_f (10^{-7})	*p W* ($M)	C_e ($M)	DOC ($M)
Nominal values							
Wing	0.95	3	30,000	0.899	30,000	45	30,045
H.Tail	0.95	3	6,000	0.899	6,000	45	6,045
V.Tail	0.95	3	3,000	0.899	3,000	45	3,045
System	---	---	39,000	2.697	39,000	135	39,135
Optimum values							
Wing	0,9949	4	28,754	2.049	28,754	60	28,814
H.Tail	0,9177	4	6,194	0.429	6,194	60	6,254
V.Tail	0,8866	3	3,205	0.219	3,205	45	3,250
System	---	---	38,153	2.697	38,153	165	38,318
Overall weight saving ≈ 847 lbs (per airplane)				**Overall cost saving ≈ 817 $M**			

Table 8. Summary of RBDO results for various penalty parameter values

p *($/lb)*	$(k_f)_W$	$(k_f)_{HT}$	$(k_f)_{VT}$	$(n_e)_W$	$(n_e)_{HT}$	$(n_e)_{VT}$	*Weight saving* (lbs)	Cost *saving* ($M)
200	0.9945	0.9059	0.8768	4	1	1	689	183
300	0.9946	0.9118	0.8769	4	2	1	744	253
500	0.9947	0.9157	0.8771	4	3	1	780	405
750	0.9947	0.9158	0.8825	4	3	2	808	606
1,000	0.9948	0.9177	0.8866	4	4	3	847	817

CONLUSIONS

In this study, a representative system composed of a wing, a horizontal tail and a vertical tail was considered, and the optimum number of structural element tests that must be performed for each component was determined by using probabilistic methods. It was assumed that the design of the wing and the tails are driven by their most critical components, which can be represented by a small region characterized by a width and thickness. The widths of the critical components were kept constant and the thicknesses of these components were designed together with their corresponding number of structural element tests. The number of structural element tests and the additional company knockdown factors for each component were selected as design variables to perform system reliability-based design optimization for minimum direct operating cost.



(a) the optimum number of element tests **(b) the optimum company knockdown factor**

(c) the weight saving **(d) the overall cost saving**

Figure 5. The variation of the optimum number of element tests and the optimum company knockdown factor as well as the overall weight and cost saving with respect to the weight penalty parameter *p*.

For reliability analysis, two alternative techniques are considered: (i) separable Monte Carlo simulations, and (ii) tail modeling. It is found for our problem that the response surface approximations generated using the results of the separable MCS technique was more accurate than the response surface approximations generated using the results of the tail modeling.

The solution of the RBDO problem indicated that by performing higher number of structural element tests and using larger company knockdown factors for the heavier components and performing lower number of structural element tests and smaller company knockdown factors for the lighter components, the direct operating cost of the system could be reduced without jeopardizing the overall system safety.

The current structural design practices for the wing and the tails do not differ much in terms of the number of tests and the additional company knockdown factors used. That is, the same number of tests is conducted for the structural elements of the wing and the tails. Similarly, the degree of conservatism in the wing structural design and the tail structural design are the same. The results of this study, on the other hand, showed that heavier components (e.g., wing) can be designed with more conservative practices but larger number of structural element tests compared to the lighter components (e.g., tails). This approach will lead to a lighter wing and heavier tails with lighter aircraft overall.

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