

**A Critical View on Closure
or
What is the Big Problem of Turbulence?**

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ABSTRACT

This short presentation firstly comprises a critical evaluation of the Turbulence problem with respect to some common solution approaches.

Secondly, some observations from simplistic solutions and experiments are presented.

Finally, based on the evaluation of the first and second part of the presentation, author's estimation of:

“where the future direction of research lies”

is explained. This research area is expected to be “a complement” at least to RANS calculations of Turbulent Shear Flows.

PREAMBLE

Hundred and twenty eight years since the introduction of Turbulence Concept by O. Reynolds; almost a century since the breakthrough with mixing length (or mixture length) type approaches to closure problem by G.I. Taylor and L.Prandtl; the advent of high speed calculating machines and high sensitivity measuring devices, thousands of dedicated young or experienced but able scientists.... It appears that we are still facing

the big problem of Turbulence,

as it was qualified by L.Prandtl !!

Or, is the expectation full of doubt of Sir H. Lamb really true?

Or, will the complain of W. Heisenberg remain an everlasting truth?

EVALUATION

It would be fair to say that, in spite of this gloomy picture depicted in the preamble, the work done by dedicated scientists and engineers accumulated a vast and valuable wealth of knowledge on turbulence which contributed to the understanding and prediction of turbulent flow behaviour.

But, if we still feel that our comprehension of turbulence is not complete, and our calculations seem to be a kind of curve fitting; perhaps we have to ask if something is missing in our thinking, hence revisit and ponder on the details of our calculations instead of repeating similar approaches again and again. Indeed, so many Turbulence models associated with RANS calculations can perhaps be taken as examples.

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This presentation is an essay to question a detail that intrigues the author.

But, first let me try to give in short terms my evaluation on “where we are with Turbulence”.

Grouping of Works on Turbulence.

A: Understanding Turbulence,

B: Computing for practical interests.

No doubt, empiricism is a necessary tool for understanding turbulence and it is at the same time a rich source to obtain practical information to complement “modelling”. Surely experimentation accompanies almost any work on turbulence and researchers will continue to do so. But the emphasis in the sequel, is rather on A and B.

A: wavelet; chaos; fractals; strange attractors,..... and DNS.

Various models such as “wavelet; chaos; fractals; strange attractors,.... “ are directed to approach Turbulence with a view to give a general physical paradigm that can be used for any Turbulent flow. This is a perfectly legitimate goal, but this goal remains questionable for any one of them.

DNS is an essentially correct approach, perfectly in harmony with the fundamental premise, that “turbulence quantities obey Navier-Stokes equations and the equation of continuity instantaneously”.

The contribution of DNS to the understanding of microscale instabilities, receptivity, bursting phenomenon, eddy formation, life cycle of eddies, energy cascading...etc. are among information that will help future formulations on turbulence. It is estimated that, presently DNS is a research tool and its use for practical needs is beyond the capabilities of existing high speed calculating machines.

B: RANS and closure models; LES and derivatives

RANS is the very first formulation to explain beyond any doubt why the mean Turbulent flow does not obey NS equations. Physical interpretation of its terms threw light to basic mechanisms existing in the turbulent flow and the interaction between mean and the fluctuations (fluctuating flow)

But the RANS formulation did not only help to clarify the difference between streamlined and turbulent flow, but it has also served to calculate turbulent flow with the help of “suitably” defined closure models.

Closure models deserve a few words of recognition. Closing RANS equations or expressing terms which contain correlations of fluctuations by means of combinations of suitable primitive variables with “adjusting constants” is a highly favoured practice. Boussinesque’s eddy viscosity and G.I. Taylor’s and L.Prandtl’s mixing length paradigms lie at the foundation of these models.

After so many years, so many articles, reports, CFD calculations, it seems it is worthwhile to take a breath and ask “where are we?; how far we are from mixing length theories of L.Prandtl, and G.I.Taylor”.

The character of these models to be noted first, is that combinations in primitive variables of “terms to be closed” in RANS formulation, are all related to small scale eddies; these are smaller than energetic eddies and much smaller than large eddies. Some of these small scale eddies have a universal character and others perhaps not so much. But, all of them (i.e. small scale eddies) are not influenced by large eddies. On the other hand, since large eddies are affected directly by the geometrical boundaries, it is clear that small eddies do not and cannot have the capability to reflect “the Boundary Effects” of the domain in which turbulent flow takes place. Therefore it is natural to expect that the

contribution of these small scale eddies to various terms in RANS equations (or in any formulation) must be universal.

If universal, constants associated with terms in closure models should not change from one flow to another one.

But they do.

This is because, these small eddies are also forced to assume the role of remaining eddies (i.e. energetic and large ones) in addition to their own; therefore, constants are not and cannot be universal constants but adjusting parameters. They naturally change from one flow to the other. These “constants” do a kind of correction to account for the effect of large eddies. This leads to a process in turbulent flow calculations, reminiscent of “curve fitting”. In addition, since smaller eddies are responsible for only a portion of the energy spectrum, the energy balance and the energy cascading process cannot be realistic.

One can assert that closure models do not have a chance to take the role but of small eddies and it is vain to expect the “adjusting constants” to be of universal nature.

LES of turbulent shear flow is relatively new compared to RANS calculations and associated closure models. Since it aims to represent complete range. i.e. all scales of eddies in the flow field, it does not bear in principle weaknesses mentioned above of RANS calculations. Yet, to include all wave numbers into calculations seems to be beyond feasible reach of present high speed calculating machines. So, to simulate high wave number range, LES relies on RANS closure models.

An intriguing question is “implication of boundary conditions” in calculating turbulent flow either using RANS or LES, as will be explained below.

SOME SIMPLE OBSERVATIONS

Let us consider logarithmic velocity distribution.

$$\frac{\bar{U}}{U_\tau} = A + B \ln \frac{yU_\tau}{\nu} \quad \text{or} \quad \frac{\bar{U}}{U_\tau} = A' + B \ln \frac{y}{k_S}$$

This distribution (one or the other one) of temporal mean velocity in a boundary layer or in a conduit flow (confined or open surface) is a bi-scale eddy representation of turbulence in simple geometries, Figure 1.

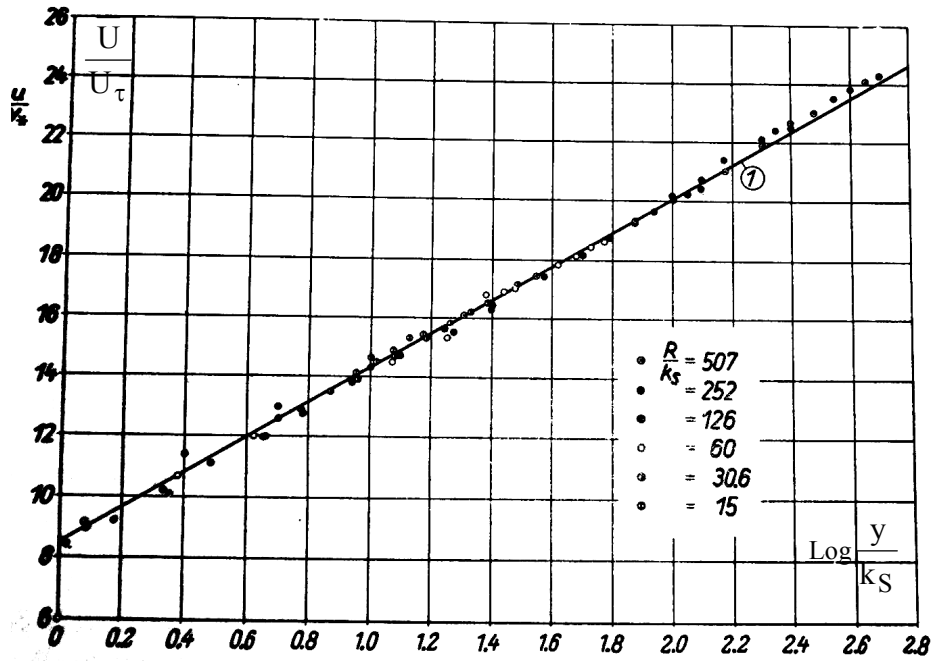


Figure 1: Velocity Distribution in Hydraulically Rough Flow

Whereas small scales are represented by the suitable mixing length formulation (second term), large eddies are taken into consideration in terms of integration constant (A or A' in the formulae).

Since both scales are **separately** present in the formula, the adjusting constant “B” of small scales is not affected by large eddies. It behaves according to the expectations from small eddies, and is considered to be a “universal constant”. Yet, if conditions of the flow are forced beyond the functional limits of B, it shows deviations from its universal value, normally taken to be 2.5 (or 5.75 if ordinary log. is used). Such a situation is demonstrated in Figures 2 and 3.

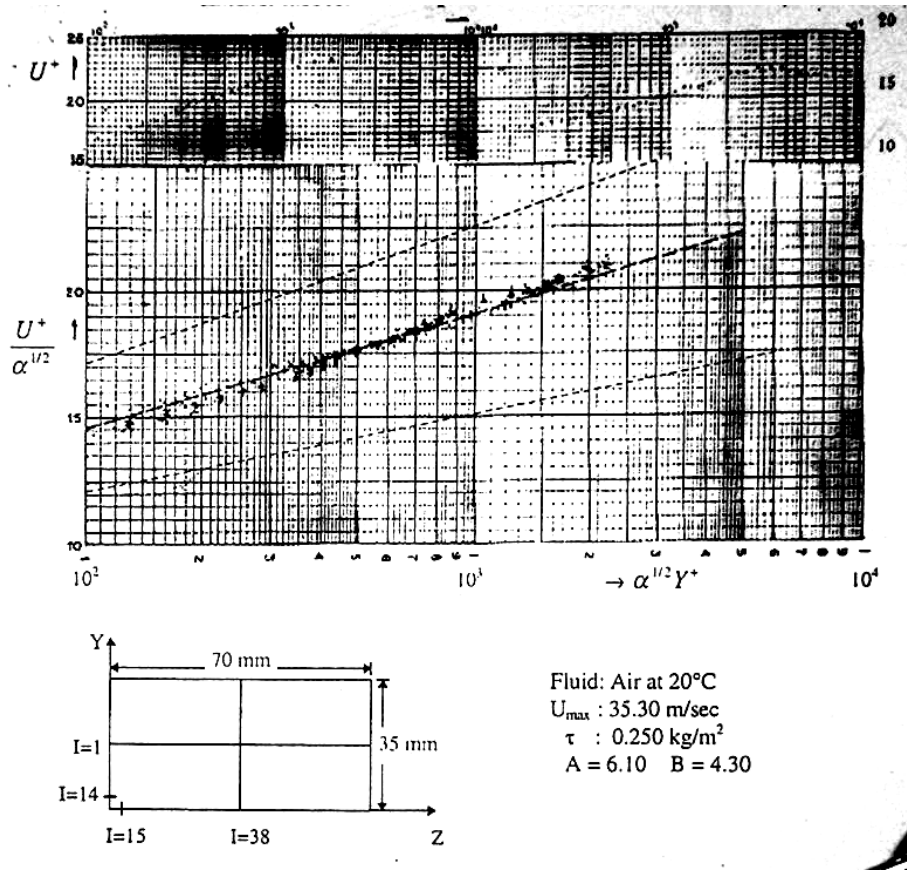


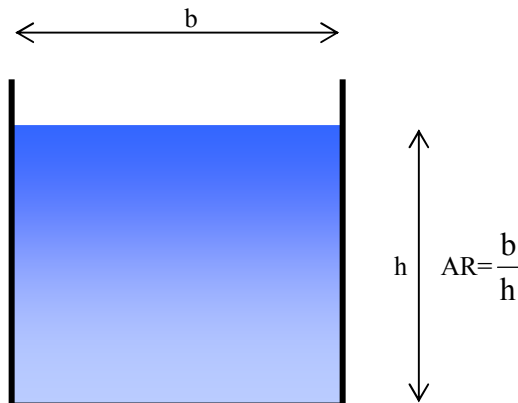
Figure 2: Velocity Distribution in a Rectangular Smooth Pipe

$$\frac{U^+(I, Y_I)}{\alpha^{1/2}(I)} = A + B \text{Log} \left(\alpha^{1/2}(I) Y_I^+ \right)$$

Note that B = 4.30 instead of 5.75 for this confined flow in a duct of aspect ratio AR=2.

On the other hand, “A” appears as an integration constant in the process of derivation of logarithmic velocity distribution. “A” is highly dependent on the geometry of the domain in which turbulent flow takes place.

The experimental results depict clearly how much the geometry of the flow domain affects “A” but does not affect “B” in this bi-scale representation of turbulent flow, Figure 3.



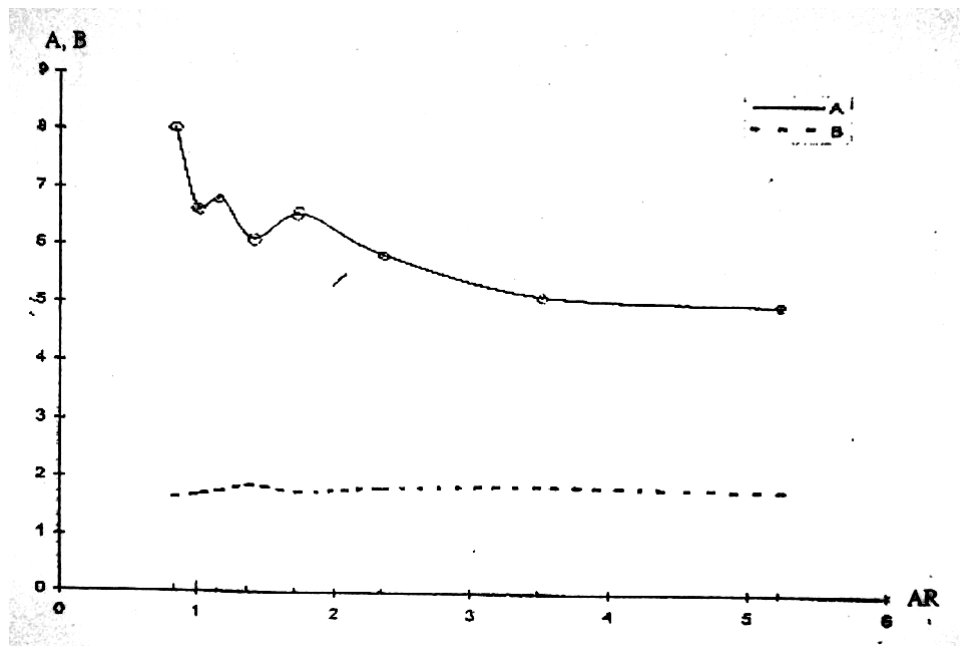


Figure 3: Variation of A and B in Open Channel Flows

Even when the flow is planar or axisymmetric, “A” varies between 5.5 and 9.5 (almost 1 to 2) depending on the:

physical conditions of rigid boundary.

Figure 3 depicts the dependence of “A” on AR in an open channel flow. This dependence shows alternating behaviour as AR decreases, as a result of increased number of larger eddies controlled by side walls and the bottom of the conduit. Thus showing the effectivity of boundaries.

What else can be added.

1: Since the constant of integration is affected by the geometry of the boundary and by the conditions at the boundary, it is reasonable to interpret “A” as the *boundary condition* completing the mathematical definition of the (this) turbulent flow.

2: This boundary condition is satisfied at y_B^+ where: $y_B^+ = 1$

This is well inside the viscous sublayer. But it is well known that the region $0 \leq y^+ < 5$ was originally considered to be “laminar” and was named “laminar sub layer”. Later, with the use of ultra microscope, this layer was discovered “NOT to be laminar”, but having slow fluctuations under the domination of viscosity. Thus, the layer was renamed as “viscous sublayer”. This character is consistent with the character of the flow for which it acts as a boundary condition.

3: As far as its effect on velocity is concerned, “A” forms approximately 60 % to 30 % of the velocity depending on y^+ in the fully turbulent zone or full length of the turbulent B/L.

4: It must be noted and underlined that, logarithmic velocity distribution does not satisfy the non-slip condition. For streamlined flows, non-slip condition serves as a successful boundary condition. But the author is inclined to think that it is misleading to expect non-slip condition to be the boundary condition for turbulent flows with rigid boundaries.

INFERENCE

It is possible that close to geometrical boundaries, a *thin boundary zone intrinsic to geometry of the flow* exists which acts as the geometrical boundaries of turbulent flow. Through this boundary zone large scale eddies introduce their contribution in terms of a finite velocity. The contribution may be reflected in terms of velocity; but the condition itself at this boundary zone may be some other physical quantity, or some entirely new physical (or mathematical) quantity to be defined.

It is the considered thought that, if this thin boundary zone intrinsic to the geometry and the condition(s) to be associated (i.e. boundary conditions of turbulent flow) can be established and used in conjunction with RANS equations, the role and contribution of large eddies will then be adequately included. Furthermore, adjusting constants within the terms which represent the effect of small scale eddies, may turn out to be real constants, perhaps universal constants in some of present closure models.

Figures from:

- 1: Boundary Layer Theory. Dr. H.Schlichting. Translated by Dr. J.Kestin. 4th. ed. McGraw Hill book Co. 1960. Library of Congress Catalog Card No: 59-15472. p: 523. Figure 20.20.
- 2: C. Çıray. On Wall Shear Stress Determination. METU J. of Pure and Applied Sciences. Vol. 3, No:1. April 1970, p: 41-54
- 3: Ahmet Bilgil. Pürüzsüz Kanallardaki Serbest Yüzlü Akımların Sürtünme Katsayısı Üzerinde Cıdar Kayma Gerilmesinin Etkisi. Doktora Tezi. KTU İnşaat fakültesi. 1998. 188 sahife.(In Turkish)